

Research on Ranging Optimization and Twice-Weighted Centroid Calculation in Wireless Sensor Network Localization Algorithm

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Abstract: In practical applications of ranging technology commonly used in wireless sensor positioning algorithm, ranging between nodes will inevitably produce errors due to environmental factors and measurement hardware effects. The wireless sensor network localization algorithm DGC-TWCL proposed in this paper takes advantage of the Cayley-Menger determinant used to provide distance geometry constraints(DGC) in two-dimensional real space to optimize and correct the ranging error, combined with the twice-weighted centroid localization(TWCL) calculation, the final position of the unknown node can be obtained. The experiment results show that the algorithm can get the positioning results with higher accuracy and better robustness.

Keywords: Localization, wireless sensor network, inaccurate ranging, Cayley-Menger determinant, twice weighted centroid localization computation.

1. INTRODUCTION

As an international hot research field attracting a great deal of attention nowadays, wireless sensor network (WSN) is a new pattern of information acquisition and processing. The networked computing devices with low price and low power provide a novel design mode to the connectivity between the physical world and human society in ubiquitous era. The node localization is one of the most important issues after many WSN system layouts have been complete. The monitoring data for many applications with unknown location have no meaning. In usual case, WSN positioning algorithm obtains the unknown node position estimation through measurement or connection between the unknown node and anchor nodes with priori position information. According to the current status, WSN positioning technology can be divided into ranging-based and non-ranging-based in view of whether the metrics between two nodes is necessary during localization process. The former positioning mode has relatively higher accuracy and cost, while the latter based on the connectivity of network has less accuracy and hardware requirement. In accordance with hops between the unknown node and anchor nodes in connection, the localization algorithm is divided into single-hop and multi-hops. From the viewpoint of processing manner to the measurement data among nodes, WSN positioning can be divided into the centralized and distributed method. The centralized algorithm uses the central processor to get all ranging information resulting in the entire network location mapping and the distributed algorithm achieves self-positioning by node acquisition of local information to the neighbor nodes. The typical

location algorithms and systems include centroid method [1], least squares estimation method [2], the MIT Cricket [3], the convex programming location algorithm of the University of California, Berkeley [4], DV-Hop [5] and DV-Distance [6] distributed localization algorithm of the Rutgers University.

The common ranging technology of WSN localization algorithm includes RSS, AoA [7], ToA and TDoA[8]. In practical application environments, ranging has the errors inevitably because of the impact of multi-path, reflection, diffraction, scattering and other environment or measurement hardware factors. Since the distance between nodes is independent from each other, the ranging error may undermine geometric topology relationship e.g. the trilateration measurements make the length of both sides less than the third one, which causes failure of the localization algorithm. This paper proposes a wireless sensor network localization algorithm DGC-TWCL which utilizes Cayley-Menger determinant in two-dimension real space to provide distance geometry constraints(DGC), makes optimization and correction to the ranging errors and obtains the final position of the unknown node combined with the twice-weighted centroid localization(TWCL) calculation.

2. CAYLEY-MENGER DETERMINANT AND OPTIMIZATION OF DISTANCE GEOMETRY CONSTRAINTS

2.1. Cayley-Menger Determinant

In the theory of distance geometry, Cayley-Menger determinant can be used to handle the geometry problem of Euler distance in invariant space [9]. Two n -points sequences $\{p_1, p_2, \dots, p_n\}$ and $\{q_1, q_2, \dots, q_n\}$ in m -dimension real space form the Cayley-Menger matrix which is defined as:

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$$M(p_1, \dots, p_n; q_1, \dots, q_n) \triangleq \begin{bmatrix} d^2(p_1, q_1) & d^2(p_1, q_2) & \dots & d^2(p_1, q_n) & 1 \\ d^2(p_2, q_1) & d^2(p_2, q_2) & \dots & d^2(p_2, q_n) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ d^2(p_n, q_1) & d^2(p_n, q_2) & \dots & d^2(p_n, q_n) & 1 \\ 1 & 1 & \dots & 1 & 0 \end{bmatrix}, \quad (1)$$

where $d(p_i, q_j), i, j \in \{1, \dots, n\}$ is the Euclidean distance between the point p_i and q_j . The bi-determinant of Cayley-Menger composed by the two n-point sequences is defined as:

$$D(p_1, \dots, p_n; q_1, \dots, q_n) \triangleq \det M(p_1, \dots, p_n; q_1, \dots, q_n). \quad (2)$$

