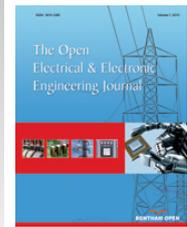




# The Open Electrical & Electronic Engineering Journal

Content list available at: [www.benthamopen.com/TOEEJ/](http://www.benthamopen.com/TOEEJ/)

DOI: 10.2174/1874129001610010141



## RESEARCH ARTICLE

# A Novel Positioning Algorithm Based on Self-adaptive Algorithm of RBF Network

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Received: February 19, 2016

Revised: July 14, 2016

Accepted: July 22, 2016

**Abstract:** Much attention has been paid to Taylor series expansion (TSE) method these years, which has been extensively used for solving nonlinear equations for its good robustness and accuracy of positioning. A Taylor-series expansion location algorithm based on the RBF neural network (RBF-TSE) is proposed before to the performance of TSE highly depends on the initial estimation. In order to have more accurate and lower cost, a new Taylor-series expansion location algorithm based on Self-adaptive RBF neural network (SA-RBF-TSE) is proposed to estimate the initial value. The proposed algorithm is analysed and simulated with several other algorithms in this paper.

**Keywords:** Angle-of-arrival (AOA), Assisted GPS (A-GPS), Position location, Self-adaptive RBF, Taylor series expansion, Time-difference-of-arrival (TDOA), Time-of-arrival (TOA).

## 1. INTRODUCTION

With the rapid development of mobile communication technology, mobile positioning technology has become the hot research topic. There have been different positioning systems, such as angle-of-arrival (AOA), time-of-arrival (TOA), time-difference-of-arrival (TDOA), assisted GPS (A-GPS) and Fingerprint [1]. Among them, TDOA method requires fewer equipment changes and does not need a mobile station (MS) and strict time synchronization between the base stations (BS), thus given immense attention. Taylor series expansion algorithm (TSE), Chan algorithm (Chan), least squares algorithm (LS). Least square and Taylor series algorithm (LS-TSE) is based on the TDOA location algorithm. The derivation process of the Chan algorithm is based on TDOA error smaller, the premise is the ideal of zero mean gaussian random variables, so for the NLOS error is bigger in the environment of TDOA measurements, it will have great influence on the performance of the algorithm. As LS algorithm does not need to consider the statistical properties of the error, in the NLOS environment, position precision is slightly better than Chan algorithm. Taylor series expansion method [2, 3] needs an early and close to the actual location of estimated position to guarantee the constringency of the algorithm, positioning accuracy is higher than that in the NLOS environment Chan algorithm and LS algorithm. But Taylor algorithm in NLOS environment positioning accuracy of positioning accuracy is far lower than LOS environment. So, how to reduce the TDOA measurements of NLOS error is important for Taylor algorithm applying in the NLOS environment. RBF network is typical of the forward neural network with functions of nonlinear continuum approximation. TDOA location algorithm is based on RBF neural network [4, 5]. Through the comprehensive utilization of multiple base station TDOA measurements, the NLOS error correction, the TDOA measurements near LOS the measured value of the environment, reusing Taylor algorithm is used to estimate position, make it has higher positioning accuracy in NLOS environment.

This paper proposed a Self-adaptive algorithm of RBF network (SA-RBF-TSE) [6, 7] to correct TDOA measurements. The algorithm does not need to make sure the number of RBF and center vector firstly. In the process,

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according to the distribution of the errors in the input space, the number of RBF adaptively increases and adjusts the center vector appropriately. Based on the corresponding deletion policy to make the number of RBF, the policy is the comprehensive evaluation of RBF network contribution firstly, and then deleting the small contributions to RBF, for the network structure always keeps it simple.

## 2. TDOA MEASUREMENTS CORRECTION BASED ON RBF NEURAL NETWORK

RBF network is a kind of three-layer feedforward network [8], composed of the input layer and output layer and hidden layer. Among them, input layers and output layers are composed of linear neurons; Hidden layer activation function (kernel functions) using center attenuation of radial symmetric nonnegative nonlinear function, its function is to input signal in response to local produce. Weights between the input layer and hidden layer are fixed to 1, and only the weights between the hidden layer and output layer are adjustable.

