The Open Automation and Control Systems Journal, 2014, *6*, 124-128

Open Access

Nonlinear Control Chua's Chaotic System

Weiping Guo and Diantong Liu*

School of Computer and Control Engineering, Yantai University, Yantai City, People's Republic of China

Abstract: The Chua's chaotic system is modeled as a nonlinear feedback cascade system and a nonlinear controller is proposed with the nonlinear recursive algorithm. The design process for the proposed controller is given in detail and the system stability is proved with the Lyapunov stability theory. Simulation results show that the Chua's chaotic system in any state can be asymptotically stabilized to the origin and the validity of the proposed control algorithm.

Keywords: Chaos control, chua's chaotic system, nonlinear control, recursive.

1. INTRODUCTION

In the last several decades, much effort has been devoted to the study of nonlinear chaotic systems. Sometimes chaos is viewed as an undesirable characteristic since the system response can be completely random. In practice, chaos in electronic, mechanical and structural systems is not desirable because it may lead to irregular operations and fatigue failure. Moreover, chaos can restrict the operating range of many mechanical, structural, optical and electronic systems. If chaos cannot be controlled, it might result in disaster and collapse for a dynamical system. Therefore, recent interests are now focused on controlling a chaotic system, i.e. bringing the chaotic state to an equilibrium point or a small limit cycle.

After the pioneering work on controlling chaos introduced by Ott *et al.* [1], domestic and foreign researchers have proposed a variety of control algorithms, such as linear and nonlinear [2], the feedback control [3], adaptive control [4], sliding mode control [5]. As a typical nonlinear system, a variety of control algorithms based on recursive algorithm have been proposed [6-11]. Two uncertain chaotic systems synchronization is implied with the adaptive backstepping design [6]. A new fuzzy adaptive control algorithm based on backstepping is proposed for the chaotic systems whose model is unknown or contain uncertain parameters and external interference [10].

Chua's circuit has been studied extensively as a prototypical electronic system [12-16]. A linear feedback controller is designed to guide the chaotic trajectory of the circuit to a limit cycle [12]. For the modified Chua's circuit, a feedback controller is proposed to drag the chaotic trajectory to fixed points or a limit cycle [13] and a Flatness-based linear output feedback controller [14] is proposed to track an output trajectory. In the past decade, adaptive control of chaos systems has undergone rapid developments [15, 16].

As a general tool, Lyapunov stability theory has also been used for the chaos systems to design the controllers. All are based on rigorous Lyapunov stability theorem and Lyapunov function methods [17]. But the construction of the Lyapunov functions remains to be a difficult task.

The aim of this paper is to introduce a simple, systematic design method for resolving the control problems of the chaos in Chua's circuit system. It is assumed that one state variable is available for implementing the feedback controller. Firstly, The Chua's chaotic system is modeled as a nonlinear feedback cascade system. Then a nonlinear recursive backstepping design method is proposed and the system stability is proved with Lyapunov stability theory. The design procedure treats the state variables as the virtual control inputs to design the virtual controllers step by step, so as to obtain the nonlinear controller to drag the system dynamics to the origin from any chaotic state. Finally, some simulations are performed.

2. THE CHUA'S CHAOTIC DYNAMICS AND THE NONLINEAR CONTROLLER DESIGN

The Chua's chaotic system [18] is modeled as

$$\begin{cases} \dot{x}_1 = p(x_2 - f(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -qx_2 \end{cases} \tag{1}$$

where,

$$f(x_1) = \begin{cases} bx_1 + a - b & x_1 > E \\ ax_1 & |x_1| \le E \\ bx_1 - a + b & x_1 < -E \end{cases}$$

In order to make the chaos control system be a normal nonlinear feedback cascade system, the following control input is inserted:

*Address correspondence to this author at the School of Computer and Control Engineering, Yantai University, Yantai City, People's Republic of China; Tel: +865356902601; Fax: +865356902601;
E-mails: diantong.liu@163.com, diantong_liu@ytu.edu.cn

$$\begin{cases} \dot{x}_1 = p(x_2 - f(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -qx_2 - u \end{cases} \tag{2}$$

In this way, the Chua's chaotic control system is a nonlinear strict feedback cascade system, so the recursive design can be utilized to obtain its nonlinear controller. The objective is to design a control law so that the chaotic dynamics of the circuit are stabilized at the origin. The design process is

Step 1

Firstly, the dynamic of state x_1 in the Chua's chaotic system (2) is considered

$$\dot{x}_1 = p(x_2 - f(x_1)) \tag{3}$$

The state x_2 can work as the virtual control input of (3) and its reference trajectory x_{2r} is defined as

$$x_{2r} = -p(x_2 - f(x_1)) + x_2 - k_1 x_1 \tag{4}$$

So the error e_2 of x_2 can be computed as

$$e_2 = x_2 - x_{2r} = p(x_2 - f(x_1)) + k_1 x_1 \tag{5}$$

where, k_1 is a positive constant to be designed.

