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Stability of Solution for Nonlinear Singular Systems with Delay

Zhang Jing*,[1] and Ling Chong[2]

[1]*Department of Basic Science, North China Institute of Aerospace Engineering, Langfang, Hebei, 065000, P.R. China*

[2]*No.4 Construction Company, China Petroleum Pipeline Bureau ng, Langfang, Hebei, 065000, P.R. China*

Abstract: In this paper, we present new definitions of stability of singular systems with delay based on the works of B.Baji'c and M.Mili'c, and establish theorem on stability. Then the sufficient conditions for uniform stability of a class of special nonlinear singular systems with delay are suggested.

Keywords: Nonlinear singular systems, singular systems with delay, stability, uniform stability.

1. INTRODUCTION

From a modeling point of view, it is perhaps more realistic to model a phenomenon by a singular such as descriptor systems, semi-state space systems, and differential-algebraic systems, use of which arise frequently in many fields such as optimal control problems, electrical circuits, neutral network, and some population growth models. Some applicable example and basic results can be found in [1, 2].

Singular systems correspond to normal systems. They have both internal logic and essential difference. But the study of singular systems had mostly referred to the given normal systems theories, which had been generalized and transplanted into singular systems so far. The methods of study for singular system are mostly geometric approach, frequency domain method and state-space techniques. And people still have different views related to the questions of singular systems, thus the research achievement of singular systems appears extremely fragmentary. The stability and asymptomatical stability still haven't have uniform distinct definitions, thus there exists confusions for some concepts as well as inconsistencies between the concept and the theorem. The consistency of the initial conditions still has two different views. On the other hand, there often happen impulses and jumps in the solutions. So, the existence and uniqueness of solutions in singular systems haven't been resolved, because the complexity of singular systems and their stability haven't achieved mutual recognition. Therefore, the research for singular systems not only has a widespread practical significance, but also its theoretical value has broad prospects for development.

As one of the major research subjects in nonlinear singular systems, the problem of stability attracts many researchers attention, for example [3-5] discussed the stability of singular systems. Since delay often occurs in singular systems such as discussed [6], therefore the research on stability of nonlinear singular systems with delay is given much importance in practice and theory. However, few studies have been done on the stability of nonlinear singular systems with delay. Therefore, in this paper we concentrate on this. The discussion on stability of singular systems, compared with that of nonsingular systems, came up with three main new difficulties: the first is that it isn't easy to satisfy the existence and uniqueness of solutions, since the initial conditions may not be consistent; the second is that it is difficult to calculate the derivatives of Lyapunov functions; the third is that there often happen impulses and jumps in the solutions. In order to overcome these difficulties, this paper presents new definitions of stability of nonlinear singular systems with delay based on the previous studies by [7, 8]. Furthermore, the stability theorem of solutions for the following nonlinear singular systems with delay has been established:

$$\begin{cases} A\dot{x}(t) = Bx(t) + Cx(t-\tau) + f(t, x(t-\tau)) \\ x_{t_0} = \varphi \end{cases} \tag{1}$$

where A is an $n \times n$ constant singular matrix, and B, C are $n \times n$ constant matrices. $\tau > 0$,

$f(t,\psi) \in C([0,+\infty) \times C([-\tau,0], R^n), R^n)$,

$f(t,0) = 0$, for any $t \ge t_0 \ge 0$. $x_{t_0} = \varphi$ is the initial condition of (1), where $\varphi \in C([-\tau,0], R^n)$.

2. PRELIMINARIES

In this paper, we assume that the solution of initial value problems for system (1) exists which is called non-perturbation solution, written as $x_u(t, t_0, \varphi)$, sometimes $x_u(t)$ for short.

