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Robust and Nonlinear Control of PWM DC-to-DC Boost Power Converters

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Abstract: The problem of controlling DC-to-DC switched power converter of Boost type is considered. The aim is to regulate the output voltage of the converter and to ensure an adequate robustness against uncertainties of the converter parameters. The control law is designed from the large-signal bilinear model of the converter using the backstepping technique. The obtained regulator is shown to meet its objectives namely a tight output voltage regulation, a fast transient response, and robustness with respect to component uncertainties.

INTRODUCTION

There are three main types of switched power converters, namely Boost, Buck and Buck-Boost. These have recently aroused an increasing deal of interest both in power electronics and in automatic control. This is due to their wide applicability domain that ranges from domestic equipments to sophisticated communication systems. They are also used in computers, industrial electronics, battery-operating portable equipments and uninterruptible power sources. From an automatic control viewpoint, a switched power converter represents an interesting case study as it is a variable-structure nonlinear system. Its rapid structure variation is coped with by using averaged models (Middlebrook 1976) [1], (Middlebrook 1989) [2], (Sira-Ramirez 1997) [3]. Based on such models, different nonlinear control techniques have been developed. These include passivity techniques (Sira-Ramirez 1997) [3], feedback linearization and, more generally, flatness methods (Fliess 1998) [4], sliding mode technique (El Fadil 2006) [5], backstepping technique (Sira – Ramirez 1996) [6], (Alvarez- Ramírez 2001) [7], (El Fadil 2003) [8], (El Fadil 2007) [9]. In this paper, the problem of controlling switched power converters is approached using the backstepping technique (Krstic 1995) [10]. While feedback linearization methods require precise models and often cancel some useful nonlinearity, backstepping designs offer a choice of design tools for accommodation of uncertain nonlinearities and can avoid wasteful cancellations. In this paper, the backstepping approach is applied to a specific class of switched power converters, namely DC-to-DC Boost converters. The full power of backstepping is exhibited in the presence of uncertain nonlinearities and unknown parameters.

The problem of controlling the output voltage of DC-DC Boost power converter is first dealt with based on a model that accounts for the converter parameters uncertainties. More precisely, the load resistance and the input voltage vary extensively. Furthermore, the inductance and capacitor can also vary with the temperature and current, and the variations are not precisely known. A robust regulator design is then performed, using the backstepping technique, to achieve closed-loop stability and output voltage reference tracking and robustness to parameter uncertainty. It is formally shown that the regulator thus obtained actually meets the performances for which it has been designed.

The paper is organized as follows: in Section 2, the Boost converter is described and modeled; Sections 3 is devoted to the controller theoretical and practical design; the controller stability and tracking performances are illustrated in Section 4. A conclusion and a reference list end the paper.

BOOST CONVERTER PRESENTATION AND MODELING

Boost converter is a circuit that is constituted of power electronics components connected as shown in figure 1. The circuit operating mode is the so-called Pulse Width Modulation (PWM). According to this principle, time is shared in intervals of length T (also called switching period). Within any period, the T-switch is on during a period fraction, say μT, for some $0 \leq \mu \leq 1$. Then, the current in the boost inductor L increases linearly and the diode D is off at that time. During the rest of the sampling period, i.e. $(1-\mu)T$, the switch T is tuned off, consequently the energy stored in the inductor is released through the diode to the output RC circuit. It is worth noting that the value of μ varies from a switching period to another. The variation law of μ determines the value of output voltage v_s.

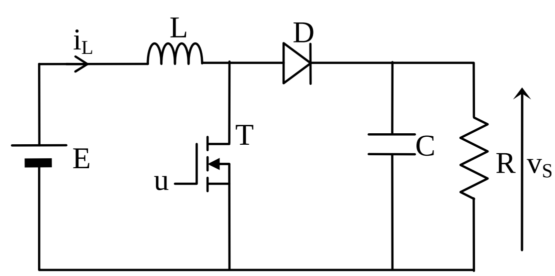

Fig. (1). Boost converter circuit.