In order to use the Lyapunov stability theory, a scalar positive definite Lyapunov function is selected as

$$V_1 = \frac{1}{2}x_1^2 + \frac{1}{2}e_2^2 \geq 0 \tag{6}$$

Its time derivative $\dot{V}_1$ is

$$\begin{aligned} \dot{V}_1 &= \dot{x}_1 x_1 + \dot{e}_2 e_2 \\ &= x_1(e_2 - k_1 x_1) + \dot{e}_2 e_2 \\ &= e_2 x_1 - k_1 x_1^2 + \dot{e}_2 e_2 \\ &= e_2(x_1 + \dot{e}_2) - k_1 x_1^2 \end{aligned} \tag{7}$$

with

$$\begin{aligned} \dot{e}_2 &= p(\dot{x}_2 - \frac{\partial f}{\partial x_1}\dot{x}_1) + k_1 \dot{x}_1 \\ &= p(x_1 - x_2 + x_3 - \frac{\partial f}{\partial x_1}\dot{x}_1) + k_1 \dot{x}_1 \end{aligned} \tag{8}$$

Thus

$$\dot{V}_1 = e_2(x_1 + p(x_1 - x_2 + x_3 - \frac{\partial f}{\partial x_1}\dot{x}_1) + k_1 \dot{x}_1) - k_1 x_1^2 \tag{9}$$

It is noted that the system state variable x_3 arises in (9). In the next step, state x_3 works as the control input.

Step 2

In Step 1, the time derivative of the Lyapunov function V_1 is obtained. In order to make the $\dot{V}_1$ be negative definite, state x_3 works as the virtual control input to proceed the recursive process. Its reference trajectory x_{3r} is defined as

$$x_{3r} = -(x_1 + p(x_1 - x_2 + x_3 - \frac{\partial f}{\partial x_1}\dot{x}_1) + k_1 \dot{x}_1) + x_3 - k_2 e_2 \tag{10}$$

where, k_2 is a positive constant to be designed. The tracking error e_3 is

$$e_3 = x_3 - x_{3r} = (1+p)x_1 - px_2 + px_3 - p\frac{\partial f}{\partial x_1}\dot{x}_1 + k_1 \dot{x}_1 + k_2 e_2 \tag{11}$$

So that the time derivative $\dot{V}_1$ is

$$\dot{V}_1 = -k_1 x_1^2 + e_2(e_3 - k_2 e_2) = -k_1 x_1^2 - k_2 e_2^2 + e_2 e_3 \tag{12}$$

We reselect the scalar positive Lyapunov function V_1 as

$$V_2 = V_1 + \frac{1}{2}e_3^2 \tag{13}$$

Differentiating V_2

$$\dot{V}_2 = \dot{V}_1 + \dot{e}_3 e_3 = -k_1 x_1^2 - k_2 e_2^2 + e_3(e_2 + \dot{e}_3) \tag{14}$$

with

$$\begin{aligned} \dot{e}_3 = {} & (1+p)\dot{x}_1 - p\dot{x}_2 + p\dot{x}_3 - p\frac{\partial f}{\partial x_1}\ddot{x}_1 \\ & - p\frac{\partial^2 f}{\partial x_1^2}\dot{x}_1 + k_1 \ddot{x}_1 + k_2 \dot{e}_2 \end{aligned} \tag{15}$$

let

$$e_2 + \dot{e}_3 = -k_3 e_3 \tag{16}$$

i.e.