At first, we introduce the following notations:

*Address correspondence to this author at the Department of Basic Science, North China Institute of Aerospace Engineering, Langfang, Hebei, 065000, P.R. China; Tel: 15831630452; E-mail: zhang_jing_9825@126.com

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$T_k=[0,t_k)$, where $0<t_k\le+\infty$; $T_{0k}(t_0)=[t_0,t_k)$ where $t_0\in T_k$. J is an open interval at R, and $T_k\subseteq J_0$. D is an open interval at R^n, $C'(J\times D)$ is the set of all continuous differentiable functions defined as $J\times D$, $q(t,x)\in C'(J\times R^n,R^m)$. $S_I(t)\subseteq C([-\tau,0],R^n)$, $S_I(t)$ is a set of all consistency initial functions of (1.1) in $[(t_0-\tau),t_k)$ through (t,ψ) at least. $S_k(t,t_k)\subseteq S_I(t)$, and for any $\psi\in S_k(t,t_k)$ there exists a continuous solution of (1.1) in $[t,t_k)$ through (t,ψ) at least.

$$B(\varphi,\delta)=\{\psi\in C([-\tau,0],R^n);\|\psi-\varphi\|<\delta\},\quad \delta>0.$$
$$Q(t,\varepsilon)=\{x\in R^n;\|q(t,x)-q(t,x_u(t))\|<\varepsilon\},\quad \varepsilon>0.$$

In this paper, we suppose $x_u(t,t_0,\varphi)$ exists in $[t_0,t_k)$, $\forall t_0\in T_k$, implies $\varphi\in S_k(t_0,t_k)$.

Definition 2.1 If $\forall t_0\in T_k$, $\forall\varepsilon>0$, there always exists $\delta(t_0,\varepsilon)>0$, such that for $\forall\psi\in B(\varphi,\delta)\cap S_k(t_0,t_k)$, $x(t,t_0,\psi)$ satisfies that $x(t,t_0,\psi)\in Q(t,\varepsilon)$, $\forall t\in[t_0,t_k)$. Then the solution $x_u(t,t_0,\varphi)$ is said to be stable in $\{q(t,x),T_k\}$. If δ is only related to ε and has nothing to do with t_0, then $x_u(t,t_0,\varphi)$ is said to be uniformly stable in $\{q(t,x),T_k\}$.

Definition 2.2 1) If $x_u(t,t_0,\varphi)$ is stable in $\{q(t,x),T_k\}$, where $t_k=+\infty$, and for $\forall t_0\in T_k$, there exists a $\Delta(t_0)>0$, such that $\forall\psi\in B(\varphi,\Delta(t_0))\cap S_k(t_0,+\infty)$,

$$\lim_{t\to+\infty}\|q(t,x(t,t_0,\psi))-q(t,x_u(t,t_0,\varphi))\|=0,$$

then $x_u(t,t_0,\varphi)$ is said to be asymptomatically stable in $\{q(t,x),T_k\}$;

2) If $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$, where $t_k=+\infty$, and there exists a $\Delta>0$, for $\forall t_0\in T_k$ and $\forall\psi\in B(\varphi,\Delta)\cap S_k(t_0,+\infty)$, $\lim_{t\to+\infty}\|q(t,x(t,t_0,\psi))-q(t,x_u(t,t_0,\varphi))\|=0$, and which is uniform at $(t_0,\psi)\in[0,+\infty)\times(B(\varphi,\Delta)\cap S_k(t_0,+\infty))$, then $x_u(t,t_0,\varphi)$ is said to be uniformly asymptomatically stable in $\{q(t,x),T_k\}$.

Remarks

1. t_k may take $+\infty$, also it is possible to take a limited number; so, the application scope of stable concept expands.

2. It is difficult to calculate the derivatives of Lyapunov functions. We introduced $q(t,x)$ to avoid the difficulty.

3. In the above notations, the initial condition is consistent and the existence of solution is satisfied in $[t_0,t_k)$. But it is not required that the uniqueness of solution is satisfied, this is different for the nonsingular systems. It is however, difficult to satisfy the uniqueness of solution for singular systems.

4. If $q(t,x)=x$, $t_k=+\infty$ and $S_k(t_0,t_k)=C([-\tau,0],R^n)$, then the above definitions become the definitions of Lyapunov stability of traditional significance.

5. If A in (1) is a symmetric matrix, then we take $q((t,x)=x^TAx$, otherwise, we take

$$q((t,x)=x^TA^TH(t)Ax$$

where $H(t)$ is a matrix function.