The averaged model of such a converter is shown to be the following (see e.g. (Middlebrook 1976) [1]:

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$$\dot{x}_1 = -(1-\mu)\frac{x_2}{L} + \frac{E}{L}$$
$$\dot{x}_2 = (1-\mu)\frac{x_1}{C} - \frac{x_2}{RC} \qquad (1)$$

Where x_1 and x_2 denote the average input current (i_L) and the average output capacitor voltage (v_s), respectively. The control input for the above model is the function μ, called duty ratio function.

The model (1) is useful to build-up an accurate simulator for the converter (this possibility will be resorted to in Section 3). However, it cannot be based upon to design a continuous control laws as it involves uncertain parameters. Indeed, the inductance L and the capacitor C are not precisely known. The load charge R and the voltage source E are in turn subject to uncertainty. More precisely, it is assumed that:

$$E = E_0(1+\Delta_E(t)) \qquad (2a)$$

$$\frac{1}{L} = \frac{1}{L_0}(1+\Delta_L(t)) \qquad (2b)$$

$$\frac{1}{C} = \frac{1}{C_0}(1+\Delta_C(t)) \qquad (2c)$$

$$\frac{1}{R} = \frac{1}{R_0}(1+\Delta_R(t)) \qquad (2d)$$

where (E_0, L_0, C_0, R_0) denote the (known) nominal values and ($\Delta_E(t)$, $\Delta_L(t)$, $\Delta_C(t)$, $\Delta_R(t)$) are unknown uncertainties satisfying:

$$\Delta_E^{\min} \le \Delta_E(t) \le \Delta_E^{\max} \qquad (3a)$$

$$\Delta_L^{\min} \le \Delta_L(t) \le \Delta_L^{\max} \qquad (3b)$$

$$\Delta_C^{\min} \le \Delta_C(t) \le \Delta_C^{\max} \qquad (3c)$$

$$\Delta_R^{\min} \le \Delta_R(t) \le \Delta_R^{\max} \qquad (3d)$$

where the bounds $\Delta^{\min}$ and $\Delta^{\max}$ are supposed to be known.

Substituting (2) in (1) and gathering appropriately the different categories of terms, one gets a new version of the model:

$$\dot{x}_1 = -(1-\mu)\frac{x_2}{L_0} + \frac{E_0}{L_0} + \varphi_1^T\Delta$$
$$\dot{x}_2 = (1-\mu)\frac{x_1}{C_0} - \frac{x_2}{R_0C_0} + \varphi_2^T\Delta \qquad (4)$$

with

$$\varphi_1^T = [-x_2 \quad 1 \quad 0 \quad 0] \qquad (5a)$$

$$\varphi_2^T = [0 \quad 0 \quad x_1 \quad -x_2] \qquad (5b)$$

$$\Delta^T = [\Delta_1 \quad \Delta_2 \quad \Delta_3 \quad \Delta_4] \qquad (5c)$$

where (φ_1, φ_2) are known vector functions. The uncertain functions ($\Delta_1, \Delta_2, \Delta_3, \Delta_4$) depend on the uncertainties (Δ_E, Δ_L, Δ_C, Δ_R), the parameters nominal values (E_0, L_0, C_0, R_0) and the duty ratio as follows

$$\Delta_1 = (1-\mu)\frac{\Delta_L}{L_0} \qquad (6a)$$

$$\Delta_2 = \frac{E_0}{L_0}(\Delta_E + \Delta_L + \Delta_E\Delta_L) \qquad (6b)$$

$$\Delta_3 = (1-\mu)\frac{\Delta_C}{C_0} \qquad (6c)$$

$$\Delta_4 = \frac{1}{R_0C_0}(\Delta_R + \Delta_C + \Delta_R\Delta_C) \qquad (6d)$$

As the uncertainties (Δ_E, Δ_L, Δ_C, Δ_R), are bounded and, furthermore, the duty ratio μ is constrained to $[0,1]$, it follows that all the uncertainties Δ_k, k = 1,…,4 in equations (6) are also bounded.

Now, the new model (4) is more convenient for control design as the nominal and uncertain parts are clearly distinguished. It will be based upon in the next section to get a regulator for boost power converters.

