

Research on Robust Control of Gas Tungsten arc Welding System with the LMI Approach

Junfeng Wu^{*}, Qiang Wang and Shengda Wang

Department of Automation, Harbin University of Science and Technology, 150080 Harbin, China

Abstract: By using the Lyapunov second method, the robust control and robust optimal control for the gas tungsten arc welding dynamic process whose underlying continuous-time systems are subjected to structured uncertainties are discussed in time-domain. As results, some sufficient conditions of robust stability and the corresponding robust control laws are derived. All these results are designed by solving a class of linear matrix inequalities (**LMIs**) and a class of dynamic optimization problem with **LMIs** constraints respectively. An example adapted under some experimental conditions in the dynamic process of gas tungsten arc welding system in which the controlled variable is the backside width and controlling variable welding speed, is worked out to illustrate the proposed results. It is shown in the paper that the sampling period is the crucial design parameter.

Keywords: robust control, linear matrix inequalities (**LMIs**), gas tungsten arc welding system.

1. INTRODUCTION

In recent years, much research concerning robust control of linear state-space models has been done. Over the past decades, two major approaches to this problem have been developed: the frequency-domain approach and the time-domain approach. Recently, a number of publications which consider the time-domain approach have appeared in the literature [1-8]. In this approach, the **Lyapunov** stability theory [9-13] is used as a criterion for the analysis and synthesis of robust control systems. Moreover, the nominal system is assumed to be stable.

In computer control application, most of the control systems are sampled-data systems, which are the continuous objects under the control of computer. So it is significant to study robust control of sampled-data systems [14-18].

In this paper, robust control and robust optimal control problems for a class of sampled-data systems with structured uncertainty are considered using the second method of **Lyapunov**. The corresponding robust control laws are given, which are derived by solving a class of **LMIs** and a class of dynamic program problem with **LMIs** constraints respectively.

2. PROBLEM FORMULATION AND PRELIMINARIES

The systems considered in this paper are assumed to be a state-space model as follows

$$x(t) = (A + \Delta A)x(t) + Bu(t) \quad (1)$$

where $x(t) \in \mathfrak{X}^n$ is the state vector, $u(t) \in \mathfrak{X}^m$ is the control vector. $A \in \mathfrak{X}^{n \times n}$, $B \in \mathfrak{X}^{n \times m}$ are constant matrices. ΔA is time-invariant matrix which represents the structured uncertainty in the system model and is assumed to be of the form:

$$\Delta A = \alpha \cdot A_p, \quad \alpha > 0 \quad (2)$$

Discretizing the equation (1), yields

$$x(k+1) = (G + \Delta G)x(k) + (H + \Delta H)u(k) \quad (3)$$

Where

$$G = \exp[AT]$$

$$H = \int_0^T \exp[A\tau] d\tau B \quad (4)$$

$$\begin{aligned} \Delta G &= \exp[(A + \Delta A)T] - \exp[AT] \\ &= \exp[AT][\exp[\Delta AT] - 1] \end{aligned}$$

$$\begin{aligned} \Delta H &= \int_0^T \exp[(A + \Delta A)\tau] d\tau B - \int_0^T \exp[A\tau] d\tau B \\ &= \int_0^T \exp[A\tau][\exp[\Delta A\tau] - I] d\tau B \end{aligned}$$

When the sampling time T is sufficient small, yields

$$\exp[\Delta AT] \approx I + \Delta AT$$

so

$$\Delta G = \exp[AT]\Delta AT = \alpha \cdot GA_p T \quad (5)$$

$$\begin{aligned} \Delta H &= \int_0^T \exp[A\tau] \tau d\tau \Delta AB \\ &= H_q \Delta AB = \alpha H_q A_p B \end{aligned} \quad (6)$$

^{*}Address correspondence to this author at the Department of Automation, Harbin University of Science and Technology, 150080 Harbin, China; Tel: +86 045186390015; E-mail: wu_jf@hrbust.edu.cn

Definition 1 If there exists a state feedback control law $u(k) = Kx(k)$ and a matrix $0 < P^T = P \in \mathfrak{R}^{n \times n}$, which make the following inequality hold

$$x(k+1)^T Px(k+1) - x(k)^T Px(k) < 0$$

then the controller is the robust controller of the discrete-time model (3).

In the proof of main results in this paper, the following lemmas are needed.

Fact 1 Consider any two square matrices Y and W such that $Y = W^T W$, and there are two matrices X and Z of appropriate dimensions. Then for any vector x with appropriate dimension and any constant $\varepsilon \in \mathfrak{R}^+$

$$2x^T X^T Y Z x \leq \varepsilon x^T X^T Y X x + \varepsilon^{-1} x^T Z^T Y Z x \quad (7)$$

Fact 2 (Schur complement) Given constant matrices $\Omega_1, \Omega_2, \Omega_3$, where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0 \quad (8)$$

3. MAIN RESULTS

In this section, according to the discrete-time model in section 2 and a given quadratic index, robust control and robust optimal control problems for sampled-data systems are studied. As a result, the corresponding robust control laws for sampled-data systems are derived.