3. MAIN RESULT

Now we propose theorem on the stability of solutions to (1) by using the Lyapunov function.

Theorem 1 Suppose that there exist two K-class wedge functions Φ_1, Φ_2 and continuous V function:

$$V(t,q(t,x)):J\times R^n\to R^+,\ V(t,q(t,x))\in C'(J\times R^n),$$

such that

i) $\Phi_1(\|q(t,x)-q(t,x_u(t,t_0,\varphi))\|)$
$\le V(t,q(t,x))\le\Phi_2(\|x-x_u(t,t_0,\varphi)\|),\forall t\in[t_0,t_k);$

ii) $\forall\psi\in S_k(t_0,t_k),\ \dot V(t,q(t,x(t,t_0,\psi)))\le 0$

Then $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$.

Proof For $t_0\in[0,t_k)$, $\forall\varepsilon>0$, there exists $\delta(\varepsilon)>0$, which is only related to ε, such that $\forall\psi\in B(\varphi,\delta)$, $\Phi_2(\|\psi-\varphi\|)<\Phi_2(\delta)\le\Phi_1(\varepsilon)$.

$$\begin{aligned}&\Phi_1(\|q(t,x)-q(t,x_u(t,t_0,\varphi))\|)\\ &\le V(t,q(t,x(t,t_0,\psi)))\le V(t_0,q(t_0,x_u(t_0,t_0,\varphi)))\\ &\le\Phi_2(\|x(t_0,t_0,\psi)-x_u(t_0,t_0,\varphi)\|)\\ &=\Phi_2(\|\psi-\varphi\|)<\Phi_1(\varepsilon).\end{aligned}$$

Thus $\|q(t,x)-q(t,x_u(t,t_0,\varphi))\|<\varepsilon$, $\forall t\in[t_0,t_k)$. Then, we have $x(t,t_0,\psi)\in Q(t,\varepsilon)$, $\forall t\in[t_0,t_k)$, hence $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$. The proof is completed.

Theorem 2 Suppose that there exist two K-class wedge functions Φ_1, Φ_2, nonnegative and non-decreasing function w, and continuous V function:

$V(t,\varphi):[0,+\infty)\times C([-r,0],R^n)\to R$,

which satisfies

i) $\Phi_1(\| q(t,x)-q(t,x_u(t,t_0,\varphi))\|)$

$\le V(t,q(t,x))\le \Phi_2(\| x-x_u(t,t_0,\varphi)\|), \forall t\in[t_0,t_k);$

ii) $\forall \psi\in S_k(t_0,t_k)$, $\dot V(t,q(t,x(t,t_0,\psi)))\le -w(\|q(t,x(t))\|)$

Then $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$, where $t_k\le +\infty$. If $w(s)>0$ when $s>0$, then $x_u(t,t_0,\varphi)$ is asymptomatically stable in $\{q(t,x(t)),[0,+\infty)\}$.

Proof 1) By conditions of theorem we get $\dot V(t,q(t,x(t,t_0,\psi)))\le 0$ .Then according to theorem 1 $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$.

2) For that $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$, we set $t_k=+\infty$, $\varepsilon=1$. Then there exists a $\delta_0>0$ such that $t_0\in[0,+\infty)$, $\psi\in(0,\delta_0)\cap S_k(t_0,+\infty)$, we have

$\| q(t,x(t,t_0,\psi))-q(t,x_u(t,t_0,\varphi))\|<1,\ \forall t\in[t_0,t_k)$.

By conditions of theorem, there exists an $L>0$, such that:

$\|q(t,x(t,t_0,\psi))\|<L,\ \forall t\in[t_0,t_k)$.

If $\lim_{t\to+\infty}\| q(t,x(t,t_0,\psi))-q(t,x_u(t,t_0,\varphi))\|=0$ is not satisfied, then there is a $\varepsilon_0\in(0,1)$ and a sequence $\{t_m\}$, $\lim_{m\to+\infty} t_m=+\infty$, such that

$\| q(t_m,x(t_m,t_0,\psi))-q(t_m,x_u(t_m,t_0,\varphi))\|>\varepsilon_0$.