ROBUST CONTROL DESIGN AND STABILITY ANALYSIS

Control Objective

The control objective consists in generating a control action μ in order to drive the output capacitor x_2 to any desired value $V_d > E$. It has already pointed out (see e.g. (Sira-Ramirez 1997) [3]) that, for the boost converter, a direct regulation of the output voltage is not achievable, due to the nonminimum phase feature of this circuit. Therefore, the above control problem is deal with following an indirect design strategy. Accordingly, the desired control purpose (i.e. output voltage regulation) will be achieved through the regulation of the input current. To this end, the desired output voltage V_d is assumed to be constant. Let $(U_d, \mathrm{I_d}, \mathrm{V_d})$ denotes the corresponding equilibrium point. This means that:

$$\mu = U_d \;\Rightarrow\; i_L = \mathrm{I_d},\; \mathrm{v_s} = \mathrm{V_d} \qquad (7)$$

From the average PWM model (1), one readily gets the relations:

$$I_d = V_d^2/(R_0E_0),\; U_d = 1 - E/V_d \qquad (8)$$

That is, regulating the (average) output voltage x_2 towards a desired value V_d amounts to regulating the (average) input current x_1 toward the corresponding equilibrium value I_d.

ROBUST REGULATOR DESIGN

Recall that the control objectives are: (i) tight regulation of the voltage v_0, (ii) fast transient response, (iii) robustness with respect to uncertainties of parameters. To this end, a robust nonlinear regulator will be designed using the backstepping approach, (Krstic 1995) [10].

The first objective is to enforce the inductor current x_1 to track a given reference signal I_d despite the system parameter uncertainties. Following closely the backstepping technique, the controller is designed in two steps.

Step 1. Let us introduce the following tracking error:

$$z_1 = x_1 - I_d \qquad (9)$$

Achieving the tracking objective amounts to enforcing the error z_1 to vanish. To this end, the dynamics of z_1 have to be clearly defined. Deriving (9), it follows from (4) that:

$$\dot{z}_1 = \dot{x}_1 = -(1-\mu)\frac{x_2}{L_0} + \frac{E_0}{L_0} + \varphi_1^T\Delta \tag{10}$$

In the above equation, the quantity x_2 / L_0 stands as a virtual control variable. Let us consider the following Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 \tag{11}$$

Its time-derivative along the trajectory of (10) is:

$$\dot{V}_1 = z_1\dot{z}_1 = z_1\left(-(1-\mu)\frac{x_2}{L_0} + \frac{E_0}{L_0} + \varphi_1^T\Delta\right) \tag{12}$$

Then, z_1 can be regulated to zero if $\frac{x_2}{L_0} = \alpha_1$ where α_1 is a stabilizing function defined by:

$$\alpha_1 = \frac{1}{1-\mu}\left(c_1 z_1 + \frac{E_0}{L_0} + k_1|\varphi_1|^2 z_1\right) \tag{13}$$

where $|\varphi_1|$ denotes the Euclidean norm of φ_1, $c_1 > 0$ is a design parameter. Note that the quantity $k_1|\varphi_1|^2 z_1$ ($k_1 > 0$) represents a nonlinear damping term introduced to dominate the uncertain term $\varphi_1^T\Delta$ in (10). As $\frac{x_2}{L_0}$ is not the actual control input, one can only seek the convergence of the error $\frac{x_2}{L_0} - \alpha_1$ to zero. We, then, define the following second error variable:

$$z_2 = \frac{x_2}{L_0} - \alpha_1 \tag{14}$$

The next step is to determine a variation law for the control signal μ so that the set of errors z_1 and z_2 vanishes asymptotically. But, let us first establish some useful equations. Equation (10) becomes, using (14) and (13):

$$\dot{z}_1 = -c_1 z_1 - (1-\mu)z_2 + \left[-k_1 z_1|\varphi_1|^2 + \varphi_1^T\Delta\right] \tag{15}$$

Also, the derivative (12) of the Lyapunov function is rewritten:

$$\dot{V}_1 = -c_1 z_1^2 - (1-\mu)z_1 z_2 + \left[-k_1 z_1^2|\varphi_1|^2 + z_1\varphi_1^T\Delta\right] \tag{16}$$