When the two sequences of points are the same, $M(p_1, \dots, p_n; p_1, \dots, p_n)$ and $D(p_1, \dots, p_n; p_1, \dots, p_n)$ can be defined simply as $M(p_1, \dots, p_n)$ and $D(p_1, \dots, p_n)$. $D(p_1, \dots, p_n)$ is called Cayley-Menger determinant which has the following classic conclusion:

(Theorem 1[10]) considering n-tuple constituted by p_1, \dots, p_n in m-dimension space, the rank of Cayley-Menger matrix $M(p_1, \dots, p_n)$ is at most $m + 1$.

According to the above theorem, it's easy to know that in two-dimension real space, $D(p_0, p_1, p_2, p_r) = 0$. p_0 can be regarded as the unknown node during the positioning process and it makes the distance measurements with the three known anchor nodes p_1, p_2, p_r , which is shown in (Fig. 1).

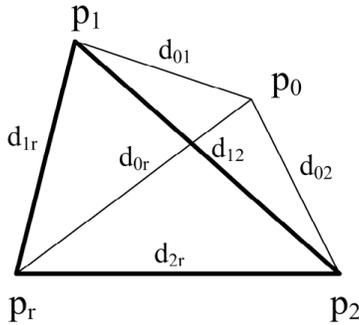


Fig. (1). Four-tuple geometric relationship in two-dimension space.

The value of Cayley-Menger determinant comprised by d_{01}, d_{02}, d_{0r} and the distances d_{12}, d_{1r}, d_{2r} among the anchor nodes is 0, that is:

$$D(p_0, p_1, p_2, p_r) = \begin{vmatrix} 0 & d_{01}^2 & d_{02}^2 & d_{0r}^2 & 1 \\ d_{01}^2 & 0 & d_{12}^2 & d_{1r}^2 & 1 \\ d_{02}^2 & d_{12}^2 & 0 & d_{2r}^2 & 1 \\ d_{0r}^2 & d_{1r}^2 & d_{2r}^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} = 0. \quad (3)$$

Due to noise influence of the environment and equipment, error ε inevitably occurs during the actual measurement. It's assumed that the accurate distance d_{0i} and measured distance \tilde{d}_{0i} between the unknown node and the anchor node have the following relationship:

$$\tilde{d}_{0i}^2 = d_{0i}^2 + \varepsilon_i, i = 1, 2, r. \quad (4)$$

When Eq.(4) is taken into Eq.(3), the following relationship can be obtained:

$$\varepsilon^T A \varepsilon + \varepsilon^T b + c = 0, \quad (5)$$

where $\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_r]^T$, the matrix A , the vector b and c are described by the distances d_{12}, d_{1r}, d_{2r} among the known anchor nodes and measured distances $\tilde{d}_{01}, \tilde{d}_{02}, \tilde{d}_{0r}$.

2.2. Ranging Optimization of Distance Geometry Constraints

If the unknown node p_0 receives the noise distance measurement information from anchor nodes p_1, \dots, p_r , in accordance with the collections of nodes $\{p_0, p_1, p_2, p_3\}, \{p_0, p_1, p_2, p_4\}, \dots, \{p_0, p_1, p_2, p_r\}$, $r-2$ independent quadratic distance constraint equations can be obtained combined with Eq.(3), Eq.(4) and Eq.(5). Each equation can be expressed as $f_i(\varepsilon_1, \varepsilon_2, \varepsilon_i) = 0, i = 3, 4, \dots, r$, where $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_r$ are the errors occurred among the unknown node and anchor nodes during the measurement process. By getting the minimized sum of squared errors under the distance constraints limits, the optimization problem can be expressed as:

$$\begin{aligned} \min J &= \varepsilon_1^2 + \varepsilon_2^2 + \dots + \varepsilon_r^2 \\ \text{s.t.} &\begin{cases} f_1(\varepsilon_1, \varepsilon_2, \varepsilon_3) = 0 \\ f_2(\varepsilon_1, \varepsilon_2, \varepsilon_4) = 0 \\ \vdots \\ f_{r-2}(\varepsilon_1, \varepsilon_2, \varepsilon_r) = 0 \end{cases} \end{aligned} \quad (6)$$

Using numerical analysis methods, the optimization solution $\varepsilon_1^*, \varepsilon_2^*, \dots, \varepsilon_r^*$ can be get. Combined with the Eq.(4), the position estimated between the unknown node and anchor node can be obtained: $\hat{d}_{0i} = \sqrt{\tilde{d}_{0i}^2 - \varepsilon_i^*}, i = 1, 2, \dots, r$.

