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*The Open Automation and Control Systems Journal*, **2014**, *6*, 165-174 

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# $H_\infty$ Filtering for Discrete-Time Neural Networks System with Time-Varying Delay and Sensor Nonlinearities

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**Abstract:** The $H_\infty$ filtering problem for a class of discrete stochastic neural networks systems with time-varying delay and nonlinear sensor is investigated. By employing the Lyapunov stability theory and linear matrix inequality optimization approach, sufficient conditions to guarantee the filtering error systems asymptotically stable are provided. By setting on the lower and upper bounds of the discrete time-varying delays, an acceptable state-space realization of the $H_\infty$ and an acceptable $H_\infty$ performance index are obtained in terms of linear matrix inequality (LMI). Numerical examples and simulations are provided to illustrate the effectiveness of the proposed methods.

**Keywords:** Asymptotically stable, discrete stochastic neural networks systems, linear matrix inequality (LMI), Sensor Nonlinearities, time-varying delay.

## 1. INTRODUCTION

Filtering problem has long been an hot topic due to its extensive application in signal processing, communication and control systems, many important research results have been reported during the past two decades, see [1-6] and the references therein.

The aim of $H_\infty$ filtering is to design a stable estimator to ensure that the $L_2$ / $l_2$ -induced gain from the noise signals is less than a prescribed level. Because the $H_\infty$ method can minimize the $H_\infty$ norm of the transfer function between the noise and the estimation error, which has an advantage in dealing with external unknown noises, so the $H_\infty$ filtering technology has been applied in diverse systems such as networked systems [7, 8], fuzzy systems [9-13], Markovian delay systems [14], singular systems [15] and discrete-time systems [2, 3, 6, 15].

As time delay is a natural phenomenon frequently encountered in various dynamic systems such as electronic, chemical systems, long transmission lines in pneumatic systems, biological systems, economic and rolling mill systems, which is very often the main sources of instability, oscillation and poor performance. So far, the stability analysis and robust control for dynamic systems with different time-delays such as independent-delay, dependent-delay, distributed-delay and discrete delay have attracted a number of researchers over the past years, many important results have been reported, see [16-18] and the references therein.

In addition, since systems in the real world are always perturbed by stochastic noises, the research on $H_\infty$ filtering of stochastic systems with time-delay or without has made much progress and various significant results have been obtained [19-25]. At the same time, it should be pointed out that the discussion about the filter research above, most of the existing results on filtering require critical assumption on the linearity of sensors. Meanwhile, in the practical application, nonlinearity is present in almost all real sensors in one form or another, which often influences the performance of the filters, so many researchers devote to the filtering problem for systems in the presence of sensor nonlinearities [26-30].

By using the Lyapunov stability theory and linear matrix inequality method, the robust $H_\infty$ filtering problem for a class of nonlinear discrete stochastic time-delay systems are concerned with [24-26]. By virtue of some matrix inequality technique, the filtering problems for stochastic time-varying delay systems with sensor nonlinearities are considered in [27-30], in which the sensor nonlinearities are assumed to be bounded by Lipschitz or sector conditions, the designed filtering and the acceptable $H_\infty$ performance level are obtained in terms of linear matrix inequality (LMI). What is more, due to the nonlinearity character of saturation, more and more scholars begin to study the senor saturation of $H_\infty$ filtering, see [31, 32] and the references therein. Recently, by decomposing the nonlinear function into a linear and a nonlinear part, the asynchronous $l_2$ / $l_\infty$ filtering for discrete-time stochastic Markov jump systems with sensor nonlinearities is studied in [33], in which the sensor nonlinearities are assumed to occur obeying Bernoulli distribution, the designed full-order filter is obtained by linear matrix inequality.

On the other hand, during the past several decades, various kinds of neural networks have received much attention due to their successful applications in signal processing, pattern classification, solving optimization problems and model identification. A lot of research results about the stability analysis, synchronization , state estimation and $H_\infty$ filter design problems for continuous or discrete neural networks with time delay have been reported [34-40].

However, so far, the $H_\infty$ filtering problem for nonlinear discrete time -delay stochastic neural networks systems with nonlinear sensor has not been fully investigated, especially for discrete-time cases, which motivates us to shorten such a gap in the present investigation. The major difficulties and challenges are how to consider the effect of stochastic noise and the nonlinearity of sensors. In our study, we assume the stochastic noise obeys to brown motion, namely, stochastic noise is chosen from a normal disturbance with mean zero and variance one. When dealing with the nonlinear sensors, by some technique, the nonlinear sensors are treated as linear sensors, so it is easy to get feed back gain filter.

Motivated by the above discussion, this paper considers the $H_\infty$ filtering problem for a class of discrete time-delay stochastic neural networks systems involving sensor nonlinearities. We aim to design a mode-dependent linear filter such that the filtering error system is not only stochastically asymptotically stable, but also satisfies a prescribed $H_\infty$ norm level. A new, simple linear matrix inequality (LMI) approach is exploited and the solvability of the desired filter is implied by the feasibility of LMI. Finally, two numerical examples are provided to show the the usefulness and effectiveness of the proposed filter design method.

Notation: Throughout this paper, if not explicit, matrices are assumed to have compatible dimensions. The notation $M>(\geq,<,\leq)0$ means that the symmetric matrix $M$ is positive-definite (positive-semidefinite, negative, negative-semidefinite). $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix; The superscript ``T'' stands for the transpose of a matrix; the shorthand diag$\{\cdots\}$ denotes the block diagonal matrix; $\|\cdot\|$ represents the Euclidean norm for vector or the spectral norm of matrices. $I$ refers to an identity matrix of appropriate dimensions. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation, $*$ means the symmetric terms. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