$$\begin{aligned} & e_2 + (1+p)\dot{x}_1 - p\dot{x}_2 + p\dot{x}_3 - p\frac{\partial f}{\partial x_1}\ddot{x}_1 - p\frac{\partial^2 f}{\partial x_1^2}\dot{x}_1 \\ & + k_1 \ddot{x}_1 + k_2 e_2 = -k_3 e_3 \end{aligned} \tag{17}$$

where, k_3 is a positive constant to be designed. From the system Eq. (2)

$$\begin{aligned} & e_2 + (1+p)\dot{x}_1 - p\dot{x}_2 + p(-qx_2 - u) - p\frac{\partial f}{\partial x_1}\ddot{x}_1 \\ & - p\frac{\partial^2 f}{\partial x_1^2}\dot{x}_1 + k_1 \ddot{x}_1 + k_2 e_2 = -k_3 e_3 \end{aligned} \tag{18}$$

The system control variable u arises in (18). Therefore, (18) can make the following equation hold

$$\dot{V}_2 = \dot{V}_1 + \dot{e}_3 e_3 = -k_1 x_1^2 - k_2 e_2^2 - k_3 e_3^2 \leq 0 \tag{19}$$

In this way, the nonlinear controller can be obtained from (18) as

$$\begin{aligned} u = {} & \frac{1}{p}[e_2 + (1+p)\dot{x}_1 - p\dot{x}_2 - pqx_2 - \\ & p\frac{\partial f}{\partial x_1}\ddot{x}_1 - p\frac{\partial^2 f}{\partial x_1^2}\dot{x}_1 + k_1 \ddot{x}_1 + k_2 e_2 + k_3 e_3] \end{aligned} \tag{20}$$