Step 2. The objective now is to enforce the error variables (z_1, z_2) to vanish. To this end, let us first determine the dynamics of z_2. Deriving (14) and using (13), (5a), (4) and (15), one obtains:

$$\dot{z}_2 = -\frac{\dot{\mu}}{(1-\mu)}\alpha_1 + \Psi + \varphi_3^T\Delta \tag{17}$$

where

$$\Psi = \left(\frac{1}{L_0} - \frac{1}{(1-\mu)}2k_1 z_1 x_2\right)\left((1-\mu)\frac{x_1}{C_0} - \frac{x_2}{R_0 C_0}\right) + \frac{1}{(1-\mu)}(c_1 + k_1|\varphi_1|^2)^2 z_1 + (c_1 + k_1|\varphi_1|^2)z_2 \tag{18}$$

$$\varphi_3^T = \left(\frac{1}{L_0} - \frac{1}{(1-\mu)}2k_1 z_1 x_2\right)\varphi_2^T - \frac{1}{(1-\mu)}(c_1 + k_1|\varphi_1|^2)\varphi_1^T \tag{19}$$

An appropriate control law for the inputs μ has now to be found for the system (15) and (17) whose state vector is (z_1, z_2). Consider the Lyapunov function V:

$$V = V_1 + \frac{1}{2}z_2^2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{20}$$

Its time derivative along trajectory of (16) and (17) is:

$$\begin{aligned}\dot{V} &= \dot{V}_1 + z_2\dot{z}_2 \\ &= -c_1 z_1^2 + z_2\left(\dot{z}_2 - (1-\mu)z_1\right) + \left[-k_1 z_1^2|\varphi_1|^2 + z_1\varphi_1^T\Delta\right]\end{aligned} \tag{21}$$

This shows that, for the (z_1, z_2)-system to be globally asymptotically stable, it is sufficient to choose the control μ so that

$$\dot{z}_2 = -c_2 z_2 + (1-\mu)z_1 \tag{22}$$

where $c_2 > 0$ is a design parameter. Combining (22) and (17) yields the following control law dynamics:

$$\dot{\mu} = \frac{1-\mu}{\alpha_1}\left\{\Psi - (1-\mu)z_1 + c_2 z_2 + k_2|\varphi_3|^2 z_2\right\} \tag{23}$$

where $k_2|\varphi_3|^2 z_2$ ($k_2 > 0$) represent nonlinear damping terms introduced to dominate the uncertain terms $\varphi_3^T\Delta$ in (17). Using (23), equation (17) becomes:

$$\dot{z}_2 = -c_2 z_2 + (1-\mu)z_1 + \left[-k_2 z_2|\varphi_3|^2 + \varphi_3^T\Delta\right] \tag{24}$$

STABILITY ANALYSIS

The stability of closed-loop system consisting of the controlled system (4) and the regulators (23) will now be analyzed. Using (21) and (24), one gets the following derivative of the Lyapunov function V:

$$\dot{V} = -c_1 z_1^2 - c_2 z_2^2 + V_\Delta \tag{25}$$

where

$$V_\Delta = \left[-k_1 z_1^2|\varphi_1|^2 + z_1\varphi_1^T\Delta\right] + \left[-k_2 z_2^2|\varphi_3|^2 + z_2\varphi_3^T\Delta\right] \tag{26}$$

We can easily show that the above expression can be overvalued as follows

$$V_\Delta \le \left[-k_1 z_1^2 |\varphi_1|^2 + |z_1| \|\varphi_1\| \|\Delta\|_\infty\right] + \left[-k_2 z_2^2 |\varphi_3|^2 + |z_2| \|\varphi_3\| \|\Delta\|_\infty\right]$$
$$\le -k_1\left(|z_1| \|\varphi_1| - \frac{\|\Delta\|_\infty}{2k_1}\right)^2 + \frac{\|\Delta\|_\infty^2}{4k_1} + \left[-k_2\left(|z_2| \|\varphi_3| - \frac{\|\Delta\|_\infty}{2k_2}\right)^2 + \frac{\|\Delta\|_\infty^2}{4k_2}\right]$$

Finally we have

$$V_\Delta \le \frac{\|\Delta\|_\infty^2}{4k_1} + \frac{\|\Delta\|_\infty^2}{4k_2} \tag{27}$$