## 2. SYSTEM DESCRIPTION

Consider the following neural networks system with time-varying interval delay and nonlinear sensor as follows:

$$\begin{cases} x(k+1)=Ax(k)+W_0f(x(k))+W_1f(x(k-\tau(k)))+D_1v(k) \\ \qquad\qquad +[E(x(k)+E_dx(k-\tau(k))+D_2v(k)]\omega(k), \\ y(k)=G(S\,x(k))+D_3v(k), \\ z(k)=Hx\,(k), \end{cases} \tag{1}$$

where $x(k)=[x_1(k),x_2(k),...,x_n(k)]^T\in\mathbb{R}^n$ is the state vector of neural network, $y(k)\in\mathbb{R}^q$ is the measurable output vector, $z(k)\in\mathbb{R}^r$ is the state combination to be estimated, The exogenous disturbance signal $v(k)\in\mathbb{R}^p$ is assumed to belong to $L_{e2}([0,\infty),\mathbb{R}^p)$, $f(x(k))=[f_1(x_1(k)),$ $f_2(x_2(k)),...,f_n(x_n(k))]^T\in\mathbb{R}^n$ denotes the neuron activation function. $G(Sx(k))$ are nonlinear sensor functions, $S$ is positive matrix, $\tau(k)$ represents the transmission time-varying delay satisfying $0<\tau_m\leq\tau(k)\leq\tau_M$, and $\tau_m$, $\tau_M$ are known positive integers denoting the minimum and maximum delays, respectively. $\omega(k)$ is Brown Motion defined on the complete probability space $(\Omega,\mathcal{F},F_t\}_t)$, which is assumed to satisfy

$$\mathbb{E}\{\omega(k)\}=0,\mathbb{E}\{\omega^2(k)\}=1,\mathbb{E}\{\omega(i)\{\omega(j)\}=0\,(i\neq j). \tag{2}$$

$A,W_0,W_1,D_1,E,E_d,D_2,S,D_3$ and $H$ are known real constant matrices.

Before proceeding further, we will state the following assumptions and well known lemma.

**Lemma 1** [35] For given proper dimensions constant matrix $\Phi_1,\Phi_2$ and $\Phi_3$, where $\Phi_1^T=\Phi_1$ and $\Phi_2^T=\Phi_2>0$, we have $\Phi_1+\Phi_3^T\Phi_2^{-1}\Phi_3<0$ such that only and only if

$$\begin{bmatrix}\Phi_1 & \Phi_3^T \\ * & -\Phi_2 \\ & \end{bmatrix}<0, or \begin{bmatrix}-\Phi_2 & \Phi_3 \\ * & \Phi_1 \\ & \end{bmatrix}<0,$$

**Assumption 1.** For $i\in\{1,2,...,n\}$, $\forall x,y\in\mathbb{R},x\neq y$, the neuron activation function $f_i(\cdot)$ is continuous, bounded and satisfies :

$$l_i^-\leq\frac{f_i(x)-f_i(y)}{x-y}\leq l_i^+, \tag{3}$$

where $l_i^-$ and $l_i^+$ are some contansts.

**Remark 1.** Assumption 1 was first introduced in Liu *et al.* (2006,2007). The constants $l_i^-,l_i^+$ in Assumption 1 are allowed to be positive, negative or zero. Hence, the resulting activation functions may be non-monotonic, more general than the usual sigmoid functions and Lipschitz-type conditions. Such a description is very precise/tight in quantifying the lower and upper bounds of the activation functions, hence very it is helpful for using an LMI-based approach to reduce the possible conservatism.

**Assumption 2.** The nonlinear sensor functions $G_i(\xi_i)$ are monotonically nondecreasing, bounded and globally Lipsichiz. That is to say there exist a pair of positive scalars $u_i$ and $\lambda_i$ such that

$$0 \le \frac{G_i(\alpha)-G_i(\beta)}{\alpha-\beta} \le u_i, \forall \alpha,\beta \in \mathbb{R}, i=1,2,...,p, \\ -\lambda_i \le G_i \le \lambda_i, i=1,2,...,p, \tag{4}$$

where $u_i$ is the magnification of the sensor, and $\lambda_i$ is the amplitude of the sensor.

In this paper, we consider the following discrete-time neural networks filter for the estimation of $z(k)$:

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + W_0 f(\hat{x}(k)) + W_1 f(\hat{x}(k-\tau(k))) \\ +[E\hat{x}(k) + E_d \hat{x}(k-\tau(k))]\omega(k) + K[y(k) - G(S\hat{x}(k))], \\ \hat{z}(k) = H\hat{x}\ (k), \end{cases} \tag{5}$$

where $\hat{x}(k) \in \mathbb{R}^n$, $\hat{z}(k) \in \mathbb{R}^r$ represent the estimates of $x(k)$ and $z(k)$, respectively, and $H$ is a constant matrix.

By defining $e(k) = x(k) - \hat{x}(k)$ and utilizing the model (1) to include the states of the filter (5), we can get the following filtering error systems:

$$\begin{cases} e(k+1) = Ae(k) + W_0\phi(k) + W_1\phi(k-\tau(k)) + D_1 v(k) \\ +[Ee(k) + E_d e(k-\tau(k)) + D_2 v(k)]\omega(k) - K\psi(Se(k)) - \\ KD_3 v(k), \\ \tilde{z}(k) = He\ (k), \end{cases} \tag{6}$$

where

$\tilde{z}(k) = z(k) - \hat{z}(k)$, $\phi(k) = f(x(k)) - f(\hat{x}(k))$, $\psi(Se(k)) = G(Sx(k)) - G(S\hat{x}(k))$, and $\psi_i(S_i e(k))$ $(i=1,2,...,p)$ satisfies the following conditions according to (4):

$$0 \le \frac{\psi_i(S_i e(k))}{S_i e(k)} \le u_i, \tag{7}$$

where $S_i$ is the $i$ th row of matrix $S$.

**Remark 2.** Nonlinearity is present in almost all real sensors in one form or another when the sensors are used in many industrial processes, so the $H_\infty$ filtering problem for various systems with sensors nonlinearities are considered recently, see [27-30] and the references therein.

The $H_\infty$ filtering problem to be investigated in this paper can be formulated as follows. For given discrete-time stochastic neural networks systems (1), a prescribed performance index $\gamma > 0$, and any $G_i (i=1,2,...,p)$, design a suitable filter in the form of (5) such that the following requirements are satisfied:

**(1)** The filtering error system (6) with $v(t) = 0$ is said to be asymptotically stable if there exists a scalar $c > 0$ such that

$$\mathbb{E}\{\sum_{k=0}^{\infty} |e(k)|^2\} \le c\mathbb{E}\{|e(0)|^2\} \tag{8}$$

where $e(k)$ denotes the solution of the system (6) with initial state error e(0).