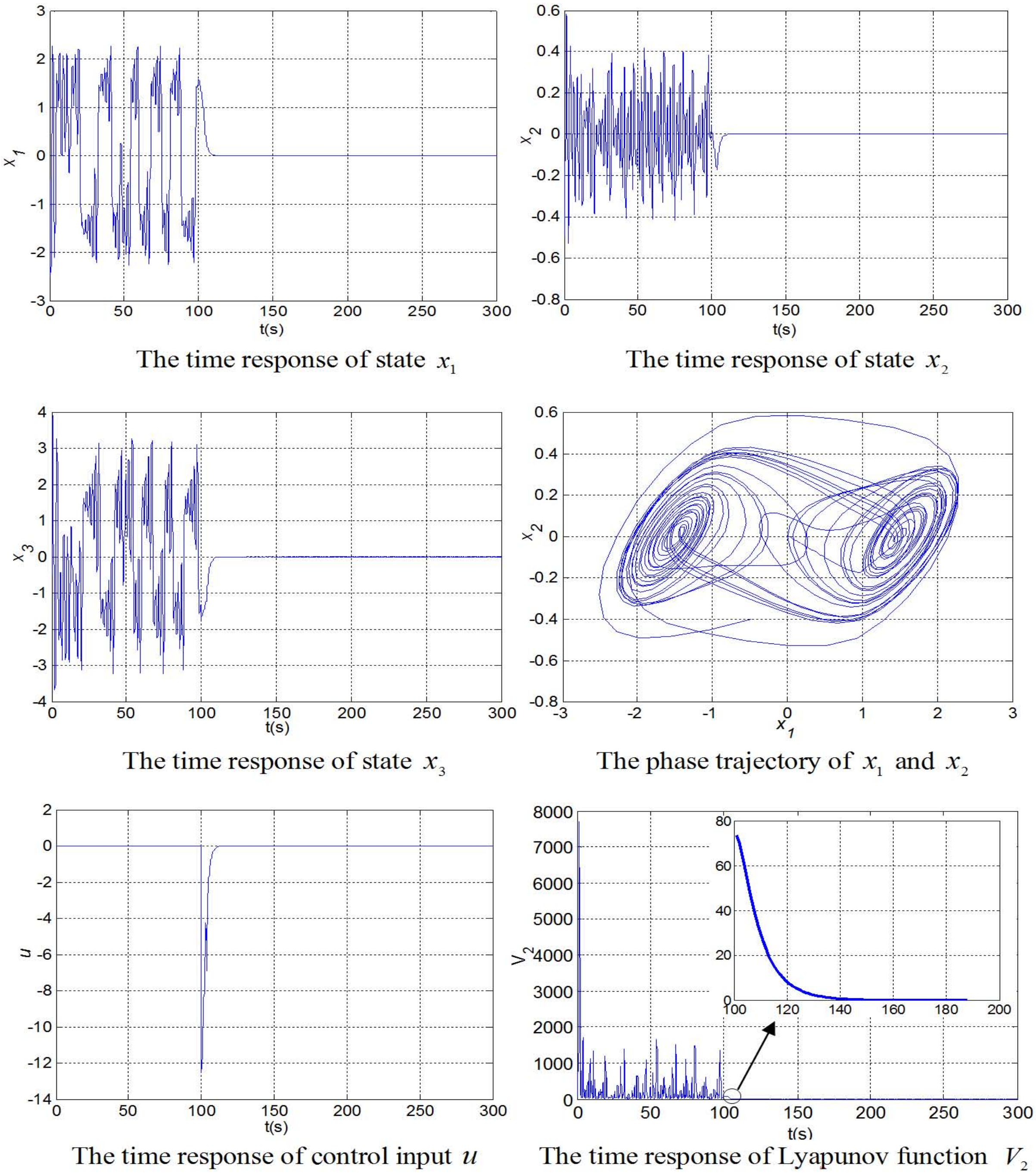

Fig. (1). The system dynamics under $x_0 = [-0.5, -0.4, -0.5]^T$.

Theorem 1: The Chua's chaotic system described as (2) is asymptotically stable under the proposed nonlinear controller, i.e. the system states (x_1, x_2, x_3) converge asymptotically to the origin (0,0,0).

Proof.

The recursive design process has proved: the time derivative of the chosen positive definite Lyapunov function

$$V_2 = \frac{1}{2}x_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \tag{21}$$

is negative definite. That is to say, the three terms of the right hand in V_2 approach asymptotically to (0,0,0), i.e. the vector (x_1, e_2, e_3) converges asymptotically to the origin (0,0,0). From Eqs. (5) and (2), $e_2 \to 0$ and $x_1 \to 0$ imply $x_2 \to 0$. From Eq. (11), $e_2 \to 0$, $e_3 \to 0$ $x_1 \to 0$ and $x_2 \to 0$ can deduce $x_3 \to 0$. That is to say, the system state vector (x_1, x_2, x_3) converges asymptotically to the origin (0,0,0).

