

Target Tracking Algorithm Based on Improved Unscented Kalman Filter

Wang Yingyan and Zeng Rui*

School of Electro-Mechanical and Information Technology, Yi Wu Industrial & Commercial College, Yi Wu, Zhejiang, 322000, P.R. China

Abstract: In order to improve the performance of target tracking and solve the defects of unscented Kalman filter, a target tracking algorithm based on improved unscented Kalman filter is proposed in this paper. Firstly, the fading factor is introduced into the filter based on strong tracking filter to avoid the filter divergence, and then wavelet transform is used to estimate the statistical characteristics of measurement noise to improve unscented Kalman filter tracking ability, finally the simulation experiment is used to test the performance of algorithm. The results show that the proposed algorithm increases adaptive ability of target tracking, and obtain good performance for weak maneuvering and non maneuvering target tracking, and fastens the tracking speed.

Keywords: Strong tracking filter, target tracking, unscented Kalman filter, wavelet transform.

1. INTRODUCTION

Target tracking refers to the use of the sensor measurement information, through the establishment of precise target model. The target motion estimation and forecasting accurately, have important applications in military, security, transportation, medical care and so on fields [1].

In view of the target tracking problem, many researchers have done a lot of work in recent years, and have achieved good results, along with many of the target tracking algorithm [2]. Target tracking is mainly including motor model and the filter algorithm; particle filter that has a good non-linear prediction performance, has also been applied in target tracking [3]. For target tracking, the particle filter is closely related to the resampling strategy, using transition prior probability density as the important density function, and the accuracy of measurement at occasions with low requirements to obtain ideal tracking results, but if the likelihood function at the end of the system state transition probability density, the deviation is more noticeable [4]. Kalman Filter algorithm is a kind of linear optimal estimation under Gaussian noise Filter algorithm [5]. But Kalman filter algorithm under the nonlinear non-Gaussian noise filter effect is not ideal or even divergence [6]. In order to better adapt to the target tracking in nonlinear environments, some scholars put forward the extended Kalman filter, the nonlinear function of the first order Taylor expansion, and assume the system noise and observation noise as Gaussian approximation. But, when the system is highly nonlinear and non-Gaussian, EKF algorithm is easy to lead to filter divergence, and EKF must calculate Jacobian matrix, increasing the difficulty to solve the problem [7]. In order to solve the problem, scholars put forward the unscented Kalman filter (UKF), using a group of parameters to determine the weights of sampling points to approximate

the nonlinear distribution to solve the non-linear problem, it does not need to linearize the nonlinear system, and has a superior performance to the EKF, it has been successfully applied in many areas [8]. UKF need to know the target of the mathematical model and the prior knowledge of the statistics of the noise when the system model parameters change; it will be out of the low accuracy of filter divergence or filter is insufficient, affecting the tracking performance in practical application [9]. Cubature Kalman filter algorithm (CKF) is presented in Ref. [10], which requires the use of integral criterion of radial spherical numerical integral method of calculation of nonlinear transformations of random variables, the mean and covariance, with simple realization and high filter precision [11]. In the practical application, the target is likely to be mutated, and the CKF algorithm is poorly adaptive estimation of the mutation state when the system is stationary [12].

In order to obtain high accuracy tracking results, in view of the current target tracking algorithms where there exist deficiencies, an improved unscented Kalman filter (IUKF) algorithm was proposed, and the performance of the experimental simulation of the IUKF is analyzed. The simulation results show that IUKF algorithm, the relative UKF algorithm, not only improves the accuracy of target tracking, and speeds up the target tracking, but also meets the demand of the real-time target tracking.

2. MATHEMATICAL MODEL OF TARGET MOTION

Set target motion equation of state is:

$$X(k+1) = F(k)X(k) + G(k)\bar{a} + W(k) \quad (1)$$

In the formula, $F(k)$ is the transition matrix, $G(k)$ is the input gating matrix, namely:

$$F(k) = \begin{bmatrix} 1 & T & (\alpha T - 1 + e^{-\alpha T}) / \alpha^2 \\ 0 & 1 & (1 - e^{-\alpha T}) / \alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix} \quad (2)$$

*Address correspondence to this author at the School of Electro-mechanical and Information Technology, Yi Wu Industrial & Commercial College, Yi Wu, Zhejiang, 322000, P.R. China; Tel: 0579-83803522; Fax: 0579-83803518; E-mail: jamse007@126.com

$$G(k) = \begin{bmatrix} (-T + \alpha T^2 / 2 + (1 - e^{-\alpha T}) / \alpha) / \alpha \\ T - (1 - e^{-\alpha T}) / \alpha \\ 1 - e^{-\alpha T} \end{bmatrix} \quad (3)$$