**(2)** For the given disturbance attention and under zero initial conditions for all $v(k) \in L_{e2}([0,\infty),\mathbb{R}^P)$, the performance index $\gamma$ satisfies the following inequality:

$$\left| z(k)_{e2} \le \gamma \right| z(k)_{e2} \tag{9}$$

## 3. MAIN RESULTS

### 3.1. Performance Analysis of $H_\infty$ Filter

Firstly, we consider system (6) with $v(k) = 0$, then system (6) becomes the following filtering error system:

$$\begin{cases} e(k+1) = Ae(k) + W_0\phi(k) + W_1\phi(k-\tau(k)) \\ \qquad +[Ee(k) + E_d e(k-\tau(k))]\omega(k) - K\psi(Se(k)), \\ \tilde{z}(k) = He\ (k), \end{cases} \tag{10}$$

In the following theorem, a sufficient condition will be derived to ensure the error system (10) is asymptotically stable.

**Theorem 1.**The filtering error system (10) is said to be asymptotically stable if there exists symmetric positive definite matrices $P$, $R$, $Q_1$, $Q_2$, diagonal positive matrices $S_1$,$S_2$, $T = diag\{t_1, t_2, ..t_p\} \ge 0$, $\Lambda = diag\{\lambda_1, \lambda_2, ..\lambda_p\} \ge 0$ and a nonzero matrix $K$ such that the following LMI is satisfied:

$$\Theta = \begin{bmatrix} \Theta_{11} & 0 & \Theta_{13} & 0 & \Theta_{15} & \Theta_{16} & \Theta_{17} \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Theta_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{55} & \Theta_{56} & \Theta_{57} \\ * & * & * & * & * & \Theta_{66} & \Theta_{67} \\ * & * & * & * & * & * & \Theta_{77} \end{bmatrix} < 0,$$

where

$\Theta_{11} = (\tau_M - \tau_m + 1)R - P + A^T \overline{P} A + E^T \overline{P} E + L^T S_1 L$,

$\Theta_{13} = E^T \overline{P} E_d$, $\Theta_{15} = A^T \overline{P} W_0$, $\Theta_{16} = A^T \overline{P} W_1$,

$\Theta_{17} = -A\overline{P}K - S^T\Lambda + S^T UT$, $\Theta_{33} = LS_2 L - R + E_d^T \overline{P} E_d$,

$\Theta_{55} = -S_1 + W_0^T \overline{P} W_0$, $\Theta_{56} = W_0^T \overline{P} W_1$, $\Theta_{57} = -W_0^T \overline{P} K$,

$\Theta_{66} = -S_2 + W_1^T \overline{P} W_1$, $\Theta_{67} = -W_1^T \overline{P} K$,

$\Theta_{77} = K^T \overline{P} K - 2T$, $\overline{P} = P + S^T \Lambda US$ and $U = diag\{u_1, u_2, ...u_p\} \ge 0$.

**Proof.** Introduce the following Lyapunov-Krasovskii functional candidate as follows:

$$V(k)=\sum_{i=1}^{5}V_i(k), \tag{12}$$

where

$$V_1(k)=e^T(k)Pe(k), \tag{13}$$

$$V_2(k)=\sum_{i=k-\tau(k)}^{k-1}e(i)^T Re(i), \tag{14}$$

$$V_3(k)=\sum_{i=k-\tau_m}^{k-1}e(i)^T Q_1 e(i)+\sum_{i=k-\tau_M}^{k-1}e(i)^T Q_2 e(i), \tag{15}$$

$$V_4(k)=\sum_{j=-\tau_M+2}^{-\tau_m+1}\sum_{i=k+j-1}^{k-1}e^T(i)Re(i), \tag{16}$$

$$V_5(k)=2\sum_{i=1}^{p}\lambda_i\psi(S_ie(k))S_ie(k), \tag{17}$$

**Remark 3.** Since 1960s, the Lyapunov second method has been a very powerful tool to deal with the stability problem of linear time-delay system or control system, especially in studying nonlinear systems, such as the stability of neural network with time-delay, because it is much easier to get the stability conditions by defining a proper Lyapunov-Krasovskii functional candidate or Lyapunov function. When the nonlinear system are represented by a linear system, the stability results can be sassily to expressed in terms of linear matrix inequality (LMIs), which can be easily solved numerically by using the Matlab LMI control toolbox. The solution cab be obtained by various convex optimization algorithms.

Calculating the difference of $V(k)$ along the filtering error system (10), we can obtain

$$\Delta V_1(k)=e^T(k+1)Pe(k+1)-e^T(k)Pe(k), \tag{18}$$

$$\begin{aligned}\Delta V_2(k) &= \sum_{i=k+1-\tau(k+1)}^{k}e(i)^T Re(i)-\sum_{i=k-\tau(k)}^{k-1}e(i)^T Re(i)\\ &= e^T(k)Re(k)-e^T(k-\tau(k))Re(k-\tau(k))+\\ &\quad \sum_{i=k+1-\tau(k+1)}^{k-1}e(i)^T Re(i)-\sum_{i=k-\tau(k)+1}^{k-1}e(i)^T Re(i)\\ &= e^T(k)Re(k)-e^T(k-\tau(k))Re(k-\tau(k))+\\ &\quad \sum_{i=k+1-\tau(k+1)}^{k-\tau_m}e^T(i)Re(i)\\ &\quad +\sum_{i=k+1-\tau_m}^{k-1}e(i)^T Re(i)-\sum_{i=k+1-\tau(k)}^{k-1}e(i)^T Re(i)\\ &\le e^T(k)Re(k)-e^T(k-\tau(k))Re(k-\tau(k))+\\ &\quad \sum_{i=k+1-\tau_M}^{k-\tau_m}e^T(i)Re(i),\end{aligned} \tag{19}$$

$$\begin{aligned}\Delta V_3(k)&=e^T(k)(Q_1+Q_2)e(k)-e^T(k-\tau_m)Q_1\\ &\quad e(k-\tau_m)-e^T(k-\tau_M)Q_2e(k-\tau_M),\end{aligned} \tag{20}$$

$$\begin{aligned}\Delta V_4(k)&=\sum_{j=-\tau_M+2}^{-\tau_m+1}[e^T(k)Re(k)+\sum_{i=k+j}^{k-1}e^T(i)Re(i)-\\ &\quad \sum_{i=k+j-1}^{k-1}e^T(i)Re(i)]\\ &=(\tau_M-\tau_m)e^T(k)Re(k)-\sum_{i=k+1-\tau_M}^{k-\tau_m}\\ &\quad e^T(i)Re(i),\end{aligned} \tag{21}$$

$$\begin{aligned}\Delta V_5(k)&=2\sum_{i=1}^{p}\lambda_i\psi_i(S_ie(k+1))S_ie(k+1)-2\psi^T\\ &\quad (Se(k))Se(k).\end{aligned} \tag{22}$$