Input layer by seven related base station is provided by the six TDOA measurements. The input vector is:  $\mathbf{x} = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [TDOA_{21}, TDOA_{31}, TDOA_{41}, TDOA_{51}, TDOA_{61}]^T$ . The role of hidden layer nodes function of the input signal in the local response, when the central range near the basis function of the input signal, the output of hidden layer nodes will produce larger, so the network has good local approximation ability. The choice of basis function is a Gaussian function:

$$\varphi_i(x) = \exp\left[-\frac{\|x - c_i\|^2}{2\sigma_i^2}\right], i = 1, 2, \dots, m \quad (1)$$

where  $\mathbf{x}$  is the input vector;  $c_i$  is the centre of the  $i$ th a basis function, and the same dimension vector of  $\mathbf{x}$ ;  $M$  determines the width of the basis function around a central point;  $m$  is the number of unit perception.  $\|x - c_i\|$  is a vector norm of  $\mathbf{x} - c_i$ .  $\varphi_i(x)$  is only a maximum value in the  $c_i$ , and  $\varphi_i(x)$  reduces quickly to zero with the increase of  $\|x - c_i\|$ . Output layer consists of six neurons, the output of the adjusted TDOA value. The output vector is:

$$\mathbf{y} = [y_1, y_2, y_3, y_4, y_5, y_6]^T = [T_{21}, T_{31}, T_{41}, T_{51}, T_{61}]^T \quad (2)$$

## 3. TDOA MEASUREMENTS CORRECTION BASED ON PROPOSED ALGORITHM

The algorithm proposed in this paper is according to the network output error in the non-uniform distribution of the input space. Through adding and deleting network parameters on adjustment strategy adaptively, changes the size of RBF network and makes network an approximation and generalization ability to achieve high-performance requirements.

Self-Adaptive RBF neural network algorithm (SA-RBF-TSE) [6, 7] of the specific process is as shown in Fig. (1). Before RBF training determines permissible error  $E_s$ , as a condition of the end of the training. The whole process, in general, can be divided into three parts of the algorithm. The first part is the centre of the hidden layer nodes and the weights between the hidden layer and output layer. In this paper, using the gradient descent method, each cycles time, adjust accordingly. The second part is to perform to add operation. Add the strategy is based on the output error in the input space distribution inhomogeneity and proposed. If you execute the operation frequency, not only can reduce the centre of the hidden layer nodes and weights to adjust speed, and can cause an excessive number of hidden layer nodes, increase the amount of calculation and lead to excessive fitting. Considering the above factors, adopt the way of intermittent performing added operation, only when  $i = 4n+1$  ( $n = 0, 1, 2, \dots$ ), add operation to perform. The third part is the deletion operation. If you execute the operation frequency, for some of the new increase of the hidden layer nodes, the centre position and weight may be able to adjust the deleted nodes, so also with the method of batch execution. When  $i = 8m + 7$  ( $m = 0, 1, 2, \dots$ ), execute the delete operation.

### 3.1. Algorithm Flow

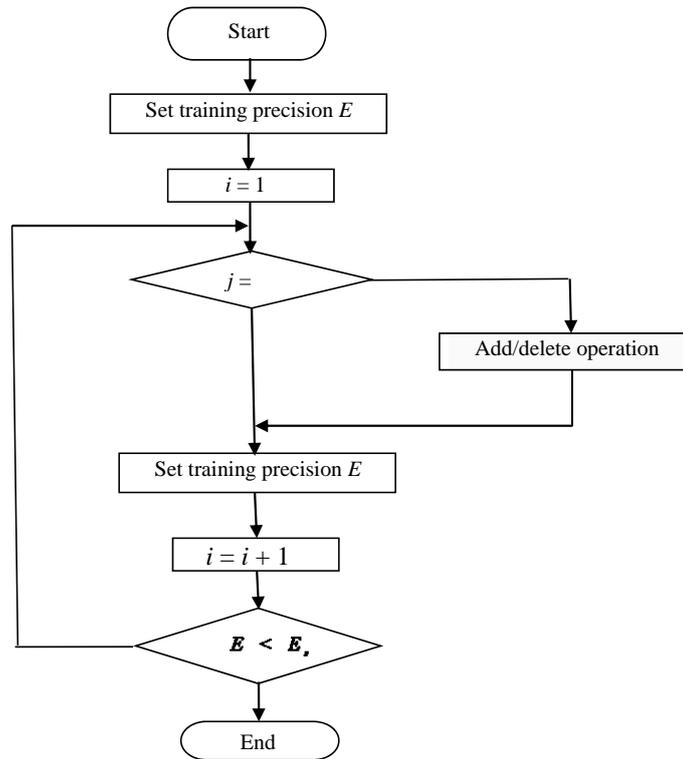


Fig. (1). Flow diagram of Adaptive RBF neural network algorithm.