3. TWICE-WEIGHTED CENTROID LOCALIZATION COMPUTATION

Through distance information with the three known anchor nodes, the definite position coordinates of the unknown node in two-dimension space theoretically can be obtained by trilateration measurement method, i.e., three anchor nodes as the centers, the relevant distance information as radiuses of the circles that intersect at one point, as shown in (Fig. 2).

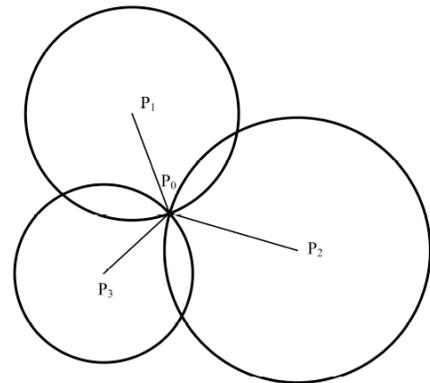


Fig. (2). Ideal model of trilateration measurement.

However, in the actual application environments, the random variables caused by the presence of noise during the distance measurement process make the unknown node located in a region (see Fig. 3), the centroid of which can be viewed as the unknown node position.

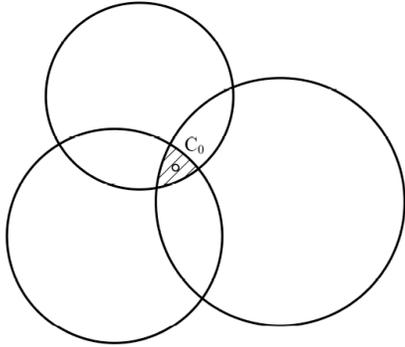


Fig. (3). The centroid of trilateration measurement in the actual application environments.

The weighted centroid localization idea is to introduce the distance between the unknown node and anchor node as a weighted factor, and the distance is inversely proportional to the factor, which reflects influence degree of the anchor node to the position of the centroid. When the unknown node receives the $k(k \geq 3)$ anchor node signals, they can constitute up to C_k^3 triangles and approximate C_k^3 coordinates of the unknown node can be calculated by using the weighted centroid calculation. The final position of the unknown node can be obtained through the second weighted centroid computation.

3.1. The Weighted Centroid Computation in Trilateration Measurement Method

It's assumed that the unknown node p_0 carries out trilateration measurement by receiving the information from the anchor nodes p_1, p_2, p_i , as shown in (Fig. 4).

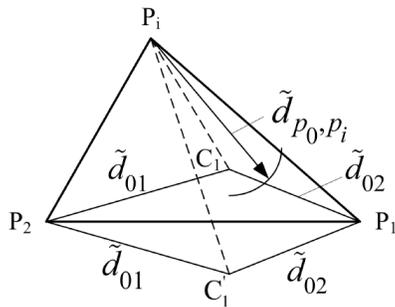


Fig. (4). The Weighted Centroid Computation in Trilateration Measurement.

According to the distance \tilde{d}_{01} (measured between p_0 and p_1) and the distance \tilde{d}_{02} (measured between p_0 and p_2), combined with the coordinates of p_1 and p_2 , two positions coordinates (c_1 and c'_1) of p_0 can be obtained. After the

distance d_{c_1, p_i} between c_1 and p_i and the distance $d_{c'_1, p_i}$ between c'_1 and p_i are calculated respectively, compared with the ranging between p_0 and p_i , the close degree for d_{c_1, p_i} and $d_{c'_1, p_i}$ to \tilde{d}_{0i} is determined, i.e., $c_1(x_1, y_1)$ corresponding to the smaller value of $|1 - \tilde{d}_{p_0, p_i} / d_{c_1, p_i}| < |1 - \tilde{d}_{p_0, p_i} / d_{c'_1, p_i}|$ can be taken as the approximate position of the unknown node. Similarly, by ranging from p_1 and p_i to p_0 as well as proximity judgment principle, the second estimated location c_2 of p_0 can be obtained; and by ranging from p_2 and p_i to p_0 as well as proximity judgment principle, the third estimated location c_3 of p_0 can be obtained.