W (k) is Gaussian white noise, and its covariance is:

$$Q(k) = E[WW'] = 2\alpha\sigma_a^2 \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12} & q_{22} & q_{23} \\ q_{13} & q_{23} & q_{33} \end{bmatrix} \quad (4)$$

In the formula, α is the autocorrelation time-constant reciprocal, T is the sampling period, σ_a^2 is the mobile acceleration variance, and $\bar{a}(k)$ is the mobile acceleration average value, namely:

$$\sigma_a^2 = \begin{cases} (4 - \pi) / \pi \cdot [a_{\max} - \bar{a}(k)]^2, \bar{a}(k) > 0 \\ (4 - \pi) / \pi \cdot [a_{-\max} + \bar{a}(k)]^2, \bar{a}(k) < 0 \end{cases} \quad (5)$$

$$\bar{a}(k) = \hat{x}(k | k - 1) \quad (6)$$

In the formula, a_{\max} , $a_{-\max}$ are the target positive and the negative maximum acceleration [12].

3. IMPROVED KALMAN FILTER ALGORITHM

3.1. Unscented Kalman Filter Algorithm

Unscented Kalman filter (UKF) is a deterministic method of particle filter, based on unscented transform (UT), using Kalman filter framework and sampling particle approximation of nonlinear distribution. UT doesn't need nonlinear state and observation models in a linearized processing. Through selecting a series of symmetric sampling points, σ , for approximation of the probability density function of the state vector, true mean and covariance of the Gaussian probability density function can be reflected [12].

Assuming the K state estimation vector and the state estimation error covariance are $\hat{X}(k | k)$ and $P(k | k)$ respectively, the UKF algorithm is as follows:

- (1) In accordance with unscented transform computing σ sampling point $\chi_i(k | k)$ and weight w_i ($i=0, 1, 2, \dots, 2n$)

$$\begin{cases} \chi_0(k | k) = \hat{X}(k | k) \\ \chi_i(k | k) = \hat{X}(k | k) + (\sqrt{(n + \lambda)P(k | k)})_i \\ \chi_{i+n}(k | k) = \hat{X}(k | k) - (\sqrt{(n + \lambda)P(k | k)})_i \end{cases} \quad (7)$$

- (2) The calculation of the state forecast and the state forecast error covariance is:

$$\begin{aligned} \chi_i(k + 1 | k) &= f(\chi_i(k | k), k) \\ i &= 0, 1, 2, \dots, 2n \end{aligned} \quad (8)$$

$$\hat{X}(k + 1 | k) = \sum_{i=0}^{2n} w_i^m \chi_i(k + 1 | k) \quad (9)$$

$$\begin{aligned} P_x(k + 1 | k) &= \sum_{i=0}^{2n} w_i^c [\chi_i(k + 1 | k) - \hat{X}(k + 1 | k)] \\ &[\chi_i(k + 1 | k) - \hat{X}(k + 1 | k)]^T + Q(k) \end{aligned} \quad (10)$$

- (3) Observations predicted estimates, the prediction error covariance and interactions with the state vector covariance observation vector are:

$$\begin{aligned} \zeta_i(k + 1 | k) &= h(\chi_i(k + 1 | k), k + 1) \\ (i &= 0, 1, 2, \dots, 2n) \end{aligned} \quad (11)$$

$$\hat{Z}(k + 1 | k) = \sum_{i=0}^{2n} w_i^m \zeta_i(k + 1 | k) \quad (12)$$

$$\begin{aligned} P_z(k + 1 | k) &= \sum_{i=0}^{2n} w_i^c [\zeta_i(k + 1 | k) - \hat{Z}(k + 1 | k)] \\ &[\zeta_i(k + 1 | k) - \hat{Z}(k + 1 | k)]^T + R(k + 1) \end{aligned} \quad (13)$$

$$\begin{aligned} P_{xz}(k + 1 | k) &= \sum_{i=0}^{2n} w_i^c [\chi_i(k + 1 | k) - \hat{X}(k + 1 | k)] \\ &[\zeta_i(k + 1 | k) - \hat{Z}(k + 1 | k)]^T \end{aligned} \quad (14)$$