### 3.2. Addition Strategy

Add strategy considering the network output error in the input space of non-uniform distribution. Need statistics each input vector to produce the output of the error, and then by comparing the find out the pointing error is relatively large, again in the near properly inserted into the hidden layer nodes.

Set  $(x_k, y_k), k = 1, 2, \dots, N$  is a set of training samples, the initial time, the number of hidden layer nodes is zero, each adding operation, according to the following guidelines determine whether add the hidden layer nodes:

$$e_k = |y_k - f(x_k)| > \tilde{\epsilon} \tag{3}$$

$$\|x_k - c_{k,nearest}\| > \frac{1}{2} \|x_k - x_{k,nearest}\| \tag{4}$$

where  $\tilde{\epsilon} = \sqrt{\frac{1}{N} \sum_{i=1}^N e_i^2}$  is the mean square error (MSE) of network output;  $c_{k,nearest}$  and  $x_{k,nearest}$  are respectively correspond to the input vector  $x_k$  closest to the center of hidden layer nodes and the input vector. If you meet the adding conditions, will be  $(x_k + x_{k,nearest})/2$  is set to the new center of hidden layer nodes,  $e_k$  set as the new node weights,  $\sqrt{\|x_k - x_{k,nearest}\|}$  set as center width.

### 3.3. Deletion Operation

Due to the RBF neural network is one kind of local awareness network, the total output of network depends on the weights between the hidden layer and output layer and hidden layer nodes center and the distance between the input vectors. In training, the selection of training samples is relatively sparse. When one hidden layer node center is far away from each input vector, even if its weight is a larger number, also won't have much of an impact on output. At the end of the training in the process of inspection, inspection data are generally more dense, if some input vector is close to the hidden layer centre, the output will be a big impact, this makes the network generalization ability becomes poor. Therefore need to develop a strategy to remove the hidden layer nodes, the deletion policy is introduced.

The deletion policy is for each hidden layer nodes of the network put forward with the contribution of different sizes. The greater contribution of nodes will be kept; smaller contribution of nodes will be deleted. For any hidden layer nodes  $i$ , use the  $\phi$  to show its contribution to the whole network.

$$\phi_i = \sum_{k=1}^N \left| \frac{w_i \varphi_i (\|x_k - c_i\|)}{y_k} \right| \tag{5}$$

Before delete execution operation to the normalized processing of  $\phi_i$ , like  $\tilde{\phi}_i = \phi_i / \phi_{\max}$ . The final judgment rule is: if,  $\tilde{\phi}_i < \theta$  the  $i$ th hidden layer nodes are removed, including  $\theta$  for decision threshold.

The gradient descent method is used to adjust the centre of the hidden layer nodes and weights in the process, need to compute each input vector of the corresponding output error  $e_k$ , and the output value of each hidden layer nodes  $\varphi (\|x_k - c_i\|)$ . And also need to compute  $e_k$  and  $\varphi (\|x_k - c_i\|)$  when perform add and remove operations. In order to reduce the amount of calculation and improve operation efficiency, can adjust the right of the centre of the hidden layer and the value in the process of saving  $e_k$  and  $\varphi (\|x_k - c_i\|)$ .

### 3.4. RBF Network Parameter Adjustment

In this paper, the gradient descent method is used to adjust the RBF centre position and the weights of hidden layer nodes. Set the number of hidden layer nodes for  $m$ , a total of  $N$  groups training sample:  $(\mathbf{x}, \mathbf{y}) = \{(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)\}$ . The actual output of neural network:  $\hat{\mathbf{y}} = \{\hat{y}_1, \hat{y}_2, \dots, \hat{y}_N\}$ . Selection of the mean square error as the error function, take  $\mu_1$  and  $\mu_2$  for vector.