We can suppose that $\forall m$, $t_m-t_{m-1}\ge 2$. Take $\delta_1\in(0,1)$, such that $L\delta_1<\frac{\varepsilon_0}{2}$, then $\forall m$, $\forall t\in[t_m-\delta_1,t_m+\delta_1]$,

$$\begin{aligned}
&\| q(t,x(t,t_0,\psi))\|\\
&\ge\| q(t_m,x(t_m,t_0,\psi))-q(t_m,x_u(t_m,t_0,\varphi))\|\\
&\quad-\| q(t_m,x(t_m,t_0,\psi))-q(t_m,x_u(t_m,t_0,\varphi))\\
&\quad-q(t,x(t,t_0,\psi))\|\\
&\ge\varepsilon_0-L\delta\ge\frac{\varepsilon_0}{2}.
\end{aligned}$$

Thus

$\dot V(t,q(t,x(t,t_0,\psi)))\le -w(\|q(t,x(t))\|)\le -w(\frac{\varepsilon_0}{2})$.

Hence there exists an m_0 big enough, such that when $t>t_{m_0}$,

$$\begin{aligned}
&V(t,q(t,x(t,t_0,\psi)))\\
&\le V(t_0,\psi)-2\sum_{m=1}^{m_0} w(\frac{\varepsilon_0}{2})\delta_1\\
&=V(t_0,\psi)-2m_0w(\frac{\varepsilon_0}{2})\delta_1<0,
\end{aligned}$$

which is in contradiction with the definition of V. Thus

$\lim_{t\to+\infty}\| q(t,x(t,t_0,\psi))-q(t,x_u(t,t_0,\varphi))\|=0$,

and the solution of (1) is asymptomatically stable in $\{q(t,x(t)),[0,+\infty)\}$. The proof is completed.

Now that we discussed the stability of solutions for nonlinear singular systems with delay by using Theorem 1

$$\begin{cases}
\dot X_1(t)=B_{11}X_1(t)+B_{12}X_2(t)\\
\qquad+C_{11}X_1(t-\tau)C_{12}X_2(t-\tau)+f_1(t-\tau)\\
\quad 0=B_{21}X_1(t)+X_2(t)+C_{21}X_1(t-\tau)\\
\qquad+C_{22}X_2(t-\tau)+f_2(t-\tau)\\
\quad x_{t_0}=\varphi
\end{cases}\tag{2}$$

Where $X_1\in R^{n_1}$, $X_2\in R^{n_2}$, $n_1+n_2=n$; B_{ij}, C_{ij} are $n_i\times n_j$ constant matrices. $f_i(t)\in C([0,t_k),R^{n_i})$, $i,j=1,2$. We can take $t_k=+\infty$; τ as a positive constant, $x=(X_1^T,X_2^T)^T=(x_1,x_2,...,x_n)^T$. $x_{t_0}=\varphi$ is the initial condition of system (2) , where $\varphi\in C([-\tau,0],R^n)$ and $t_0\ge 0$. If the initial function φ satisfies the following consistency condition, (2) has a unique continuous solution in $[t_0-\tau,+\infty)$ through (t_0,φ),

$$\begin{aligned}
0=&B_{21}\varphi_1(0)+\varphi_2(0)+C_{21}\varphi_1(-\tau)\\
&+C_{22}\varphi_2(-\tau)+f_2(t_0-\tau).
\end{aligned}\tag{3}$$

Thus for $\forall t_0\in[0,+\infty)$,

$S_k(t_0,t_k)=\{\psi\in C([-\tau,0],R^n);\varphi \text{ satisfies } (3)\}$.

Let $\varphi\in S_k(t_0,t_k)$, the solution of initial value problems (2) can be written as:

$x_u(t,t_0,\varphi)=(X_{u1}^T(t),X_{u2}^T(t))^T$.

Then, we have the following theorem.