Consider the following notations:

$$\beta = 2\min(c_1, c_2)\,,\ \kappa = \min(k_1, k_2) \tag{28}$$

Then (25) can be rewritten, using (27) and (28), as follows:

$$\dot{V} \le -\beta V + \frac{1}{2\kappa}\|\Delta\|_\infty^2 \tag{29}$$

The later equation shows that:

$$V(t) \le e^{-\beta(t-t_0)} V(t_0) + \frac{1}{2\beta\kappa}\|\Delta\|_\infty^2 \tag{30}$$

which yields that V(t) is bounded and $\lim_{t\to\infty} V(t) \le \frac{1}{2\beta\kappa}\|\Delta\|_\infty^2$. This immediately implies that the error vector $z(t) = [z_1, z_2]^T$ is in turn globally uniformly bounded. Furthermore, it follows from (9) and (14) that the components of the state vector $x(t) = [x_1, x_2]^T$ are smooth functions of the components of $z(t)$. Hence, $x(t)$ is in turn globally uniformly bounded and, furthermore, it converges to the compact residual set:

$$\Omega = \left\{ x \in IR^2 \,/\, z_1^2 + z_2^2 \le \frac{1}{\beta\kappa}\|\Delta\|_\infty^2 \right\} \tag{31}$$

Since the bound $\|\Delta\|_\infty$ is finite, the size of the above set may be made arbitrarily small by increasing the values of the design parameters k_i and c_i (i= 1, 2). The results thus established are summarized in the following theorem:

Theorem 1. Consider the closed-loop system consisting of a boost power converter represented by (1) where the parameters are not precisely known but are bounded, subject to model uncertainties described by (2), and the regulator defined by the control laws (23). Then, one has:

i) All the closed-loop signals remain bounded,

ii) The tracking error $\varepsilon = x_2 - V_d$ is bounded and can be arbitrarily reduced by choosing the design parameters k_i and c_i sufficiently large. This propriety ensures tight regulation under uncertainties.

iii) If $\Delta(t) = 0$ the tracking error vanishes asymptotically

SIMULATION RESULTS

The performances of the proposed robust control design are illustrated through simulations. The parameters nominal values of the boost converter are illustrated in Table **1**.

Table 1. Parameters of the Boost Power Converters

Parameter	Symbol	Value
Inductance value	L	$20mH$
Capacitor value	C	$68\mu F$
Load resistor	R	30Ω
Input voltage	E	$15V$
Switching frequency	f_s	$25kHz$

Table 2. Design Parameters of the Regulator

Deign Parameter	Value
c_1	100
c_2	100
k_1	10-3
k_2	10-5

The experimental bench is described by Fig. **2** and is simulated using the MATLAB software. The design parameters are summarized in Table **2**. The behavior of the closed loop system is illustrated by figures **3** to **7**.

Fig. **3** shows the closed-loop behavior in ideal conditions (invariant parameters). The desired output voltage is $V_d = 40V$. It can be seen from the figure that the output signal tracks perfectly its reference.

Fig. **4** illustrates the closed-loop behavior in presence of a change in the input source. The input voltage is perturbed by a stochastic noise source of significant voltage amplitude, approximately 20% of the nominal value $E_0 = 15V$. Note that the changes come on in the simulation model only. In the regulator, the parameter E is kept constant (equal to its nominal value $E_0 = 15V$). It is seen from the figures that the regulator is robust against these variations.

Fig. **5**, Fig. **6** and Fig. **7** illustrate the controller robustness with respect to the uncertainties of inductance, capacitor and load resistance, respectively. Also, in the regulator, the parameters are kept constant and equal to their nominal values. As can be seen from the figures, the regulator actually compensates these variations and stabilizes the output voltage.

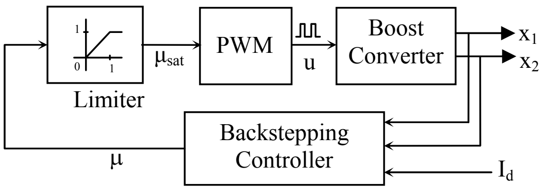

Fig. (2). Experimental bench for Boost Converter control.