(1) Adjust the weights of hidden layer nodes

$$e_k(t) = \hat{y}_k(t) - y_k(t), k = 1, 2, \dots, N \tag{6}$$

$$E_k(t) = e_k^2(t) \tag{7}$$

$$E(t) = \frac{1}{N} \sum_{k=1}^N E_k(t) \tag{8}$$

$$\frac{\partial E(t)}{\partial w_i(t)} = \frac{\partial E(t)}{\partial y_k(t)} \frac{\partial y_k(t)}{\partial w_i(t)} = \frac{2}{N} e_k \varphi_i (\|x_k - c_i\|) \tag{9}$$

$$w_i(t+1) = w_i(t) - \mu_1 \frac{\partial E(t)}{\partial w_i(t)} \tag{10}$$

(2) Adjust the center of the hidden layer nodes

$$\frac{\partial E(t)}{\partial w_i(t)} = \frac{\partial E(t)}{\partial y_k(t)} \frac{\partial y_k(t)}{\partial w_i(t)} = \frac{2}{N} e_k \frac{(x_k - c_i)}{\sigma_i^2} \varphi_i (\|x_k - c_i\|) \tag{11}$$

$$c_i(t+1) = c_i(t) - \mu_2 \frac{\partial E(t)}{\partial c_i(t)} \tag{12}$$

### 4. TAYLOR SERIES EXPANSION LOCALIZATION ALGORITHM

First use trained RBF network to simulate TDOA measurements, then Taylor algorithm [9] is adopted to improve the position estimation using the revised TDOA value. Set MS coordinates to  $(x, y)$ , participate in the positioning of BS <sub>$i$</sub>  coordinates to  $(x_i, y_i)$ , the number is  $M$ , BS <sub>$i$</sub>  is the service station,  $c$  is the wave propagation speed, according to the measured wave propagation time (TOA) to establish the following distance equation:

$$r_i^2 = (ct_i)^2 = (x_i - x)^2 + (y_i - y)^2 = K_i - 2x_i x - 2y_i y + x^2 + y^2 \tag{13}$$

where  $K_i = x_i^2 + y_i^2$

Let  $r_{i,1}$  be the distance between the BS<sub>*i*</sub> and BS<sub>1</sub>:

$$r_{i,1} = r_i - r_1 = \sqrt{(x_i - x)^2 + (y_i - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2} \tag{14}$$

Equation (14) in selected MS initial position makes Taylor expansion, ignore the second order more weight, Equation (14) will be got:

$$\psi = h_i - G_i \delta \tag{15}$$

where,

$$\delta = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}, h_i = \begin{bmatrix} r_{2,1} - (r_2 - r_1) \\ r_{3,1} - (r_3 - r_1) \\ \vdots \\ r_{M,1} - (r_M - r_1) \end{bmatrix}$$

$$G_i = \begin{bmatrix} (x_1 - x_0)/r_1 - (x_2 - x_0)/r_2 & (y_1 - x_0)/r_1 - (y_2 - x_0)/r_2 \\ (x_1 - x_0)/r_1 - (x_3 - x_0)/r_3 & (y_1 - x_0)/r_1 - (y_3 - x_0)/r_3 \\ \vdots & \vdots \\ (x_1 - x_0)/r_1 - (x_M - x_0)/r_M & (y_1 - x_0)/r_1 - (y_M - x_0)/r_M \end{bmatrix}$$

where  $r_i, i = 1, 2, \dots, M$  is the initial position and the distance between the base station. Weighted least-square solution is in Equation (16):

$$\delta = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = (G_i^T Q^{-1} G_i)^{-1} G_i^T Q^{-1} h_i \tag{16}$$

where  $Q$  is the covariance matrix of TDOA measurements,  $r_i, i = 1, 2, \dots, M$  can be computed with  $x = x_0, y = y_0$ . In the next recursive,  $x'_0 = x_0 + \Delta x, y'_0 = y_0 + \Delta y$ . Repeat the process until the  $\Delta x, \Delta y$  is small enough to satisfy a set threshold:  $|\Delta x| + |\Delta y| < \varepsilon$ . At this time,  $(x'_0, y'_0)$  is the estimate position  $(x, y)$  of MS.