Taking advantage of the three estimated positions $c_1(\hat{x}_1, \hat{y}_1), c_2(\hat{x}_2, \hat{y}_2), c_3(\hat{x}_3, \hat{y}_3)$ of p_0 , the weighted factors reflecting the influence degree of anchor nodes are introduced:

$$w_1 = \frac{1 / (\tilde{d}_{01} + \tilde{d}_{02})}{1 / (\tilde{d}_{01} + \tilde{d}_{02}) + 1 / (\tilde{d}_{01} + \tilde{d}_{0i}) + 1 / (\tilde{d}_{02} + \tilde{d}_{0i})} \tag{7}$$

$$w_2 = \frac{1 / (\tilde{d}_{01} + \tilde{d}_{0i})}{1 / (\tilde{d}_{01} + \tilde{d}_{02}) + 1 / (\tilde{d}_{01} + \tilde{d}_{0i}) + 1 / (\tilde{d}_{02} + \tilde{d}_{0i})} \tag{8}$$

$$w_3 = \frac{1 / (\tilde{d}_{02} + \tilde{d}_{0i})}{1 / (\tilde{d}_{01} + \tilde{d}_{02}) + 1 / (\tilde{d}_{01} + \tilde{d}_{0i}) + 1 / (\tilde{d}_{02} + \tilde{d}_{0i})} \tag{9}$$

Using the weighted centroid calculation through the anchor nodes p_1, p_2, p_i , the final estimated position of the unknown node p_0 can be obtained: $\hat{x}_i = w_1 \hat{x}_1 + w_2 \hat{x}_2 + w_3 \hat{x}_3$, $\hat{y}_i = w_1 \hat{y}_1 + w_2 \hat{y}_2 + w_3 \hat{y}_3$. When solving the weighted factors in DGC-TWCL algorithm, the estimated value \hat{d}_{0i} is adopted after optimization and correction.

3.2. Twice-weighted Centroid Computation

As described in Section 2.2, when the unknown node p_0 has received ranging information from p_1, \dots, p_r , the distances of the estimated position $\hat{d}_{01}, \hat{d}_{02}, \dots, \hat{d}_{0r}$ between the unknown node and anchor nodes can be obtained by distance geometry constraints optimization of Cayley-Menger determinant. Consider the following set $\{p_1, p_2, p_3\}, \{p_1, p_2, p_4\}, \dots, \{p_1, p_2, p_r\}$ in which each element consists of three anchor nodes, the weighted centroids collection $\{O_1(\hat{x}_1, \hat{y}_1), O_2(\hat{x}_2, \hat{y}_2), \dots, O_{r-2}(\hat{x}_{r-2}, \hat{y}_{r-2})\}$ can be calculated respectively through the trilateration measurement method and the final unknown node position can be obtained by using the second weighted centroid computation as below:

$$\hat{x} = \frac{\sum_{k=3}^r [\hat{x}_{k-2} / (\hat{d}_{01} + \hat{d}_{02} + \hat{d}_{0k})]}{\sum_{k=3}^r [1 / (\hat{d}_{01} + \hat{d}_{02} + \hat{d}_{0k})]} \tag{10}$$

$$\hat{y} = \frac{\sum_{k=3}^r [\hat{y}_{k-2} / (\hat{d}_{01} + \hat{d}_{02} + \hat{d}_{0k})]}{\sum_{k=3}^r [1 / (\hat{d}_{01} + \hat{d}_{02} + \hat{d}_{0k})]} \quad (11)$$

4. DGC-TWCL ALGORITHM DESCRIPTION AND EXPERIMENT ANALYSIS

4.1. Algorithm Description

The DGC-TWCL algorithm includes two parts, one is solving the ranging optimization value provided by distance geometry constraints(DGC) of Cayley-Menger determinant, and the other is twice weighted centroid (TWC) computation.