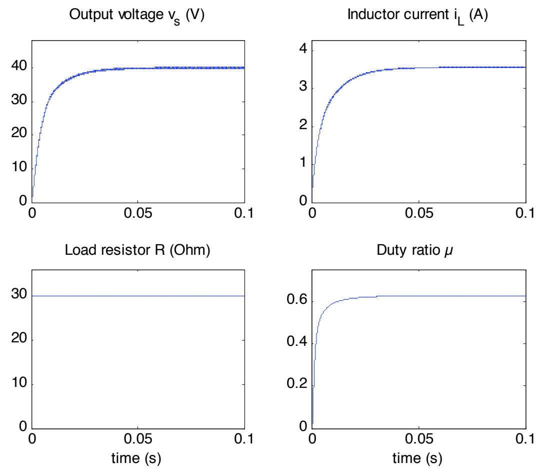

Fig. (3). Controller behavior in presence of step reference V_d = 40V and invariant parameters

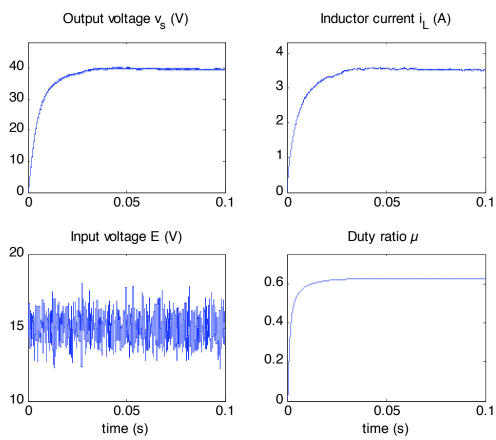

Fig. (4). Controller behavior for input voltage change.

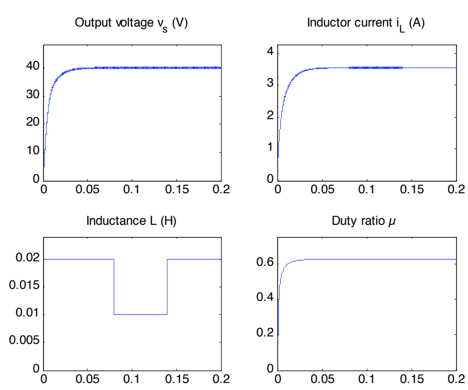

Fig. (5). Controller behavior for inductance change.

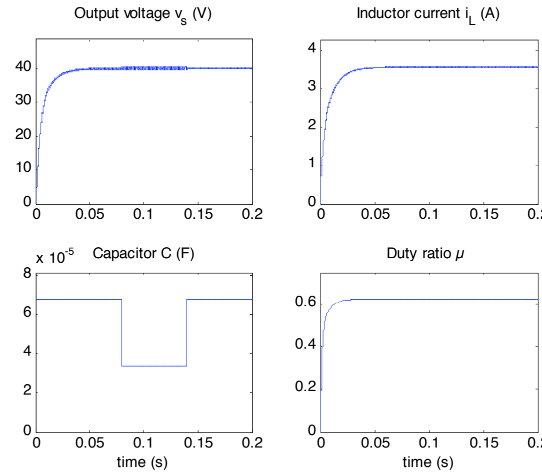

Fig. (6). Controller behavior for capacitor change.

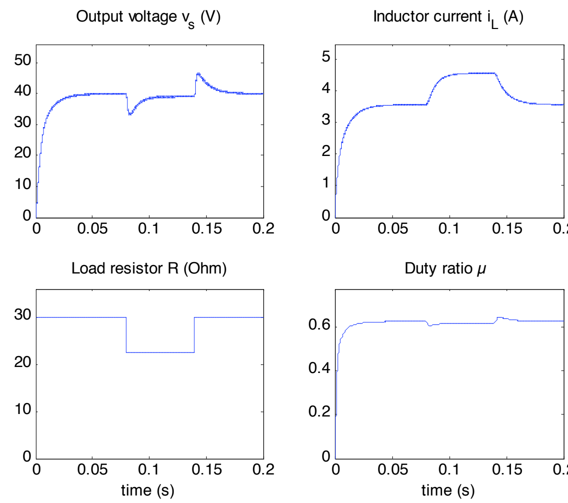

Fig. (7). Controller behavior for load resistance change.

CONCLUSIONS

The problem of controlling boost power converters has been considered. The regulator is obtained from the average large-signal bilinear model of the converter using a robust version of the backstepping approach. It is established, using a formal analysis and a simulation study, that the obtained regulator meets its objectives namely a tight output voltage regulation, a fast transient response, and robustness with respect to component uncertainties.

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