- (4) The covariance of the gain matrix, the updated state estimation vector and the state estimation error are:

$$K(k + 1) = P_{xz}(k + 1 | k) P_z^{-1}(k + 1 | k) \quad (15)$$

$$\begin{aligned} \hat{X}(k + 1 | k + 1) &= \hat{X}(k + 1 | k) + K(k + 1) \\ &[Z(k + 1) - \hat{Z}(k + 1 | k)] \end{aligned} \quad (16)$$

$$\begin{aligned} P(k + 1 | k + 1) &= P_x(k + 1 | k) - \\ &K(k + 1) P_z(k + 1 | k) K^T(k + 1) \end{aligned} \quad (17)$$

3.2. Improvement of Unscented Kalman Filter Algorithm

Although UKF has good filter characteristics, yet it still can show the phenomenon of filter divergence, *i.e.* filter precision is not high, when the system noise and measurement noise statistical characteristics are unknown or inaccurate, and model parameters are changed [13].

Therefore, this article on UKF algorithm has presented an improved unscented Kalman filter algorithm (IUKF), improved tracking performance. IUKF is based on: the Strong tracking filter (STF) thoughts; the fading factor is introduced into the UKF algorithm step prediction covariance matrix; reduce the data to the negative impact of the current filter value adaptive adjustment of gain matrix; achieve the purpose of stable tracking; and wavelet transform is used to estimate the statistical properties of the measurement noise tracking ability of the UKF.

3.2.1. Strong Filter

As far as state of mutations in the process of filter is concerned, the data used for the current filter values influence is bigger, so often it cannot well respond to the current state estimation, therefore it is difficult to realize the

adaptive tracking. Strong tracking filter algorithm based on output residuals orthogonality principle, with the introduction of fading factor, will be included in the residual sequence of effective information extracted, real-time adjustment of gain matrix; it has the ability of adaptive tracking state changes [14]. Strong tracking filter will be introduced to the fading factor step prediction covariance matrix, to reduce the data for the current filter; the fading factor λ_{k+1} calculation method is as follows:

$$\lambda_{k+1} = \begin{cases} \lambda_0, \lambda_0 > 1 \\ 1, \lambda_0 \leq 1 \end{cases} \quad (18)$$

Among

$$\lambda_0 = \frac{\text{tr}[N_{k+1}]}{\text{tr}[M_{k+1}]} \quad (19)$$

In the formula, $\text{tr}[\cdot]$ indicates the trace of the matrix, and is:

$$N_{k+1} = V_{0,k+1} - R_{k+1} - H_{k+1} Q_{k+1} H_{k+1}^T \quad (20)$$

$$M_{k+1} = H_{k+1} \Phi P_{k+1} \Phi^T H_{k+1}^T \quad (21)$$

In the formula, $V_{0,k+1}$ is the covariance matrix of the residuals, and

$$V_{0,k+1} = E[v_{k+1} v_{k+1}^T] = \begin{cases} v_1 v_1^T & k = 0 \\ \frac{\rho V_{0,k} + v_{k+1} v_{k+1}^T}{1 + \rho} & k \geq 1 \end{cases} \quad (22)$$

In the formula, ρ is the forgetting factor, v_{k+1} is the filter residuals, $v_{k+1} = z_{k+1} - \hat{z}_{k+1|k}$.

3.2.2. Wavelet Estimation for Measurement Noise

When the amount of the system is not accurate, the filter performance will deteriorate seriously. In order to reduce the harmful effect of the inaccurate measurement noise to the UKF, UKF algorithm is used for measurement of noise by wavelet estimation. The noise containing signal $y(k)$ can be decomposed into the real signal $h(k)$ and the measurement noise $d(k)$.

$$y(k) = h(k) + d(k) \quad (23)$$

$\varphi(k)$ is a wavelet function, and its scale and time shift is:

$$\varphi_{s,\tau}(k) = \frac{1}{\sqrt{s}} \varphi\left(\frac{k-\tau}{s}\right) \quad (24)$$

Contains noise measurement signal $y(k)$ of wavelet transform is:

$$W_y(s, \tau) = y(k) * \varphi_{s,\tau}(k) = W_h(s, \tau) + W_d(s, \tau) \quad (25)$$

In the formula, "*" indicates convolution.