The DGC portion is described as follows:

Step 1: After the unknown node p_0 has received ranging information from the known anchor nodes p_1, \dots, p_r , the sequence $\tilde{d}_{01} < \tilde{d}_{02} < \dots < \tilde{d}_{0r}$ with descending order is set. Let the ranging errors be $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_r$;

Step 2: Taking p_0 and $\{p_1, p_2, p_i\}, i=3, \dots, r$ into the Cayley-Menger determinant $D(p_0, p_1, p_2, p_i) = 0$, the function $f_i(\varepsilon_1, \varepsilon_2, \varepsilon_i) = 0$ is determined whose variables are $\varepsilon_1, \varepsilon_2, \varepsilon_i$;

Step 3: Taking the minimum value of $J = \varepsilon_1^2 + \varepsilon_2^2 + \dots + \varepsilon_r^2$ as objective function, $r-2$ equations from Step 2 as constraints conditions, nonlinear optimization solutions $\varepsilon_1^*, \varepsilon_2^*, \dots, \varepsilon_r^*$ are obtained;

Step 4: According to $\hat{d}_{0i} = \sqrt{\tilde{d}_{0i}^2 - \varepsilon_i^*}, i=1, 2, \dots, r$, the estimated position of the unknown node to the anchor node can be obtained;

The TWC part is described as follows:

Step 5: by using $\{p_1, p_2, p_i\}, i=3, \dots, r$ to constitute a collection of $r-2$ triangles, in accordance with the method in Section 3, three position coordinates can be calculated by trilateration measurement and proximity principle in each triangle, and then the weighted centroid position coordinates can be obtained by making use of the factors w_1, w_2, w_i inversely proportional to the estimated distance value \hat{d}_{0i} within each triangle;

Step 6: By adopting the method in Section 3.2 and introducing the second centroid weighted factors, the final estimated position coordinate $p_0(\hat{x}, \hat{y})$ of the unknown node p_0 can be obtained.

4.2. Experiment Analysis

Against the DGC-TWCL positioning method proposed in this paper, the positioning error, positioning error sample variance, the scalability and robustness of the algorithm have been evaluated. Experiment simulation scene has been set in a rectangular area of 8m * 8m. Consider positioning of the unknown node p_i located at the grid point under the configuration cases of three, four and five anchor nodes respectively. Suppose the ranging error is within $\pm 10\%$ random value of the true distance. When three anchor nodes

$\{b_1, b_2, b_3\}$ are placed in the points of (4, 8), (0, 0), (8, 0) (see Fig. 5), the testing nodes set is $\{p_1, p_2, \dots, p_{22}\}$. The four anchor nodes configuration is shown in (Fig. 6) and the testing nodes set is $\{p_1, p_2, p_6, p_7, p_8, p_{10}, p_{11}, p_{13}, p_{14}, p_{16}, p_{17}, p_{19}, p_{20}\}$. The five anchor nodes configuration is shown in (Fig. 7), and the test nodes set is $\{p_1, p_2, p_4, p_6, p_7, p_{10}, p_{12}\}$.

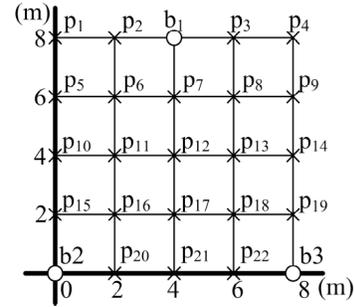


Fig. (5). Experiment simulation scene with three anchor nodes in DGC-TWC.

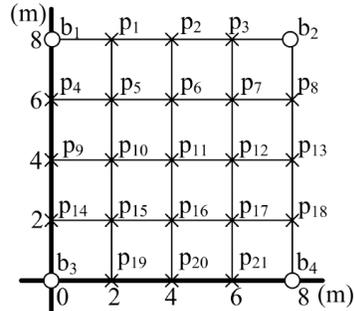


Fig. (6). Experiment simulation scene with four anchor nodes in DGC-TWC.

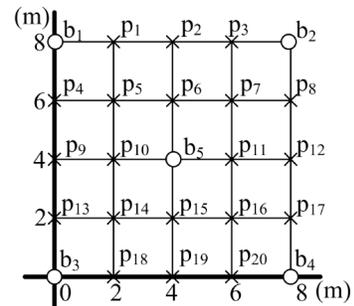


Fig. (7). Experiment simulation scene with five anchor nodes in DGC-TWC.