From condition (7), we can obtain

$$\begin{aligned}&2\sum_{i=-1}^{p}\lambda_i\psi_i(S_ie(k+1))S_ie(k+1)\le 2\sum_{i=-1}^{p}\lambda_i\rho_i(S_ie(k+1))\\ &S_ie(k+1),\end{aligned} \tag{23}$$

So from (22)-(23), we can get

$$\begin{aligned}&\Delta V_5(k)\le 2e^T(k+1)S^T\Lambda USe(k+1)\\ &-2\psi^T(Se(k))\Lambda Se(k),\end{aligned} \tag{24}$$

Combining (18)-(21) and (24), we obtain that

$$\begin{aligned}\Delta V(k)= &\ \Delta V_1(k)+\Delta V_2(k)+\Delta V_3(k)+\Delta V_4(k)+\Delta V_5(k)\\ &\le e^T(k+1)\overline{P}e(k+1)-e^T(k)Pe(k)+\\ &(\tau_M-\tau_m+1)e^T(k)Re(k)\\ &+e^T(k)(Q_1+Q_2)e(k)-e^T(k-\tau_m)Q_1e(k-\tau_m)-\\ &e^T(k-\tau_M)Q_2e(k-\tau_M)\\ &-e^T(k-\tau(k))Re(k-\tau(k)-2\psi^T(Se(k))Se(k),\end{aligned} \tag{25}$$

similarly from (7), we have

$$-2\psi_i(S_ie(k))t_i[\psi_i^T(S_ie(k))-u_iS_ie(k)]\ge 0 \tag{26}$$

Then there exist $T=diag\{t_1,t_2,..t_p\}\ge 0$ and $U=diag\{u_1,u_2,..u_p\}\ge 0$ such that

$$-2\psi(Se(k))T\psi^T(Se(k))+2\psi^T(Se(k))TU(Se(k))\ge 0, \tag{27}$$

In addition, it can be deduced from assumption 1 that there exist two positive diagonal matrices $S_1$ and $S_2$ such that the following inequality holds:

$$\begin{aligned}&\phi^T(k)S_1\phi(k)=[f(x(k))-f(\hat{x}(k))]^T S_1[f(x(k))\\ &\qquad -f(\hat{x}(k))]\\ &\le e^T(k)LS_1Le(k),\end{aligned} \tag{28}$$

This means that

$$e^T(k)LS_1Le(k)-\varphi^T(k)S_1\varphi(k)\ge 0, \tag{29}$$

where $L$ are constant matrix.

By the same way, the following inequality can be obtained

$$e^T(k-\tau(k))L \quad S_2Le(k-\tau(k))-\phi^T(k-\tau(k))S_2\phi \\ (k-\tau(k))\geq 0, \tag{30}$$

By adding the left side of (27) and (29)-(30) into the right side (25), we have

$$\begin{aligned}\Delta V(k) \quad &\leq e^T(k+1)\overline{P}e(k+1)-e^T(k)Pe(k)+ \\ &(\tau_M-\tau_m+1)e^T(k)Re(k) \\ &+e^T(k)(Q_1+Q_2)e(k)-e^T(k-\tau_m)Q_1e(k-\tau_m) \\ &-e^T(k-\tau_M)Q_2e(k-\tau_M) \\ &-e^T(k-\tau(k))Re(k-\tau(k)-2\psi^T(Se(k))Se(k) \\ &-2\psi^T(Se(k))T\psi^T(Se(k))+2\psi^T(Se(k))TU(Se(k)) \\ &+e^T(k)LS_1Le(k) \\ &-\phi^T(k)S_1\phi(k)+e^T(k-\tau(k))LS_2Le(k- \\ &\tau(k))-\phi^T(k-\tau(k))S_2\phi(k-\tau(k))\end{aligned} \tag{31}$$

By using (2) and taking the mathematical expectation on both side of (31), we can get

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\}\leq \quad &\mathbb{E}\{e^T(k+1)\overline{P}e(k+1)-e^T(k)Pe(k)+(\tau_M-\tau_m+1)e^T(k)Re(k) \\ &+e^T(k)(Q_1+Q_2)e(k)-e^T(k-\tau_m)Q_1e(k-\tau_m)-e^T(k-\tau_M)Q_2e(k-\tau_M) \\ &-e^T(k-\tau(k))Re(k-\tau(k)-2\psi^T(Se(k))Se(k) \\ &-2\psi^T(Se(k))T\psi^T(Se(k))+2\psi^T(Se(k))TU(Se(k))\}+e^T(k)LS_1Le(k) \\ &-\varphi^T(k)S_1\varphi(k)+e^T(k-\tau(k))LS_2Le(k-\tau(k))-\varphi^T(k-\tau(k))S_2\varphi(k-\tau(k)) \\ = \quad &[Ae(k)+W_0\varphi(k)+W_1\varphi(k-\tau(k))-K\psi(Se(k))]^T \\ &\times\overline{P}[Ae(k)+W_0\varphi(k)+W_1\varphi(k-\tau(k))-K\psi(Se(k))] \\ &+[E(e(k)+E_de(k-\tau(k))]\overline{P}[E(e(k)+E_de(k-\tau(k))]^T-e^T(k)Pe(k) \\ &+(\tau_M-\tau_m+1)e^T(k)Re(k)-e^T(k-\tau(k))Re(k-\tau(k)-2\psi^T(Se(k))\Lambda Se(k) \\ &-2\psi^T(Se(k))T\psi^T(Se(k))+2\psi^T(Se(k))TU(Se(k))+e^T(k)LS_1Le(k) \\ &-\varphi^T(k)S_1\varphi(k)+e^T(k-\tau(k))LS_2Le(k-\tau(k))-\varphi^T(k-\tau(k))S_2\varphi(k-\tau(k)) \\ &=\xi^T(k)\Theta\xi(k),\end{aligned} \tag{32}$$

where

$$\xi^T(k)=[e^T(k)\ e^T(k-\tau_m)\ e^T(k-\tau(k))\ e^T(k-\tau_M)\ \phi^T(k) \\ \phi^T(k-\tau(k))\ \psi^T(Se(k))], \tag{33}$$