Theorem 3: For initial value problems (2), if matrix

$$\begin{pmatrix} U & C_{11}-B_{12}C_{21} & C_{12}-B_{12}C_{22} \\ (C_{11}-B_{12}C_{21})^T & 0 & 0 \\ (C_{12}-B_{12}C_{22})^T & 0 & 0 \end{pmatrix}$$

is semi-negative definite, where

$U=(B_{11}-B_{12}B_{21})^T+B_{11}-B_{12}B_{21}$.

Then $x_u(t,t_0,\varphi)$ is uniformly stable in $\{(X_1-X_{u1})^T(X_1-X_{u1}),[0,t_k)\}$. Furthermore if $\|C_{22}\|<1$ then $x_u(t,t_0,\varphi)$ is uniformly stable in $\{x(t)-x_u(t),[0,t_k)\}$, where t_k can be $+\infty$.

Proof Set

$q(t,x)=(X_1-X_{u1})^T(X_1-X_{u1})$,

$V(t,q(t,x))=q(t,x)$.

Let wedge function

$\Phi_1(s)=s$, $\Phi_2(s)=s^2$, $s\in[0,+\infty)$.

Then

$$\begin{aligned}&\Phi_1(\|q(t,x)-q(t,x_u(t))\|)\\&=\|q(t,x)\|=(X_1-X_{u1})^T(X_1-X_{u1})\\&=V(t,q(t,x))=\Phi_2(\|X_1-X_{u1}(t)\|).\end{aligned}\tag{4}$$

Because of

$\|x-x_u(t)\|^2=\|X_1-X_{u1}(t)\|^2+\|X_2-X_{u2}(t)\|^2$,

we have

$$\begin{aligned}&\Phi_1(\|q(t,x)-q(t,x_u(t))\|)\\&=\Phi_2(\|X_1-X_{u1}(t)\|)\le\Phi_2(\|x-x_u(t)\|).\end{aligned}\tag{5}$$

For $\forall\psi\in S_k(t_0,t_k)$, we can get that

$$\begin{aligned}&\dot V(t,q(t,x(t,t_0,\psi)))\\&=(\dot X_1(t)-\dot X_{u1}(t))^T(X_1(t)-X_{u1}(t))\\&\quad+(X_1(t)-X_{u1}(t))^T(\dot X_1(t)-\dot X_{u1}(t))\\&=[B_{11}X_1(t)+B_{12}X_2(t)+C_{11}X_1(t-\tau)\\&\quad+C_{12}X_2(t-\tau)-B_{11}X_{u1}(t)-B_{12}X_{u2}(t)\\&\quad-C_{11}X_{u1}(t-\tau)-C_{12}X_{u2}(t-\tau)]^T(X_1(t)-X_{u1}(t))\\&\quad+(X_1(t)-X_{u1}(t))^T[B_{11}X_1(t)+B_{12}X_2(t)\\&\quad+C_{11}X_1(t-\tau)+C_{12}X_2(t-\tau)\\&\quad-B_{11}X_{u1}(t)-B_{12}X_{u2}(t)-C_{11}X_{u1}(t-\tau)-C_{12}X_{u2}(t-\tau)].\end{aligned}\tag{6}$$

From the second equation of system (2) we have

$$\begin{aligned}&X_2(t)-X_{u2}(t)\\&=-B_{21}(X_1(t)-X_{u1}(t))-C_{21}(X_1(t-\tau)-X_{u1}(t-\tau))\\&\quad-C_{22}(X_2(t-\tau)-X_{u2}(t-\tau)).\end{aligned}\tag{7}$$

Then (6) can be written as

$$\begin{aligned}&\dot V(t,q(t,x(t,t_0,\psi)))\\&=D\begin{pmatrix} U & C_{11}-B_{12}C_{21} & C_{12}-B_{12}C_{22} \\ (C_{11}-B_{12}C_{21})^T & 0 & 0 \\ (C_{12}-B_{12}C_{22})^T & 0 & 0 \end{pmatrix}D^T\\&=DED^T,\end{aligned}$$

where

$U=(B_{11}-B_{12}B_{21})^T+B_{11}-B_{12}B_{21}$,

and

$$\begin{aligned}D=[&(X_1(t)-X_{u1}(t))^T,(X_1(t-\tau)-X_{u1}(t-\tau))^T,\\&(X_2(t-\tau)-X_{u2}(t-\tau))^T].\end{aligned}$$