### 5. SIMULATION

Assume that the 7 base station coordinates are  $(0,0), (0, \sqrt{3}R), (\frac{3}{2}R, \frac{\sqrt{3}}{2}R), (\frac{3}{2}R, -\frac{\sqrt{3}}{2}R), (0, -\sqrt{3}R), (-\frac{3}{2}R, -\frac{\sqrt{3}}{2}R), (-\frac{3}{2}R, \frac{\sqrt{3}}{2}R)$ , respectively, where  $R$  is the cell radius, value is 2000m. The simulation of distance measurement  $d_i = R_i + R_i^{noise} + R_i^{nlos}$ , where  $R_i^{noise}$  is measurement error, can be thought of as zero mean Gaussian variables and the variance  $\sigma_{LOS}^2$  of in general, assuming that the standard deviation of 10 m here;  $R_i^{nlos}$  is NLOS error and the variance of  $\sigma_{NLOS}^2$  Gaussian distribution, unable to carry on the accurate modeling, so that it is evenly distributed between 0 and MAX random variables, including MAX to determine a value [10].

Under the condition of LOS,  $R_i^{nlos}$  is zero, which shows measuring distance is the sum of the actual distance and the measurement noise. In NLOS condition,  $R_i^{nlos}$  is not zero and is the dominant position, and measuring distance mainly inclusion of the actual distance and NLOS error.

In this paper, the proposed Taylor-series expansion location algorithm based on Self-adaptive RBF neural network (SA-RBF-TSE) algorithm is compared with Chan algorithm (Chan), LS algorithm (LS), Taylor series expansion algorithm (TSE), Least square and Taylor series algorithm (LS-TSE) and Taylor-series expansion location algorithm based on the RBF neural network (RBF-TSE), performance results are obtained by  $N$  times Monte Carlo simulation, and adopt the following root mean square error (RMSE) as positioning error evaluation indexes:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^n \left( (x - \hat{x}_i)^2 + (y - \hat{y}_i)^2 \right)} \tag{17}$$

where  $(x,y)$  is the true location of the unknown node,  $(\hat{x}_i, \hat{y}_i)$  is the  $i$ th experimental estimate of the unknown node position, and  $n$  is simulation times.

Fig. (2) shows the relationship between Measurement noise standard deviation  $\sigma_{LOS}$  and RMSE. With the increase of  $\sigma_{LOS}$ , an error of all position the algorithm increases, compared with Chan, LS, TSE, LS-TSE and RBF-TSE, the positioning precision of SA-RBF-TSE on average increased by 55.26%, 76.57%, 73.80%, 51.60% and 10.81% respectively. Fig. (3) is RMSE performances of positioning Algorithms based on the NLOS error  $\sigma_{NLOS}$  effect. Fig. (3) shows, with the increase of NLOS error  $\sigma_{NLOS}$ , a sharp rise in the positioning error of Chan, LS and TSE. SA-RBF-TSE under the influence of NLOS error  $\sigma_{NLOS}$  is minimal and has the highest positioning accuracy than other algorithms. When the number of NLOS error  $\sigma_{NLOS}$  is 500m, SA-RBF-TSE positioning accuracy compared with Chan, LS, TSE, LS-TSE and RBF-TSE is increased by 83.41%, 76.74%, 73.80%, 45.08% and 15.14% respectively.

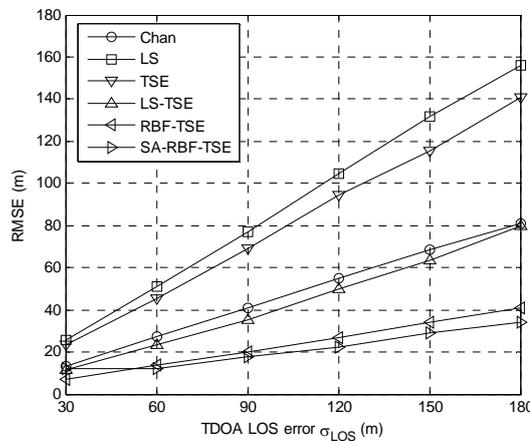


Fig. (2). RMSE performance comparison of positioning Algorithms on Measurement noise standard deviation  $\sigma_{LOS}$ .

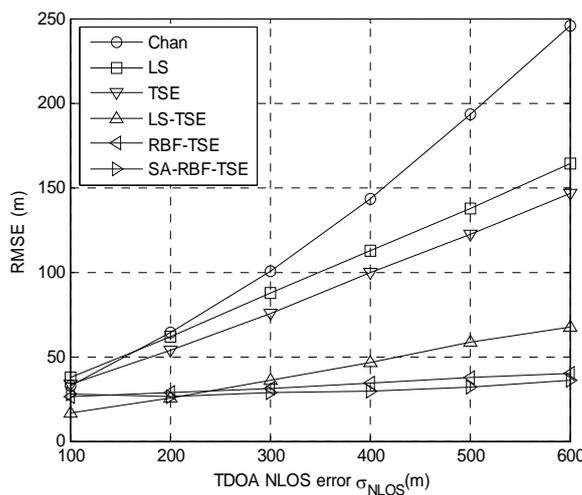


Fig. (3). RMSE performance comparison of positioning Algorithms on NLOS error  $\sigma_{NLOS}$ .