According to the Weierstrass approximation theorem, to measure the signal $y(k)$ in a measurement interval, a lower order polynomial or a fragment of low-order polynomial approximation to arbitrary accuracy can be used; the polynomial is [15]:

$$y(k) = \sum_{i=0}^M a_i k^i \quad (26)$$

If $\varphi(k)$ vanishing matrix is $p(p > M)$, $y(k)$ wavelet transform suppresses the signal and noise components are retained:

$$W_y(s, \tau) = W_d(s, \tau) \quad (27)$$

The standard deviation of the noise can be estimated by the absolute value of the wavelet coefficient:

$$\sigma \approx \frac{1}{0.6745} \text{Med}(|W_y(s, t_h)|) \quad (28)$$

In the formula, S scale is 0.5; t_h is in the details of the scale of discrete τ .

In measuring unknown noise, first select the length L of a signal, then the measurement sequence of wavelet transform, according to type (28) to estimate the standard deviation of the noise by using unscented Kalman filter (UKF) for state estimation.

4. SIMULATION EXPERIMENT

4.1. Simulation Environment

In the computer of Intel 4 core CPU 4GB, RAM 2.8GHZ, XP Windows operating system, the simulation experiment is realized by VC++ programming and compared with the UKF algorithm.

The target is located in the two-dimensional plane, the maneuvering frequency is 0.05, the maximum maneuvering acceleration is $\alpha_{\max} = 120 \text{m/s}^2$, and the simulation time is 100 times. The target trajectories are shown in Fig. (1). In order to overcome the influence of the initial state to the filter accuracy, the initial state of the filter is assumed:

$$x_0 = [x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}] = [-5750, 100, 0, -800, 0, 0]^T, \text{ system noise } w_k \text{ and process noise } v_k \text{ in line with the Gaussian distribution, } w_k \in N(0, Q_k), v_k \in N(0, R_k) \text{ initial state estimation is } \hat{x}_{00} = x_0, \text{ and the initial covariance matrix is: } P_{00} = \text{diag}\{4000, 900, 4, 4000, 900, 4\}.$$

4.2. Results and Analysis

4.2.1. Tracking Comparison (Simulation)

In order to test the performance of the UKF and IUKF algorithms, take the average 100 independent simulations. UKF and IUKF tracking trajectories are shown in Fig. (2). Fig. (2) shows that, compared with the UKF algorithm, IUKF tracking error is smaller. In non-maneuvering targets and weak maneuvering, IUKF maintained good tracking performance, especially in a mutation in the moment and tracking stability and faster convergence speed.

4.2.2. Tracking Comparison (Qualitative)

To quantitatively compare the performance of the filter algorithm, the definition of RMS error (RMSE) is:

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=0}^N (x_k - \hat{x}_{k|k})^2} \tag{29}$$

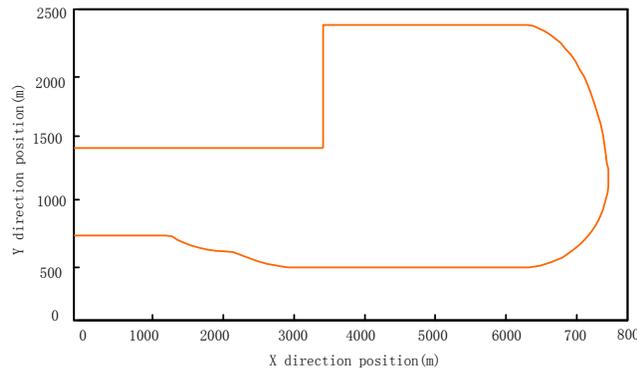


Fig. (1). Target trajectory.

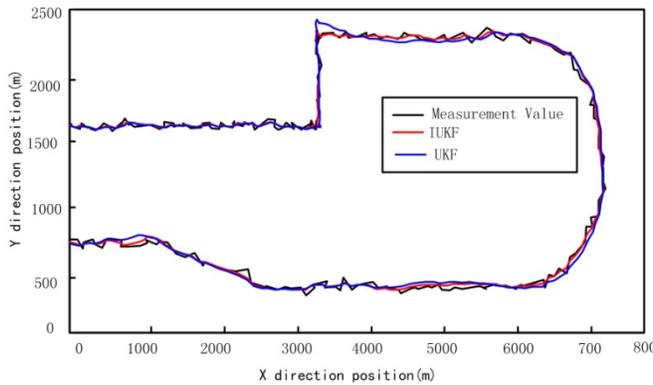


Fig. (2). Comparison of tracking trajectories of UKF and IUKF algorithm.

Among them, $\hat{x}_{k|k}$ is the filter estimate of x_k .