If E is semi-negative definite, we get

$\dot V(t,q(t,x(t,t_0,\psi)))\le 0$.

from Theorem 1, $x_u(t,t_0,\varphi)$ is uniformly stable in $\{(X_1-X_{u1})^T(X_1-X_{u1}),[0,t_k)\}$. By the second equation of system (2),

$$\begin{aligned}&\|X_2(t)-X_{u2}(t)\|\\&=\|-B_{21}(X_1(t)-X_{u1}(t))\\&\quad-C_{21}(X_1(t-\tau)-X_{u1}(t-\tau))\\&\quad-C_{22}(X_2(t-\tau)-X_{u2}(t-\tau))\|\\&\le\|B_{21}\|\ \|X_1(t)-X_{u1}(t)\|\\&\quad+\|C_{21}\|\ \|X_1(t-\tau)-X_{u1}(t-\tau)\|\\&\quad+\|C_{22}\|\ \|X_2(t-\tau)-X_{u2}(t-\tau)\|\\&=\|B_{21}\|\sqrt{q(t,x(t))}+\|C_{21}\|\sqrt{q((t-\tau),x(t-\tau))}\\&\quad+\|C_{22}\|\ \|X_2(t-\tau)-X_{u2}(t-\tau)\|.\end{aligned}\tag{8}$$

Since $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$, therefore $\forall\varepsilon>0$, there exists $\delta(\varepsilon)>0$. We can take $\delta<\varepsilon$, such that for $\forall t_0\in T_k$, $\forall\psi\in B(\varphi,\delta)\cap S_k(t_0,t_k)$ and $\forall t\in[t_0,t_k)$, we have

$$\begin{aligned}&\|B_{21}\|\sqrt{q(t,x(t))}+\|C_{21}\|\sqrt{q((t-\tau),x(t-\tau))}\\&=\|B_{21}\|\sqrt{\|q(t,x(t))-q(t,x_u(t))\|}\\&\quad+\|C_{21}\|\sqrt{\|q((t-\tau),x(t-\tau))-q((t-\tau),x_u(t-\tau))\|}\\&<(\|B_{21}\|+\|C_{21}\|)\sqrt{\varepsilon},\end{aligned}$$

Where $x(t)=x(t,t_0,\psi)$. Because ε is arbitrary and B_{ij}, C_{ij} are constant matrices, we can conclude:

$$\|B_{21}\|\sqrt{q(t,x(t))}+\|C_{21}\|\sqrt{q((t-\tau),x(t-\tau))}<\varepsilon. \tag{9}$$

And we also have

$$\|X_2(t-\tau)-X_{u2}(t-\tau)\|\le\|\psi-\varphi\|<\delta<\varepsilon,$$
$$\forall t\in[t_0,t_0+\tau)\cap[t_0,t_k).$$

So from (8) and (9), we have

$$\|X_2(t)-X_{u2}(t)\|\le\varepsilon+\|C_{22}\|\varepsilon.$$

Suppose that for $\forall t\in[t_0+(l-1)\tau,t_0+l\tau]\cap[t_0,t_k)$, then

$$\|X_2(t)-X_{u2}(t)\|\le\varepsilon+\|C_{22}\|\varepsilon+...+\|C_{22}\|^l\varepsilon.$$

Thus when

$$t\in[t_0+l\tau,t_0+(l+1)\tau]\cap[t_0,t_k),$$

and

$$t-\tau\in[t_0+(l-1)\tau,t_0+l\tau]\cap[t_0,t_k),$$

by inductive supposition and (8), (9), we get that:

$$\begin{aligned}&\|X_2(t)-X_{u2}(t)\|\\&\le\varepsilon+\|C_{22}\|\ \|X_2(t-\tau)-X_{u2}(t-\tau)\|\\&\le\varepsilon+\|C_{22}\|\varepsilon+...+\|C_{22}\|^{l-1}\varepsilon.\end{aligned}$$