Fig. (4) shows the number of hidden layer nodes for RBF-TSE and SA-RBF-TSE on different Measurement noise standard deviation  $\sigma_{LOS}$ . Fig. (5) shows the number of hidden layer nodes for RBF-TSE and SA-RBF-TSE on different NLOS error  $\sigma_{NLOS}$  under Measurement noise standard deviation  $\sigma_{LOS}$  is 30m. From Figs. (4 and 5), the RBF training determines permissible error threshold needed for the number of hidden layer nodes, SA-RBF-TSE is less than RBF-

TSE obviously. Therefore, nodes loss of SA-RBF-TSE is better than RBF-TSE under the condition of LOS/NLOS.

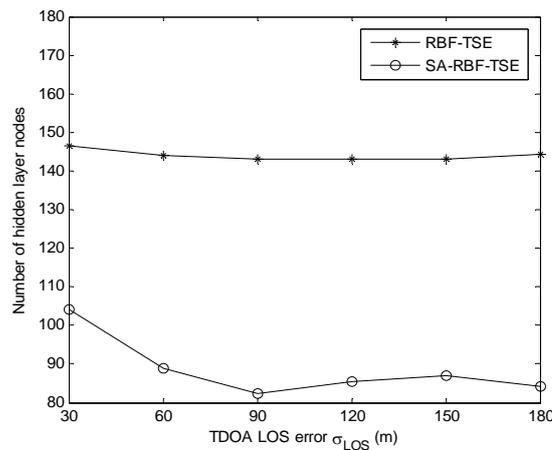


Fig. (4). Number of hidden layer nodes for RBF-TSE and SA-RBF-TSE on Measurement noise standard deviation  $\sigma_{LOS}$ .

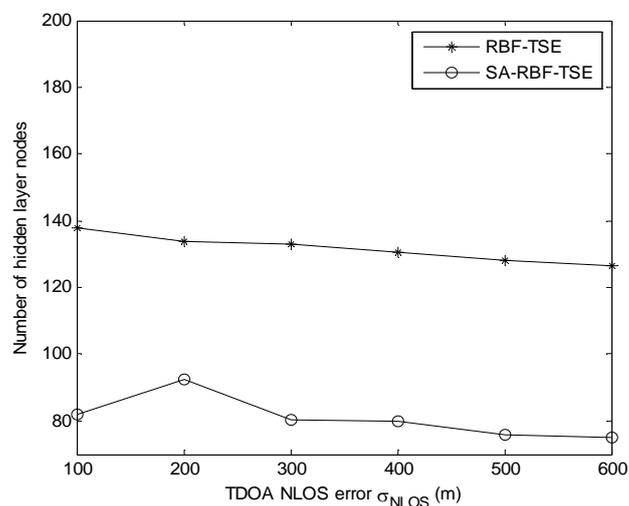


Fig. (5). Number of hidden layer nodes for RBF-TSE and SA-RBF-TSE on NLOS error  $\sigma_{NLOS}$ .

### CONCLUSION

In this paper, a Novel Positioning Algorithm is proposed. The algorithm using Self-Adaptive RBF neural network algorithm to correct TDOA measurements, then Taylor algorithm adopted the revised TDOA value to improve the position estimation. It shows that the proposed Positioning algorithm has a strong inhibition of LOS/NLOS error of simulation results. Through the neural network of the LOS/NLOS error correction in NLOS channel environment, this algorithm has high location accuracy and reliability of the positioning performance is better than Taylor algorithm, LS algorithm, Chan algorithm and RBF-TSE algorithm, and the required Hidden layer nodes for getting performance threshold are less than the RBF-TSE algorithm.

### CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

### ACKNOWLEDGEMENTS

This work was supported by 2013 Technology Foundation for Selected Overseas Chinese Scholar (No.401053740756), Ministry of Personnel of Beijing and The Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry and Training program for Outstanding Young Scholars (No.14085), 2016 annual “Outstanding Young Teacher Training Program” project of North China University of

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