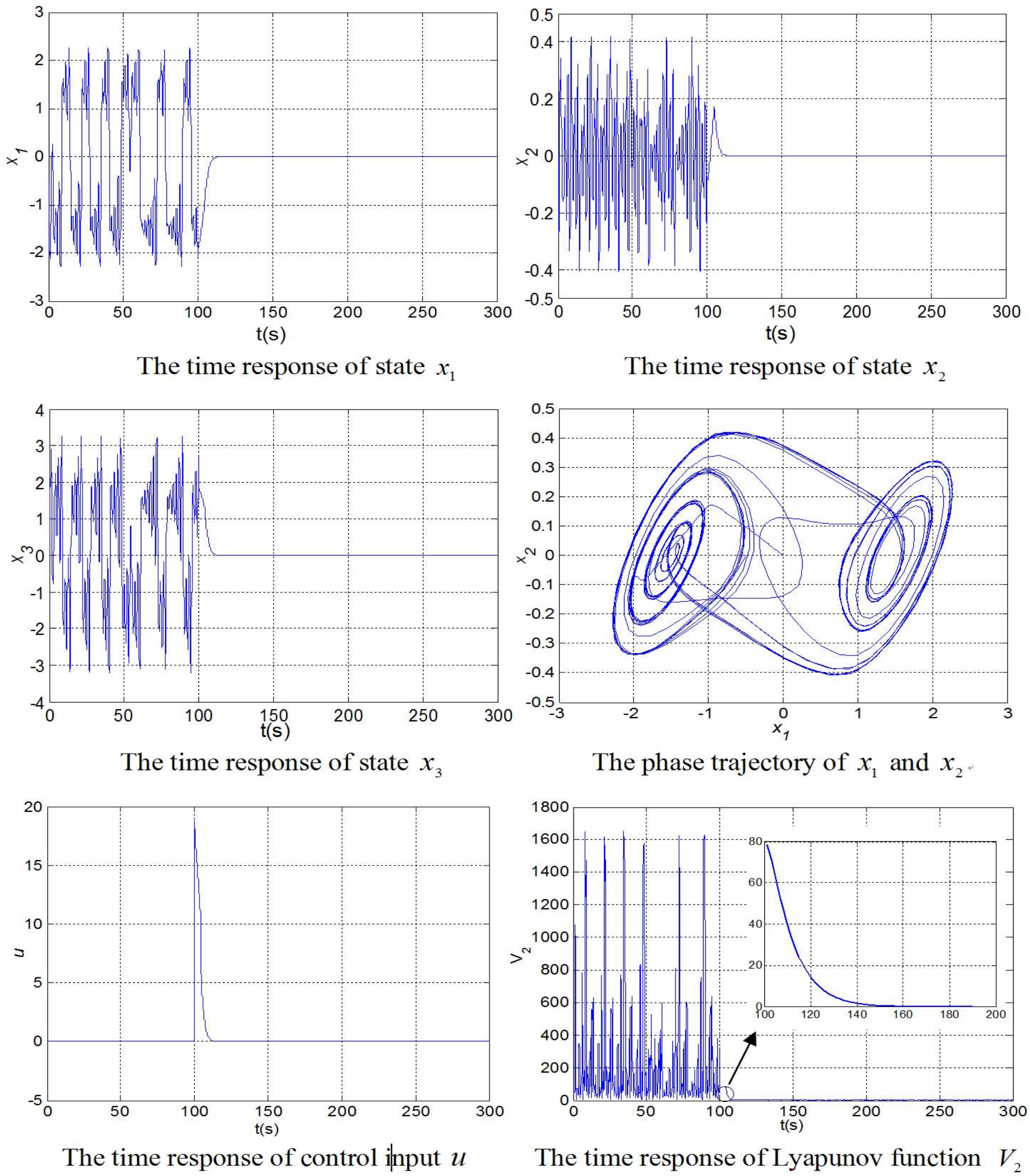

Fig. (2). The system dynamics under $x_0 = [-0.5, 0, 0]^T$.

Remark 1: It can be seen that the design procedure also is a procedure to find the suitable Lyapunov function. A major advantage of the proposed method is that it has the flexibility to choose the control law so that the goals of stabilization are achieved with reduced control effort.

Remark 2: If the control input can not be placed in the state x_3, it can be placed in the state x_2. At this time, the first step in the design procedure is same as the above and the second step should treat the states (x_2, x_3) as a second order system (the design method can refer to [19]).

3. SIMULATION STUDIES

In order to test the proposed nonlinear control algorithm, the following system parameters [18] and controller parameters are used: $p = -1/7$, $q = 2/7$, $a = 10$, $b = 15$, $k_1 = 4$, $k_2 = 4$, $k_3 = 4$.

In order to compare the system dynamics with and without the control, the control input is inserted at 100th second. The simulation results are shown in Fig. (**1**) when the system initial state is $x_0 = [-0.5, -0.4, -0.5]^T$ and in Fig. (**2**) when the system initial state is $x_0 = [-0.5, 0, 0]^T$. From the simulation results, it can be seen that the Chua's chaotic system exhibits the double-scroll attractor under the given initial states and the system can be asymptotically dragged the origin with the proposed control algorithm. The proposed control algorithm can stabilize the Chua's chaotic system to a fixed point under any initial states from the view of simulations and theory. The proposed method is a systematic design procedure and the system stability can be assured. It is shown from the simulation results that the designed Lyapunov function V_2 defined in (21) are asymptotically convergent to zero. Moreover, based on recursive application of Lyapunov's direct method, the backstepping enables to drive the chaotic motion towards any desired trajectory.

The control performance can be improved through adjusting the parameters of the proposed controller. Lots of simulation experiments show that the parameters k_1, k_2, k_3 respectively correspond to the system states x_1, x_2, x_3 and it is easy to adjust the parameters in the proposed controller for an improved system performance. The parameters in the controller can be obtained by some optimization algorithms, such as genetic algorithm.

CONCLUSION

A Lyapunov-based nonlinear controller is proposed for controlling the Chua's chaos. The Chua's chaotic system is modeled as a nonlinear feedback cascade system. The nonlinear controller is obtained with recursive design method and the design procedure is a systematic design procedure. Simulation results verify the effectiveness of the proposed control algorithm. The recursive approach as well as the procedures of control law design does not use specific features of chaos. Along with its advantages, the design procedure has certain drawbacks. One of them is that for high-order systems the nonlinear expression of the controller becomes increasingly complex. Moreover, if the system parameters can not been obtained, some adaptive technology or soft computing techniques can be used.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

This work is supported by a Project of Shandong Province Higher Educational Science and Technology Program (No. J13LN25) and the National Natural Science Foundation of China (No.61175086).