If $\|C_{22}\|<1$, from induction, $\forall t\in[t_0,t_k)$

$$\begin{aligned}&\|X_2(t)-X_{u2}(t)\|\\&\le\varepsilon+\|C_{22}\|\varepsilon+...+\|C_{22}\|^l\varepsilon+... \qquad (10)\\&\le\frac{\varepsilon}{1-\|C_{22}\|}.\end{aligned}$$

Thus

$$\begin{aligned}&\|x(t)-x_u(t)\|\\&\le\|X_1(t)-X_{u1}(t)\|+\|X_2(t)-X_{u2}(t)\| \qquad (11)\\&=\sqrt{\|q(t,x(t))-q(t,x_u(t))\|}+\|X_2(t)-X_{u2}(t)\|.\end{aligned}$$

from (10), (11) and $x_u(t,t_0,\varphi)$ is uniformly stable in $\{q(t,x),T_k\}$ and

$$\|x(t)-x_u(t)\|\le\varepsilon+\frac{\varepsilon}{1-\|C_{22}\|}.$$

Therefore, we see that if $\|C_{22}\|<1$, $x_u(t,t_0,\varphi)$ is uniformly stable in $\{x,T_k\}$. The proof is completed.

EXAMPLE

Example 3.3 Let us discuss the following nonlinear singular system with delay

$$\begin{cases}\dot{X}_1(t)=\begin{pmatrix}-1&0\\0&1\end{pmatrix}X_1(t)+\begin{pmatrix}1&0\\0&2\end{pmatrix}X_2(t)\\ \qquad+\begin{pmatrix}1&1\\0&2\end{pmatrix}X_1(t-\tau)\begin{pmatrix}1&0\\0&1\end{pmatrix}X_2(t-\tau)+3(t-\tau)\\ 0=\begin{pmatrix}1&0\\0&1\end{pmatrix}X_1(t)+X_2(t)+\begin{pmatrix}1&1\\0&1\end{pmatrix}X_1(t-\tau)\\ \qquad+\begin{pmatrix}1&0\\0&\frac{1}{2}\end{pmatrix}X_2(t-\tau)+(t-\tau)^3\end{cases} \tag{12}$$

The initial condition of system (10) is

$$x_{t_0}=\theta,\quad \theta\in C([-\tau,0],R^6),\quad t_0\ge0. \tag{13}$$

Where

$$B_{11}=\begin{pmatrix}-1&0\\0&1\end{pmatrix},\ B_{12}=\begin{pmatrix}1&0\\0&2\end{pmatrix},\ B_{21}=\begin{pmatrix}1&0\\0&1\end{pmatrix},\ C_{11}=\begin{pmatrix}1&1\\0&2\end{pmatrix},$$
$$C_{12}=\begin{pmatrix}1&0\\0&1\end{pmatrix},C_{21}=\begin{pmatrix}1&1\\0&1\end{pmatrix},\ C_{22}=\begin{pmatrix}1&0\\0&\frac{1}{2}\end{pmatrix}.$$

the matrix

$$\begin{pmatrix}U&C_{11}-B_{12}C_{21}&C_{12}-B_{12}C_{22}\\(C_{11}-B_{12}C_{21})^T&0&0\\(C_{12}-B_{12}C_{22})^T&0&0\end{pmatrix}$$
$$=\begin{pmatrix}-4&0&0&0&0&0\\0&-2&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}$$

is semi-negative definite, and $\|C_{22}\|=\frac{1}{2}<1$. By Theorem 3, $x_u(t,t_0,\theta)$ is uniformly stable in $\{x(t)-x_u(t),[0,+\infty)\}$.

CONCLUSION

It is difficult to discuss the stability of nonlinear singular systems with delay, because of impulse and delay. In this paper, the stability is considered by using new definitions and Lyapunov function. Some stable criteria are proposed. These criteria are algebraic that's why they are convenient to use. Using these criteria the stable problem for a class of nonlinear singular systems with delay can be solved easily. Moreover, further studies on stability of solution for nonlinear singular systems with delay will be summarized in our next study.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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