UKF and IUKF algorithms are used to estimate the direction of the axis of the state estimation mean, RMSE as shown in Fig. (3). x , y , \dot{x} and \dot{y} axis of the RMSE are shown in Table 1. Fig. (3) and Table 1 show that, compared with the UKF algorithm, in IUKF for each coordinate direction, filter error is greatly reduced, especially for the step of maneuvering target position and velocity. The UKF algorithm, with the strongest tracking ability, can adaptively track the state in the larger changes.

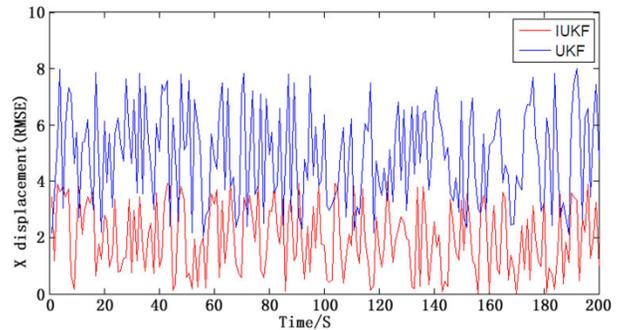
4.2.3. Tracking Speed Comparison

Tracking speed is one of the important indexes to measure the performance of target tracking algorithm, and the tracking speed is evaluated by operation time. UKF and IUKF computing time is shown in Fig. (4). Fig. (4) shows that, relative to the UKF, IUKF's operation is relatively small, increase the tracking speed, can satisfy the real-time target tracking, and the value is higher.

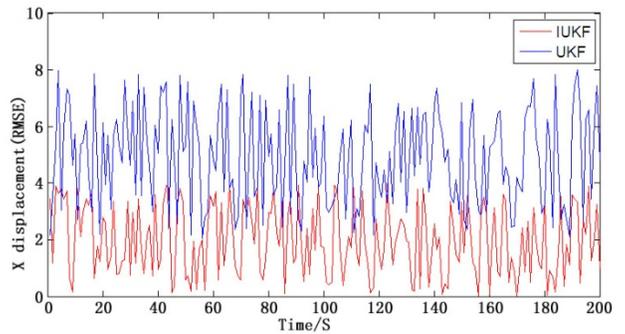
CONCLUSION

Nonlinear target tracking is advisable in order to obtain a more accurate result, UKF deficiencies led to an improved Unscented Kalman filter (IUKF) algorithm for target tracking. First model the strong tracking filter, followed by the fading

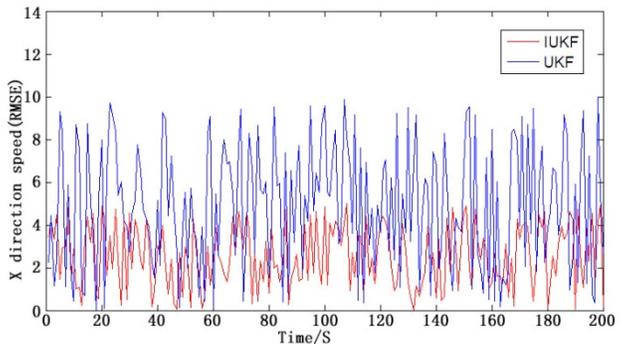
into the filter process, avoiding divergence that occurred, then measurement of noise statistical properties of wavelet and improved tracking of Unscented Kalman filter and simulation experiments show that the IUKF algorithm significantly outperforms the UKF algorithm in target tracking accuracy and robustness, and has a wider range of applications.



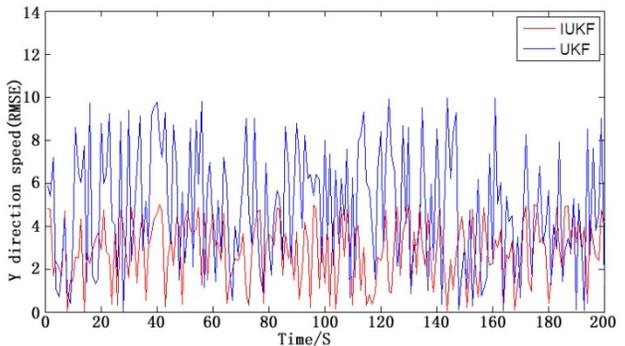
(a) X displacement RMSE



(b) Y displacement RMSE



(c) X direction speed RMSE



(d) Y direction speed RMSE

Fig. (3). Each direction axis RMSE.