REFERENCES

[1] E. O. Grebogic, and J. A. Yorke, "Controlling chaos", *Physics Review Letter*, vol. 64, no. 11, pp. 1196-1199, 1990.

[2] K.S. Tank, K.F. Man and G.Q. Zhong, "Generating chaos via x|x|", *IEEE Trans. on Circuits and System*, vol. 48, no. 5, pp. 635-641, 2001.

[3] M.T. Yessen, "Controlling chaos and synchronization for new chaotic system using linear feedback control", *Chaos, Solution and Fractals*, vol. 26, no. 3, pp. 913-920, 2005.

[4] S.H. Chen, and J.H Liu, "Synchronization of an uncertain unified chaotic system via adaptive control", *Chaos, Solitons and Fractals*, vol. 14, no. 4, pp. 643-647, 2002.

[5] M. Feki, "Sliding mode control and synchronization of chaotic systems with parametric uncertainties", *Chaos, Solitons and Fractals*, vol. 41, no. 3, pp. 1390-1400, 2009.

[6] Y. Yu, and S. Zhang, "Adaptive backstepping synchronization of uncertain chaotic system", *Chaos, Solution and Fractals*, vol. 21, no. 3, pp. 643-649, 2004.

[7] J.H. Park, "Synchronization of genesio chaotic system via backstepping approach", *Chaos, Solution and Fractals*, vol. 27, no. 5, pp. 1369-1375, 2006.

[8] M.T. Yessen, "Controlling, synchronization and tracking chaotic Liu system using active backstepping design", *Physics Letters A*, vol. 360, no. 4-5, pp. 582-587, 2007.

[9] C. Wang, and S.S. Ge, "Adaptive synchronization of uncertain chaotic system via backstepping design", *Chaos, Solution and Fractals*, vol. 12, no. 7, pp. 1199-1206, 2001.

[10] X.P. Guan, C.L. Chen and Z.P. Fan, "Adaptive fuzzy control for a chaotic system based on the backstepping approach", *Acta Physica Sinica*, vol. 51, no. 4, pp. 753-758, 2002.

[11] G.G. LIU, "Synchronization of strict-feedback chaotic system via Backstepping control", *Computer Engineering and Applications*, vol.43, no. 23, pp. 200-202, 2002.

[12] G. Chen, and X. Dong, "From Chaos to order: perspectives and methodologies in controlling chaotic nonlinear dynamical system", *International Journal of Bifuration and Chaos*, vol. 3, no. 6, pp. 1363-1369, 1993.

[13] C. C. Hwang, J. Y. Hsieh, and R. S. Lin, "A linear continuous feedback control of chua's circuit", *Chaos, Solitons and Fractals*, vol. 8, no. 9, pp. 1507-1515,1997.

[14] H. Sira-Ramírez , A. Luviano-Juárez and J. Cortés-Romero, "Flatness-based linear output feedback control for disturbance rejection and tracking tasks on a Chua's circuit", *International Journal of Control*, vol. 85, no,5, pp.594-602, 2012.

[15] F. Moez, "An adaptive feedback control of linearizable chaotic systems", *Chaos, Solitons and Fractals*, vol. 15, no. 5, pp. 883-890, 2013.

[16] N.B. He, Q. Gao, C.L. Gong and Y. Feng. "Adaptive tracking control for a class of Chua's chaotic systems", *in Proceedings of Conference on Control and Decision*, pp. 4241-4243, 2009.

[17] S. S. Ge, C. Wang and T. H. Lee, "Adaptive backstepping control of a class of chaotic cystems"' *International Journal of Bifurcation and Chaos*, vol. 10, no. 5, pp.1149-1156, 2000.

[18] S. M. YU, "*Chaotic Systems and Chaotic Circuits*", Xi'an University of Electronic Science and Technology Press, 2011.

[19] L. Diantong, W. Guo, " Nonlinear backstepping design for the underactuated TORA system", *Journal of Vibroengineering*, vol. 16, no. 2, pp. 552-559, 2001.

Received: November 22, 2013 Revised: June 27, 2014 Accepted: July 08, 2014