Then from (11), there exists a small scalar $\beta>0$ such that

$$\Theta<\begin{bmatrix}-\beta I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \tag{34}$$

thus it follows from (34) that

$$\mathbb{E}\{V(K+1)\}-\mathbb{E}\{V(K)\}<-\beta\mathbb{E}\{|e(k)|^2\}, \tag{35}$$

So, by summing up both sides of (35) from 0 to $N$ for any integer $N>1$, we have

$$\mathbb{E}\{V(N+1)\}-\mathbb{E}\{V(0)\}<-\beta\mathbb{E}\{\sum_{k=0}^{N}|e(k)|^2\}, \tag{36}$$

which is equal to

$$\begin{aligned}\mathbb{E}\{\sum_{k=0}^{N}|e(k)|^2\}< \quad &\frac{1}{\beta}[\{V(0)\}-\mathbb{E}\{V(N+1)\}] \\ \leq \quad &\frac{1}{\beta}\mathbb{E}\{V(0)\} \\ \leq \quad &c\mathbb{E}\{|e(0)|^2\},\end{aligned} \tag{37}$$

where $c=\frac{1}{\beta}\lambda_{max}(P)$.

Taking $N\to\infty$, by (8) and (37), we can conclude that the filtering error system (10) is asymptotically stable for $v(k)=0$. This complete the proof.

Next, we will show that for all nonzero $v(k)\in L_{e_2}([0,\infty),\mathbb{R}^p)$, the filtering error systems (10) satisfies

$$\|\widetilde{z}(k)\|_{e_2}<\gamma\|v(k)\|_{e_2} \tag{38}$$

Before showing the proof, we define

$$J(N)=\mathbb{E}\{\sum_{k=1}^{N}|\widetilde{z}(k)|^2-\gamma^2|v(k)|^2\} \tag{39}$$

with any integer $N>0$. Then for any nonzero $v(k)$, we have the following Theorem 2.

**Theorem 2.** Given constants $\tau_M$ and $\tau_m$, for the discrete stochastic neural networks system with nonlinear sensor in (1), a filter of form (5), the filter error system (6) is asymptotically stable with performance index $\gamma$, if there exists symmetric positive definite matrices $P$, $R$, $Q_1$, $Q_2$, diagonal positive matrices $S_1,S_2$, $T=diag\{t_1,t_2,..t_p\}\geq 0$, $\Lambda=diag\{\lambda_1,\lambda_2,...\lambda_p\}\geq 0$ and a nonzero matrix $K$ such that the following LMI holds

$$\Gamma=\begin{bmatrix}\Gamma_{11} & 0 & \Gamma_{13} & 0 & \Gamma_{15} & \Gamma_{16} & \Gamma_{17} & \Gamma_{18} \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_{33} & 0 & 0 & 0 & 0 & \Gamma_{38} \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Gamma_{55} & \Gamma_{56} & \Gamma_{57} & \Gamma_{58} \\ * & * & * & * & * & \Gamma_{66} & \Gamma_{67} & \Gamma_{68} \\ * & * & * & * & * & * & \Gamma_{77} & \Gamma_{78} \\ * & * & * & * & * & * & * & \Gamma_{88}\end{bmatrix}<0, \tag{40}$$

where

$\Gamma_{11} = (\tau_M - \tau_m + 1)R + Q_1 + Q_2 - P + A^T \overline{P} A +$
$E^T \overline{P} E + H^T H + LS_1 L,$

$\Gamma_{13} = E^T \overline{P} E_d$ , $\Gamma_{15} = A^T \overline{P} W_0$ , $\Gamma_{16} = A^T \overline{P} W_1$ , $\Gamma_{17} = -A\overline{P}K +$
$S^T \Lambda + S^T UT,$

$\Gamma_{18} = A^T \overline{P}(D_1 - KD_3) + E^T \overline{P} D_2$ , $\Gamma_{33} = LS_2 L - R + E_d^T \overline{P} E_d$ ,

$\Gamma_{55} = -S_1 + W_0^T \overline{P} W_0$ , $\Gamma_{56} = W_0^T \overline{P} W_1$ , $\Gamma_{57} = W_0^T \overline{P} K$ ,

$\Gamma_{38} = E_d^T \overline{P} D_2$, $\Gamma_{58} = W_0^T \overline{P}(D_1 - KD_3)$ , $\Gamma_{66} = -S_2 + W_1^T \overline{P} W_1$ ,

$\Gamma_{67} = -W_1^T \overline{P} K$ , $\Gamma_{68} = W_1^T \overline{P}(D_1 - KD_3)$ ,

$\Gamma_{77} = K^T \overline{P} K - 2T$ , $\Gamma_{78} = -K^T \overline{P}(D_1 - KD_3),$

$\Gamma_{88} = (D_1 - KD_3)^T \overline{P}(D_1 - KD_3) + D_2^T \overline{P} D_2 - \gamma^2 I$ .

**Proof.** By taking the same method as Theorem 1 and using the result of Theorem 1, we have

$$\begin{aligned} J(N) = {} & \mathbb{E}\{\sum_{k=1}^{N} |\widetilde{z}(k)|^2 - \gamma^2 |v(k)|^2 + \Delta V(k)\} - \mathbb{E}\{V(N+1)\} \\ & \le \mathbb{E}\{\sum_{k=1}^{N} |\widetilde{z}(k)|^2 - \gamma^2 |v(k)|^2 + \Delta V(k)\} \\ & = \mathbb{E}\{\overline{\xi}^T(k)\Gamma\overline{\xi}(k)\}, \end{aligned} \tag{41}$$

where

$$\overline{\xi}^T(k) = [e^T(k)\ e^T(k-\tau_m)\ e^T(k-\tau(k))\ e^T(k-\tau_M)$$
$$\phi^T(k)\ \phi^T(k-\tau(k))\ \psi^T(Se(k))\ v^T(k))],$$

From (40), we can obtain $J(N) < 0$ , that is

$\|\widetilde{z}(k)\|_{e_2} < \gamma \|v(k)\|_{e_2}$ .

