

Cellular Neural Networks Model on Real Estate Investment Systems and Its Stability and Chaos Synchronization

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Abstract: This paper presents the complex networks chaos model on real estate investment systems and brings attention to Chaos synchronization between real estate investment drive system and economic increase response system. The sufficient conditions for achieving the synchronization of two systems are derived based on Lyapunov stability theory. If theory's condition is met, the risk of investment is low, or it is high relative, when the real estate investment appears chaos phenomena. The result can be realized by Matlab.

Keywords: Cellular neural networks, chaos, complex, real estate investment, stability, time-delay.

1. INTRODUCTION

Chaos theory is a break through changing the way of thinking in the fields of classical economics, since it is applied to the fields of economic in the 1980s. It can announce the ordered and regular structure at the back of economic phenomena, which seems random. And the chaos theory can broaden the horizon of people's study of economic problems, so it drew great attention of scholars. If the economic systems have chaos traits, then it can fluctuate in a wider band. On the one hand, the research workers applied chaos theory to analyze the experimental data, which aimed to illustrate the regularity of economic problems [1, 2]. On the other hand, they applied the theory of chaos to study the economic growth and period and made a new interpretation of classical economic theory of growth and period [3-6].

Real estate is an industry of high risk, high investment and high return. It gives great return to investments, draws great risks and perhaps suffers a great loss. Because the real estate commodities are affected by natural, society, and economic administration and psychology, so the investment faces great uncertainties. The real estate development needs longer period. Additionally, if the land resource is limited, the real estate applies take on hypoelasticity, whereas if the market demand is great, the real estate demands take on hyperelasticity [7].

Appraise, analyze, evaluate and manage the risk of building works include in the theory from parametric estimation to subjective judgment. But the real estate has a certain found period, which is the time-lag effect where the front certainties can affect the back investment. This paper provides the complex networks chaos model on real estate

investment systems and brings attention to Chaos synchronization between real estate investment drive system and economic increase response system. The sufficient conditions for achieving the synchronization of two systems are derived based on Lyapunov stability theory.

2. RESULTS

Suppose it has N developers of the real estate in the markets, $y_i(t)$ is the amount of investment of i th developers at the time of t ; $y_i(t-\tau)$ is the amount of investment of i th developers at the time of $t-\tau$ ($i=1,2,\dots,N$); $f(y(t))$ represent price function, τ is the period of founded, which is time delay; $f_i(y(t)), f_i(y(t-\tau))$ is the price of the houses of the i th developers at the time of t and $t-\tau$. Because the investment amount at the time of t take on time-lag to the price of the houses at the time of $t-\tau$, so we use the following neural networks model of time delay to describe the real estate investment amount:

$$\dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^N a_{ij} f_j(y_j(t)) + \sum_{j=1}^N b_{ij} f_j(y_j(t-\tau)) + j_i \quad (1)$$

Or else, it's equivalent form is:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t-\tau)) + J \quad (2)$$

Where $y(t) = [y_1(t), y_2(t), \dots, y_N(t)]^T$, $A = [a_{ij}]_{N \times N}$, $B = [b_{ij}]_{N \times N}$, $J = [j_1, j_2, \dots, j_N]^T$ is output, and C is positive diagonal matrix.

It has been established that [8], systems (2) can translate the equilibrium point to origin of coordinates, so we can found following

complex networks chaos model:

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$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^N a_{ij} f_j(x_j(t)) + \sum_{j=1}^N b_{ij} f_j(x_j(t-\tau)) \quad (3)$$

Where N is the amount of development or, $x_i(t), x_i(t-\tau)$ represent the amount of the real estate investment at the time of $t, t-\tau$ each $f_j(x_j(t)), f_j(x_j(t-\tau))$ represent the price of the houses in the market at the time of $t, t-\tau$, a_{ij}, b_{ij} represent the affection factors of the j th development or houses selling price at the time of $t, t-\tau$ to the i th investor. (3) is equivalent to (4).

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau)) \quad (4)$$

Hypothesis 1: $f_i^2(x_i(t)) \leq k_i x_i(t) f_i(x_i(t)), \forall x \in R, f(0) = 0$

Theorem 1: under the hypothesis 1, if exist symmetric positive definite matrix P, Q , and positive semidefinite diagonal matrix $T_j = \text{diag}(t_{j1}, t_{j2}, \dots, t_{jN}) \geq 0$, and

$D = \text{diag}(d_1, d_2, \dots, d_N) \geq 0$, meet the following inequality (5):

$$\Omega = \begin{bmatrix} N_1 & 0 & N_2 & PB \\ * & -Q_{11} & 0 & N_3 \\ * & * & N_4 & DB \\ * & * & * & N_5 \end{bmatrix} < 0 \quad (5)$$

$$N_1 = -2PC + Q_{11}, N_2 = PA - CD + Q_{12} + KT_1 \quad N_5 = -Q_{22} - 2T_2$$

$$N_4 = DA + A^T D^T + Q_{22} - 2T_1$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \text{ then the systems (4) is global asymptotical stable.}$$

Proof: found up Lyapunov function $V(t) = V_1 + V_2 + V_3$

$$V_1 = x^T(t)Px(t), \quad V_2(t) = 2 \sum_{i=1}^N d_i \int_0^{x_i(t)} f_i(s) ds,$$

$$V_3 = \int_{t-\tau}^t [x^T(s), f^T(s)] \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ f(s) \end{bmatrix} ds$$