Table 1. State reckon mean square error.

Filter Algorithm	RMSE Mean Value			
	x	\dot{x}	y	\dot{y}
UKF	5.3467	6.7164	5.7390	7.5485
IUKF	3.8435	4.9594	3.9612	4.9682

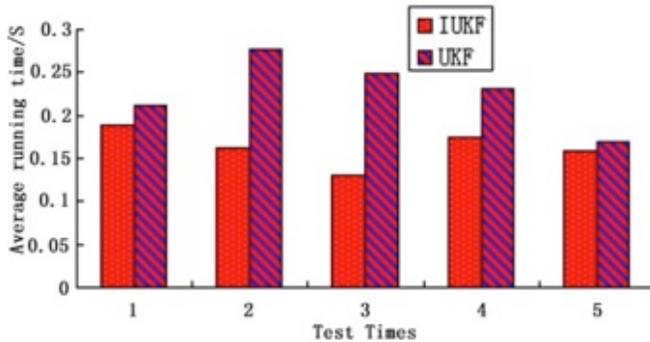


Fig. (4). Running time comparison between IUKF and UKF.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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REFERENCES

- [1] H.Q. Fan, S. Wang, and Q. Fu, "Overview of target mobile detection algorithm," *Systems Engineering and Electronic Technology*, vol. 31, no. 5, pp. 1064-1070, 2009.
- [2] Y. Zhou, J.X. Li, and D.L. Wang, "Target tracking in wireless sensor networks using adaptive measurement quantization," *Science China Information Sciences*, vol. 55, no. 4, pp. 827-838, 2012.
- [3] F.W. Yang, Y.M. Li, and X.H. Liu, "Robust error square constrained filter design for systems with non-Gaussian noises," *IEEE Signal Processing Letters*, vol. 15, pp. 930-933, 2008.
- [4] M.Y. Li, G.Y. Shi, Y.Q. Wen, "Improved high-mobility Jerk model tracking algorithm," *Electric and Control*, vol. 20, no. 5, pp. 40-43, 2013.
- [5] W.S. Liu, Y.A. Li, and L. Cui, "Based on the current statistical model of strong adaptive maneuvering target tracking algorithm," *Journal of Systems Engineering and Electronics*, vol. 9, pp. 1937-1940, 2011.
- [6] B.L. Guan, X.F. Tang, and X.L. Xu, "Noise related to sensor network bandwidth constraint fusion algorithm," *Journal of Henan University*, vol. 43, no. 2, pp. 200-203, 2013.
- [7] Y.B. Kong, X.X. Feng, and C.G. Lu, "Based on the improved Gaussian mixture particle filter in bearings only target tracking algorithm," *Journal of Astronautics*, vol. 33, no. 7, pp. 971-976, 2012.
- [8] J. Xu, J.X. Li, and S. Xu, "Data fusion for target tracking in wireless sensor networks using quantized innovations and Kalman filter," *Science China Information Sciences*, vol. 55, no. 3, pp. 530-544, 2012.
- [9] J.W. Wang, H.T. Shui, and H.X. Ma, "The objective function of optimal kalman filter algorithm selection," *Journal of Systems Engineering and Electronics*, vol. 31, no. 1, pp. 200-201, 2009.
- [10] G. Huo, D.H. Li, and J. Li, "Based on strong tracking kalman filter volume of single station passive track algorithm," *Journal of Modern Radar*, vol. 35, no. 11, pp. 52-57, 2013.
- [11] C.L. Wu, and C.Z. Han, "Quadrature quadrature kalman filter," *Electronic Journal*, vol. 37, no. 5, pp. 987-992, 2009.
- [12] W.J. Zou, and H.M. Bo, "Difference linearization EKF filter method research," *Computer Engineering and Application*, vol. 25, no. 9, pp. 64-66, 2009.
- [13] B.F. Wang, and G. Guo, "Kalman filter with partial Markovian packet losses," *International Journal of Automation and Computing*, vol. 6, no. 4, pp. 395-399, 2009.
- [14] X.X. Wang, Z. Lin, Q.X. Xia, and Y. Hao, "Unscented based strong track filter," *Control and Decision*, vol. 25, no. 7, pp. 1063-1068, 2010.
- [15] Y. Gao, and J.Q. Zhang, "Kalman filter algorithm for estimating the observed noise variance and its application in data fusion," *Electronic Journal*, vol. 35, no. 1, pp. 108-111, 2007.

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