### 3.2. Design of $H_\infty$ Filter

**Theorem 3.** Consider the discrete-time stochastic neural networks systems (1) with nonlinear sensor and constants $\tau_M, \tau_m$ , the filtering error systems (6) is asymptotically stable with performance index γ, if there exist positive definite matrices $P$, $Q_1$, $Q_2$, $S_1$, $S_2$, diagonal positive definite matrices $T$, $\Lambda$ , and matrix $X$ such that the following LMI is satisfied:

$$\Phi = \begin{bmatrix} \Phi_{11} & 0 & 0 & 0 & 0 & 0 & \Phi_{17} & 0 & A^T\overline{P} & E^T\overline{P} \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & E_d^T\overline{P} \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -S_1 & 0 & 0 & 0 & W_0^T\overline{P} & 0 \\ * & * & * & * & * & -S_2 & 0 & 0 & W_1^T\overline{P} & 0 \\ * & * & * & * & * & * & -2T & 0 & -X & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I & D_1^T - D_3^T X & D_2^T\overline{P} \\ * & * & * & * & * & * & * & * & -\overline{P} & 0 \\ * & * & * & * & * & * & * & * & * & -\overline{P} \end{bmatrix} < 0, \tag{42}$$

where

$\Phi_{11} = (\tau_M - \tau_m + 1)R + Q_1 + Q_2 - P + H^T H + LS_1 L$ , $\Phi_{17} = -A\overline{P}K +$ $S^T \Lambda + S^T UT$, $\Phi_{33} = LS_2 L - R$ , Moreover, if the previous condition is satisfied, an acceptable state-space realization of the $H_\infty$ filter is given by

$$K = [(P + 2S^T \Lambda US)^{-T} X. \tag{43}$$

**Proof.** By lemma (1), the matrix inequality (40) is equivalent to

$$\begin{bmatrix} \Gamma_{11} & 0 & 0 & 0 & 0 & 0 & \Gamma_{17} & 0 & A^T\overline{P} & E^T\overline{P} \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & LS_2L - R & 0 & 0 & 0 & 0 & 0 & 0 & E_d^T\overline{P} \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -S_1 & 0 & 0 & 0 & W_0^T\overline{P} & 0 \\ * & * & * & * & * & -S_2 & 0 & 0 & W_1^T\overline{P} & 0 \\ * & * & * & * & * & * & -2T & 0 & -K^T\overline{P} & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I & (D_1 - KD_3)^T\overline{P} & D_2^T\overline{P} \\ * & * & * & * & * & * & * & * & -\overline{P} & 0 \\ * & * & * & * & * & * & * & * & * & -\overline{P} \end{bmatrix} < 0. \tag{44}$$

By defining

$$K^T \overline{P} = X, \tag{45}$$

and then substituting (45) into (44), we can get (42) , so the proof is completed.

**Remark 2.** When $\tau_M, \tau_m$ are given, the matrix inequality (42) is linear matrix inequality, if matrix variables $P > 0$, $R > 0$, $Q_1 > 0$, $Q_2 > 0$, $T > 0$, $\Lambda > 0$ , then $X$ can be efficiently solved by the developed interior point algorithm [38]. At the same tine, the minimal performance index γ can be found out easily.

When neglecting stochastic disturbance, the system (1) will become the following one:

$$\begin{cases} x(k+1) = Ax(k) + W_0 f(x(k)) + W_1 f(x(k - \tau(k))) + D_1 v(k) \\ y(k) = G(S\ x(k)) + D_3 v(k), z(k) = Hx\ (k), \end{cases} \tag{46}$$

then for system (46), the following corollary can be obtained from Theorem 2.

**Corollary 1.** For the discrete neural networks system (46), the corresponding filtering error system is asymptotically stable with performance γ, if there exist positive definite matrices $P$, $Q_1$, $Q_2$, $S_1$, $S_2$, diagonal positive definite matrices $T$, $\Lambda$ and matrix $X$ such that the following LMI holds:

$$\begin{bmatrix} \Phi_{11} & 0 & 0 & 0 & 0 & 0 & \Phi_{17} & 0 & A^T\overline{P} \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -S_1 & 0 & 0 & 0 & W_0^T\overline{P} \\ * & * & * & * & * & -S_2 & 0 & 0 & W_1^T\overline{P} \\ * & * & * & * & * & * & -2T & 0 & -X \\ * & * & * & * & * & * & * & -\gamma^2 I & D_1^T - D_3^T X \\ * & * & * & * & * & * & * & * & -\overline{P} \end{bmatrix} < 0, \tag{47}$$

where $\Phi_{11}$, $\Phi_{17}$, $\Phi_{33}$ and $\overline{P}$ have the same definition as in Theorem 2.

What is more, if the condition above is satisfied, an acceptable state-space realization of the $H_\infty$ filter is given by

$$K=(P+S^T\Lambda US)^{-T}X^T. \tag{48}$$

## 4. NUMERICAL EXAMPLES

In this section, two numerical examples with simulation results are provided to demonstrate the effectiveness of the proposed filter design approaches and their performances.