Derivation to (4) along the rail line systems:

$$\dot{V}_1 = 2x^T(t)P[-Cx(t) + Af(x(t)) + Bf(x(t-\tau))],$$

$$\dot{V}_2 = 2f^T(x(t))D[-Cx(t) + Af(x(t)) + Bf(x(t-\tau))],$$

$$\dot{V}_3(t) = \begin{bmatrix} x^T(t) & f^T(x(t)) \end{bmatrix} Q \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} - \begin{bmatrix} x^T(t-\tau) & f^T(x(t-\tau)) \end{bmatrix} Q \begin{bmatrix} x(t-\tau) \\ f(x(t-\tau)) \end{bmatrix}$$

From hypothesis 1, we get the following inequality easily:

$$\begin{aligned} 0 &\leq 2 \sum_{i=1}^N [k_i t_{ii} f_i(x_i(t)) x_i(t) - t_{ii} f_i^2(x_i(t))] + \\ &2 \sum_{i=1}^N [k_i t_{ii} f_i(x_i(t-\tau)) x_i(t-\tau) - t_{ii} f_i^2(x_i(t-\tau))] \\ &= 2f^T(x(t))KT_1x(t) - 2f^T(x(t))T_1f(x(t)) + \\ &2f^T(x(t-\tau))KT_2x(t-\tau) - 2f^T(x(t-\tau))T_2f(x(t-\tau)) \end{aligned}$$

So, $\dot{V} \leq \xi^T(t)\Omega\xi(t) < 0$, where $\xi(t) = [x(t), x(t-\tau), f(x(t)), f(x(t-\tau))]$ regards (3) as driving systems, the response systems corresponding is:

$$\dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^N a_{ij} f_j(y_j(t)) + \sum_{j=1}^N b_{ij} f_j(y_j(t-\tau)) \quad (6)$$

Hypothesis 2: $\|f_j(y_j(t)) - f_j(x_j(t))\| \leq L_j \|y_j(t) - x_j(t)\|$, $\|f_j(y_j(t-\tau)) - f_j(x_j(t-\tau))\| \leq L_j \|y_j(t-\tau) - x_j(t-\tau)\|$

lemma 1: giving appropriate dimension matrix Y, D and E, F , then $Y + DFE + E^T F^T D^T < 0$ is founded to all of matrix F which meet with $FF^T \leq I$, if and only if exists constant $\lambda > 0$, s.t $Y + \lambda DD^T + \lambda^{-1} E^T E < 0$.

Theorem 2: if it meets with the inequality, $N(\sigma_1 + \sigma_2 + 2) \leq 2c_i$, then the driving system (3) is chaos synchronization to the response system (6).

Proof: definite the systems errors as:

$$e_i(t) = y_i(t) - x_i(t)$$

Then its derivative is: $\dot{e}_i(t) = -c_i e_i(t) +$

$$\sum_{j=1}^N a_{ij} [f_j(y_j(t)) - f_j(x_j(t))] + \sum_{j=1}^N b_{ij} [f_j(y_j(t-\tau)) - f_j(x_j(t-\tau))]$$

Found Lyapunov function $V(x(t)) = \sum_{i=1}^N e_i^T(t) e_i(t) +$

$$N \sum_{i=1}^N \int_{t-\tau}^t e_i^T(s) e_i(s) ds$$

$$\dot{V} = \sum_{i=1}^N -2c_i e_i^T(t) e_i(t) + \sum_{i=1}^N \sum_{j=1}^N 2e_i^T a_{ij} L_j e_j(t)$$

$$+ \sum_{i=1}^N \sum_{j=1}^N 2e_i^T b_{ij} L_j e_j(t-\tau)$$

$$+ N \sum_{j=1}^N e_j^T(t) e_j(t) - N \sum_{j=1}^N e_j^T(t-\tau) e_j(t-\tau),$$

Use lemma1 we get:

$$\sum_{i=1}^N \sum_{j=1}^N 2e_i^T(t) a_{ij} L_j e_j(t) \leq \sum_{i=1}^N \sum_{j=1}^N a_{ij}^2 L_j^2 e_i^T(t) e_i(t) +$$

$$N \sum_{j=1}^N e_j^T(t) e_j(t)$$

For the same reason,

$$\sum_{i=1}^N \sum_{j=1}^N 2e_i^T(t) b_{ij} L_j e_j(t-\tau) \leq$$

$$\sum_{i=1}^N \sum_{j=1}^N b_{ij}^2 L_j^2 e_i^T(t) e_i(t) +$$

$$N \sum_{j=1}^N e_j^T(t-\tau) e_j(t-\tau)$$

where

$$\sigma_1 = \lambda_{\max}(L_j^2) \max_{1 \leq i, j \leq N} \{a_{ij}^2\}, \quad \sigma_2 = \lambda_{\max}(L_j^2) \max_{1 \leq i, j \leq N} \{b_{ij}^2\}$$

$$\dot{V} \leq \sum_{i=1}^N e_i^T(t) (-2c_i + N\sigma_1 + N\sigma_2 + 2N) e_i(t)$$

If it meet with $N(\sigma_1 + \sigma_2 + 2) \leq 2c_i$, then $\dot{V}(t) < 0$, then the driving system (3) is chaos synchronization to the response system (6).

CONCLUSION

If the theory's condition is met, the risk of investment is low, or it is high relative when the real estate investment appears as chaos phenomena. And it provides the basis for the real estate investment, whereas how to erase and control the chaos phenomena by establishing a policy are hot topics for further research.

CONFLICT OF INTEREST

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