Besides, the positioning states of the linear least squares(LLS) estimation, mean centroid and weighted centroid location are calculated under the same ranging condition. The calculated indicators include the biggest error(BE), the smallest error(SE), the mean error(ME) and the error sample variance(SV), where $e_i = \sqrt{(\hat{x}_i - x)^2 + (\hat{y}_i - y)^2}, i=1, 2, \dots, n$ is one point error in the testing nodes set, and $BE = \max\{e_i\}$, $SE = \min\{e_i\}$, $ME = \left(\sum_{i=1}^n e_i\right) / n$, $SV = \frac{1}{n-1} \sum_{i=1}^n (e_i - \bar{e})^2$. \bar{e} is the averaged

measurement error of the testing nodes set. Ranging error optimization process takes advantage of *fsolve* in Matlab optimization toolbox. The experiment results with three, four and five anchor nodes are shown in (Table 1, Table 2 and Table 3) respectively. The corresponding graphic plotting according to the data from (Table 1 to Table 3) is shown in (Fig. 8).

As it can be seen from Table 1 to Table 3 and Fig. (8), under the three anchor nodes configuration, the smaller smallest error(SE) can be obtained in DGC-TWCL with respect to the other positioning methods. On the aspect of the mean error (ME), except to obtain a value slightly larger than the mean centroid under five anchor nodes configuration, the other values are lower than the other three positioning algorithm. Compared with the indicators of the biggest error(BE) and sample variance(SV), the values in DGC-TWCL are medium, which shows that distance fluctuation degree between the positioning result and true position of the algorithm is moderate. In addition, under the three anchor node

configuration program, the performance of the above four statistical indicators is stable in DGC-TWCL, which indicates the scalability and robustness of the algorithm. On the other hand, DGC-TWCL algorithm increases the calculation amount and time because of the ranging calibration and the twice weighted factors computation process, which is more suitable for the positioning occasions of centralized single-hop and medium anchor nodes density. Table 4 shows the comparison of the several positioning algorithms in computation overhead, positioning accuracy, stability and scalability. Wherein r is the number of anchor nodes, T_{inv} is the time-consuming for calculating the generalized inverse matrix in LLS algorithm, T_{cl} is the time overhead for seeking the centroid of one trilateration measurement, T_w is the time consumption for computing the weighted factors and T_g is the time spent on the measurement correction. By increasing a certain amount of calculation seen from Table 4, the positioning accuracy and stability in DGC-TWCL can be improved.

Table 1. Indicators Statistics of Experiment Results Under 3 Anchor Nodes

Localization Algorithm	3 Anchor Nodes			
	BE	SE	ME	SV
LLS	2.2139	1.2086	1.5443	0.0622
Mean Centroid	3.5577	0.4706	1.5313	0.9767
Weighted Centroid	3.4981	0.5147	1.5081	0.9900
DGC-TWCL	3.6956	0.3355	1.4203	0.7920

Table 2. Indicators Statistics of Experiment Results Under 4 Anchor Nodes

Localization Algorithm	4 Anchor Nodes			
	BE	SE	ME	SV
LLS	0.0563	0.0028	0.0288	0.2211×10^{-3}
Mean Centroid	0.1214	0.0026	0.0341	0.9341×10^{-3}
Weighted Centroid	0.0508	0.0025	0.0334	0.9511×10^{-3}
DGC-TWCL	0.0560	0.0018	0.0240	0.3469×10^{-3}

Table 3. Indicators Statistics of Experiment Results Under 5 Anchor Nodes

Localization Algorithm	4 Anchor Nodes			
	BE	SE	ME	SV
LLS	0.0563	0.0028	0.0288	0.8936
Mean Centroid	0.1214	0.0026	0.0341	0.1124
Weighted Centroid	0.0508	0.0025	0.0334	0.0531
DGC-TWCL	0.0560	0.0018	0.0240	0.2397