**Example 1.** Consider the discrete stochastic neural networks systems with parameters as follows [37]:

$$A=\begin{bmatrix}0.4&0&0\\0&0.3&0\\0&0&0.3\end{bmatrix},W_0=\begin{bmatrix}0.2&-0.2&0.1\\0&-0.3&0.2\\-0.2&-0.1&-0.2\end{bmatrix},$$

$$W_1=\begin{bmatrix}-0.2&0.1&0\\-0.2&0.3&0.1\\0.1&-0.2&0.3\end{bmatrix},H=\begin{bmatrix}0.1&0&0\\0&0.1&0\\0&0&0.1\end{bmatrix},$$

$$U=\begin{bmatrix}0.2&0&0\\0&0.2&0\\0&0&0.2\end{bmatrix},L=\begin{bmatrix}0.2&0&0\\0&0.2&0\\0&0&0.2\end{bmatrix},$$

$$D=\begin{bmatrix}0.1&-1&0.2\end{bmatrix}.$$

From the Theorem 2, we know that for different $\tau_M$ and $\tau_m$, we can get different $\gamma_{min}$, In this case, we assume that $\tau(k)=3-2sin(\frac{\pi}{2}k)$, by using the Matlab LMI toolbox and solving (42), the feasible solutions can be obtained as follows:

$$P=\begin{bmatrix}7.7786&-0.6921&-2.8354\\-0.6921&4.9820&-0.5467\\-2.8354&-0.5467&7.3521\end{bmatrix},Q1=\begin{bmatrix}0.0126&-0.0057&0.0078\\-0.0057&0.0466&-0.0766\\0.0078&-0.0766&0.1600\end{bmatrix},$$

$$Q2=\begin{bmatrix}0.0126&-0.0057&0.0078\\-0.0057&0.0466&-0.0766\\0.0078&-0.0766&0.1600\end{bmatrix},S1=\begin{bmatrix}3.1234&0&0\\0&5.2403&0\\0&0&11.1715\end{bmatrix},$$

$$S2=\begin{bmatrix}2.7544&0&0\\0&2.6200&0\\0&0&5.6739\end{bmatrix},R=\begin{bmatrix}0.8096&-0.0671&-0.3249\\-0.0671&0.5261&-0.0941\\-0.3249&-0.0941&0.4861\end{bmatrix},$$

$$T=\begin{bmatrix}29.8167&0.1&0\\0&6.6348&0\\0&0&136.6493\end{bmatrix},\Lambda=\begin{bmatrix}0.2405&0&0\\0&0.0148&0\\0&0&0.2109\end{bmatrix}.$$

By virtue of the developed interior point algorithm, the minimum reachable performance index is $\gamma_{min}=0.6593$, and the corresponding $H_\infty$ filter parameters are as follows:

$$K=\begin{bmatrix}-0.1013&-0.0615&-0.1041\\0.0052&-0.1611&0.0417\\0.1212&-0.2012&0.1313\end{bmatrix}.$$

**Remark 4.** In this example, the sensor nonlinear functions satisfy (4), in which $u_1=u_2=2$ and $\lambda_1=\lambda_2=2$, In Theorem 2, the matrix inequality (40) can not be solved by Matlab LMI toolbox directly, but by Lemma 1 and simple transformation of (43), the LMI (42) can be solved by Matlab LMI toolbox and $K$ can be easily found out.

**Remark 5.** In this example, when the value $\tau_M-\tau_m$ is less than 4, different parameters $u_1=u_2$, $L$ are chosen to do experiment, for example, when $L=diag\{0.1-0.1-0.2\}$, $H=diag\{-0.1-0.1\ 0.2\}$, $u_1=diag\{-0.1-0.1\ 0.2\}$, different $K$ can be found out and the feasible solution has not been affected. That is to say, some parameters are not sensitive to on the results.

**Remark 6.** In example 1, we have taken the system parameters as that in [37], by Matlab LMI Control Toolbox, we have obtained the $H_\infty$ performance index $\gamma_{min}=0.6593$, while in [37], only the $H_\infty$ parameters $K$ has been found out, the performance index $\gamma_{min}$ is not mentioned.

By choosing the proper initial condition, for example, $x(t)=[0.8-0.8\,0.2]^T$, $\hat{x}(t)=[-0.3\,0.2\,0.8]^T$, respectively, the activation functions are given as $f(x)=\frac{|x-1|-|x+1|}{2}$, the nonlinear sensor functions are taken by *tanh*(*Sx*(*k*)), and the exogenous disturbance signal *v*(*k*) is given by *exp*(-*k*), by applying the previous $H_\infty$ filter parameter $K$ to system (1), the simulation results can be obtained as Figs. (**1-3**). Fig. (**1**) shows the state response $x(k)$ under the initial condition; Fig. (**2**) shows the estimation of filter; Fig. (**3**) shows the error response *e*(*k*). From these simulation results we can see that the designed $H_\infty$ filter can stabilize the discrete-time stochastic neural works system (1) with nonlinear sensors and time-varying delay.

**Example 2.** Consider the discrete neural networks systems (46) with parameters as follows :

$$A=\begin{bmatrix}0.2&0\\0&-0.5\end{bmatrix},W_0=\begin{bmatrix}0.4&-0.2\\-0.2&0.4\end{bmatrix},W_1=\begin{bmatrix}-0.3&-0.6\\-0.2&0.3\end{bmatrix},C=\begin{bmatrix}0.1&-0.1\\-0.1&0.2\end{bmatrix},$$

$$D2=\begin{bmatrix}0.1\\0.1\end{bmatrix},H=\begin{bmatrix}0.1&0\\0&0.1\end{bmatrix},U=\begin{bmatrix}0.1&0\\0&0.1\end{bmatrix},L=\begin{bmatrix}0.1&0\\0&0.1\end{bmatrix},$$

In this example, we assume that $\tau(k)=3.5-2.5sin(\frac{\pi}{2}k)$, by using the Matlab LMI toolbox and solving the LMI (47), we can obtain the feasible solutions as follows:

$$P=\begin{bmatrix}9.3744&1.6707\\1.6707&12.3118\end{bmatrix},R=\begin{bmatrix}1.1171&0.1939\\0.1939&0.9304\end{bmatrix},$$

$$T=\begin{bmatrix}7.1577&0\\0&6.3967\end{bmatrix},S1=\begin{bmatrix}16.3782&0\\0&24.1831\end{bmatrix},$$

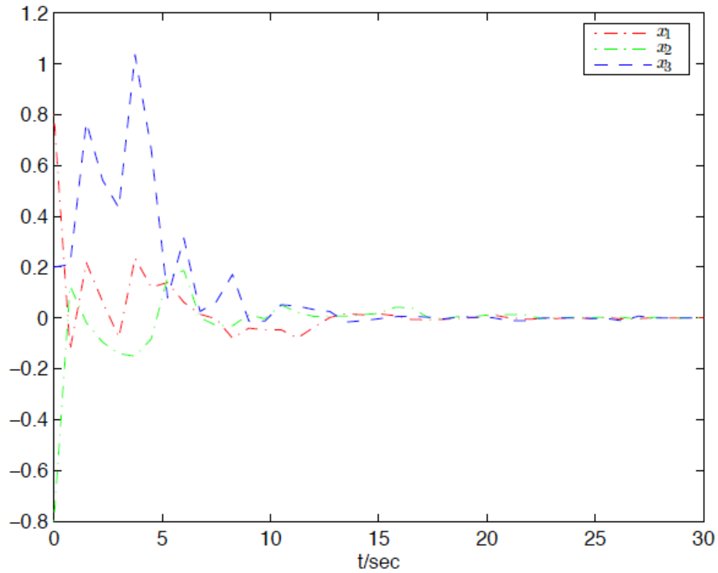


**Fig. (1).** The true state response of $x(t)$.

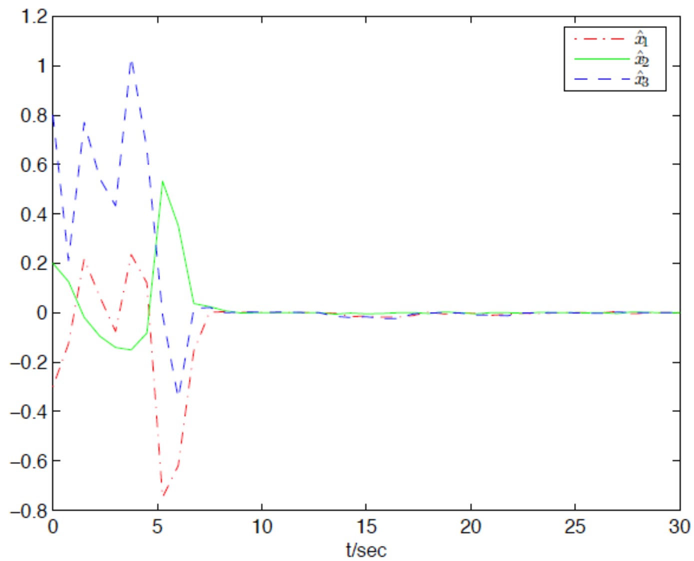


**Fig. (2).** The estimation of $x(t)$.

$$S2=\begin{bmatrix} 11.9083 & 0 \\ 0 & 12.2956 \end{bmatrix}, Q1=\begin{bmatrix} 0.3265 & 0.0362 \\ 0.0362 & 0.2072 \end{bmatrix},$$

$$Q2=\begin{bmatrix} 0.3265 & 0.0362 \\ 0.0362 & 0.2072 \end{bmatrix}, \Lambda=\begin{bmatrix} 5.5281 & 0 \\ 0 & 0.5925 \end{bmatrix},$$

therefore, the concerned discrete neural network with time-varying delay and nonlinear sensor is asymptotically stable. Meanwhile, for different $\tau_m$ and $\tau_M$, different $\gamma_{min}$ can be obtained. In this case, we can get that $1 \leqslant \tau(k) \leqslant 6$, then the minimal $H_\infty$ performance index is $\gamma_{min}=1.0322$, and the corresponding filter matrix is

$$K=\begin{bmatrix} 0.5171 & 0.0001 \\ -0.0212 & -0.0000 \end{bmatrix}. \tag{49}$$

**Remark 7.** In spite of the considerable advantages of the $H_\infty$ filtering design results, it still entails some appreciable amount of conservatism due to the majorization procedure in filter design.

## CONCLUSION

In this paper, the problem of $H_\infty$ filtering for a class of discrete stochastic neural networks systems with nonlinear sensor and time-varying delay have been studied. By

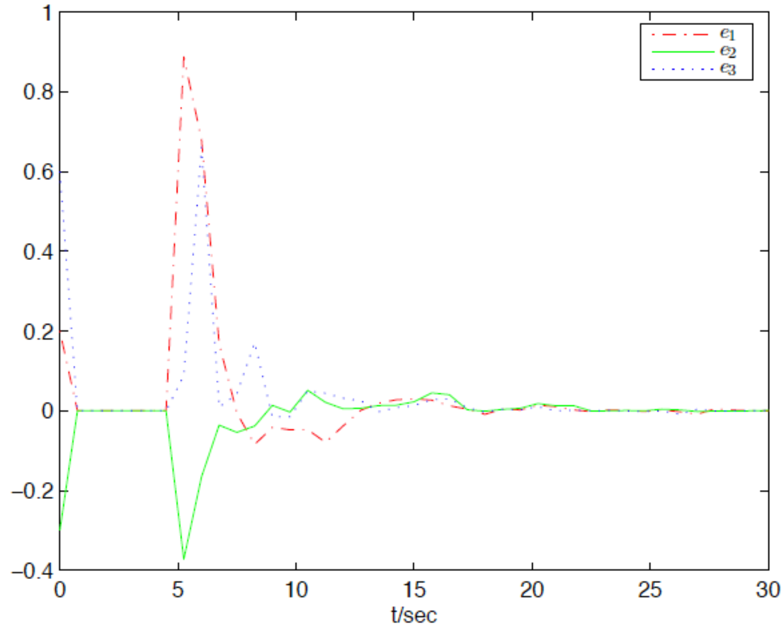


**Fig. (3).** Response of the filtering error $e(t)$ of $H_\infty$.

employing the Lyapunov stability theory and linear matrix inequality optimization approach, sufficient conditions to guarantee the filtering error systems asymptotically stable are provided. By setting the lower and upper bounds of the discrete time-varying delays, an acceptable state-space realization of the $H_\infty$ performance index is obtained in terms of linear matrix inequality (LMI). Finally, two numerical examples and simulations have been exploited to show the usefulness and effectiveness of the proposed filter design method. Our future research directions would extend to the investigation on more general nonlinear systems, more complex discrete-time systems, fuzzy neural networks systems with different delays and the reduction of conservatism brought by the $H_\infty$ filter technology.

## CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.


## ACKNOWLEDGEMENTS

Declared none.


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Received: February 13, 2014 Revised: October 05, 2014 Accepted: October 24, 2014