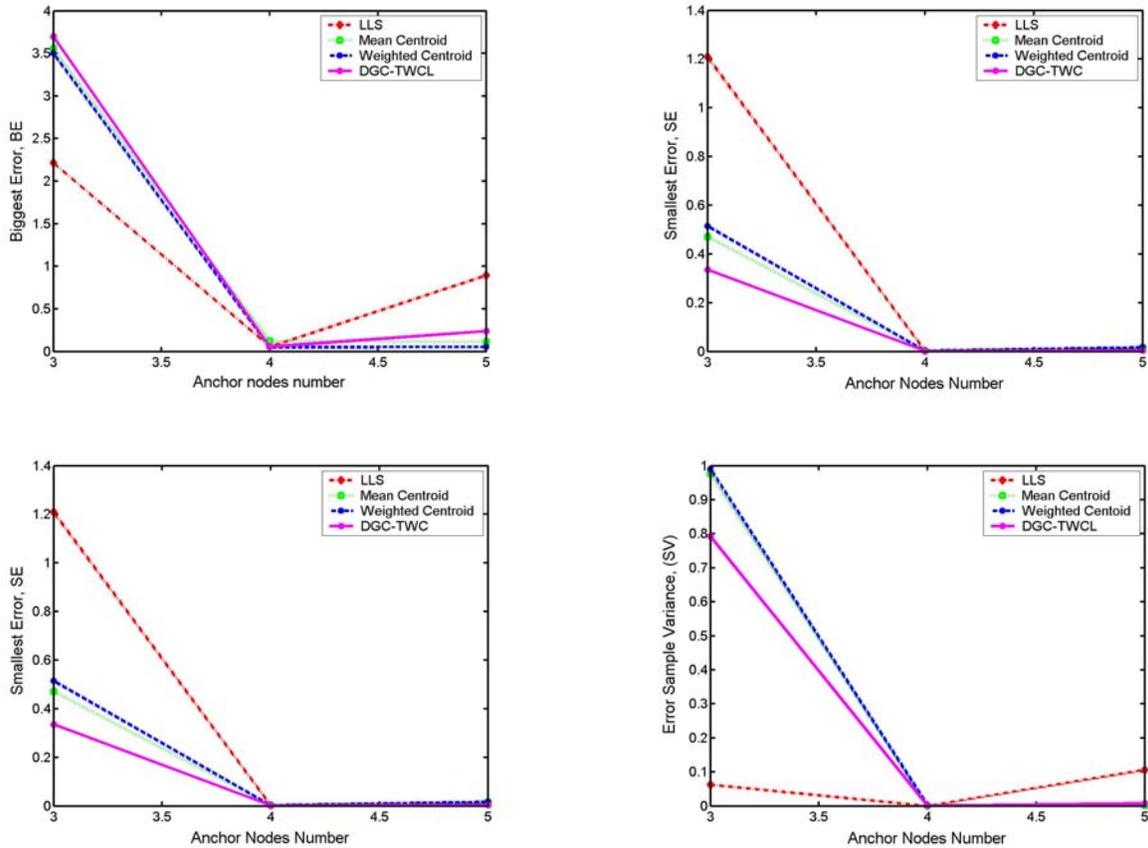


Fig. (8). The corresponding graphic plotting to experiment indicators statistics.

Table 4. Performance Comparison of Several Positioning Algorithm

Computation Overhead	LLS	Mean Centroid	Weighted Centroid	DGC-TWCL
	T_{inv}	$(r-2)*T_{cl}$	$(r-2)*T_{cl}+T_w$	$(r-2)*T_{cl}+T_w+T_g$
Positioning Accuracy	Lower	Lower	Lower	Better
Positioning Stability	Lower	Lower	Lower	Better
Positioning Scalability	Better	Better	Better	Better

5. CONCLUSIONS

For node localization in wireless sensor network, because of its low-cost feature, it is impossible for each node to have the ability to directly access to the location information such as GPS global positioning system. Most nodes localization firstly makes distance measurements with anchor nodes, and due to the presence of measurement noise and estimation error, the ranging value is non-precise. In this paper, the DGC-TWCL algorithm proposed uses geometrical relationship among the nodes in wireless sensor network formed by Cayley-Menger determinant, takes the minimum of ranging squared error sum as the optimization target and corrects the non-exact distance measurement values. Combined with the twice weighted centroid calculation, the high accuracy and robustness positioning results can be achieved.

CONFLICT OF INTEREST

The author(s) confirm that this article content has no conflicts of interest.

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