3.1. Design of General Controller

Theorem 1 System (3) is robust stable via state feedback if there exist matrices

$$0 < Q^T = Q \in \mathfrak{R}^{n \times n} \quad \text{and} \quad Y \in \mathfrak{R}^{m \times n} \quad (9)$$

satisfying the following **LMI**

Furthermore, the control law is given by

$$u(k) = Kx(k), \quad K = YQ^{-1} \quad (10)$$

Proof (9) is equivalent to

$$(1+\varepsilon)(GQ+HY)^T Q^{-1} (GQ+HY) + (1+\varepsilon^{-1})\alpha^2 \left[\begin{pmatrix} G & H_q \\ & A_p & 0 \\ & 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \right]^T Q^{-1} \quad (11)$$

$$\left[\begin{pmatrix} G & H_q \\ & A_p & 0 \\ & 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \right] - Q < 0$$

that is

$$(1+\varepsilon)(GQ+HY)^T Q^{-1} (GQ+HY) + (1+\varepsilon^{-1}) (\Delta GQ + \Delta HY)^T Q^{-1} (\Delta GQ + \Delta HY) - Q < 0 \quad (12)$$

By fact 1, (12) hold if and only if

$$(GQ+HY)^T Q^{-1} (GQ+HY) + 2(GQ+HY)^T Q^{-1} (\Delta GQ + \Delta HY) + (\Delta GQ + \Delta HY)^T Q^{-1} (\Delta GQ + \Delta HY) - Q < 0 \quad (13)$$

arranging it, yields

$$[(G + \Delta G)Q + (H + \Delta H)Y]^T Q^{-1} [(G + \Delta G)Q + (H + \Delta H)Y] - Q < 0 \quad (14)$$

let $K = YQ^{-1}$, yields

$$[(G + \Delta G) + (H + \Delta H)K]^T Q^{-1} [(G + \Delta G) + (H + \Delta H)K] - Q < 0 \quad (15)$$

let $V_k = x(k)^T Q^{-1} x(k)$, $u(k) = Kx(k)$, then

$$\begin{aligned} \Delta V_k &= V_{k+1} - V_k \\ &= x(k+1)^T Q^{-1} x(k+1) - x(k)^T Q^{-1} x(k) \\ &= x(k)^T \{ [(G + \Delta G) + (H + \Delta H)K]^T Q^{-1} \\ &\quad [(G + \Delta G) + (H + \Delta H)K] - Q^{-1} \} x(k) < 0 \end{aligned}$$

so the state feedback controller is the robust controller of the system (3), and the sampled-data system (shown in Fig. 1) under the control of (10) is also asymptotically stable.

3.2. Design of Guaranteed Cost Controller

Consider the performance index

$$\left[\begin{array}{ccc} -Q & (GQ+HY)^T \left[\begin{pmatrix} G & H_q \\ & A_p & 0 \\ & 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \right]^T \\ (GQ+HY) & -(1+\varepsilon)^{-1}Q & 0 \\ \left[\begin{pmatrix} G & H_q \\ & A_p & 0 \\ & 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \right] & 0 & -(1+\varepsilon)^{-1}\alpha^2 Q \end{array} \right] < 0$$

$$J = \sum_{k=0}^{\infty} [x(k)^T Sx(k) + u(k)^T Ru(k)] \quad (16)$$

and the controller

$$u(k) = Kx(k) \quad (17)$$

According to **Lyapunov** stability theory, the following theorem is derived.

Theorem 2 The controller (17) is guaranteed cost controller if there exists matrix $P^T = P > 0$ satisfying

$$\begin{aligned} &[(G + \Delta G) + (H + \Delta H)K]^T P \\ &[(G + \Delta G) + (H + \Delta H)K] - P + S + K^T R K \leq 0 \end{aligned} \quad (18)$$

Proof let

$$\begin{aligned} &x(k+1)^T Px(k+1) - x(k)^T Px(k) \\ &\leq -(x(k)^T Sx(k) + u(k)^T Ru(k)) \end{aligned} \quad (19)$$

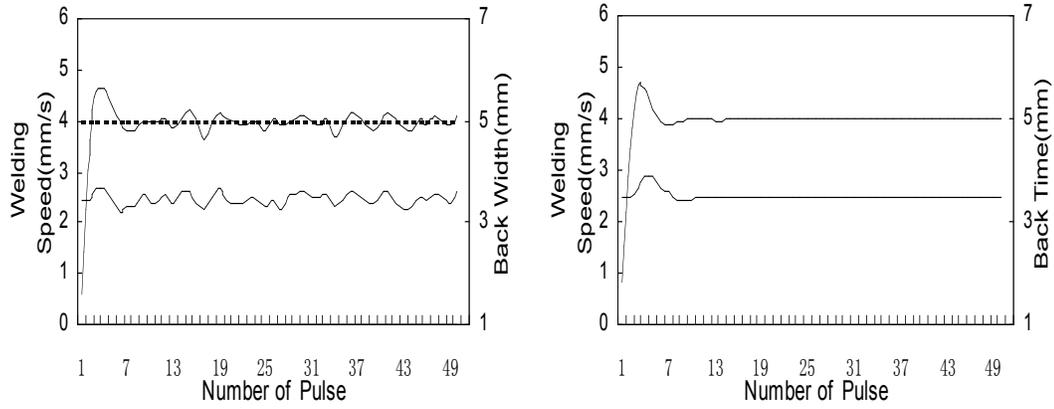


Fig. (1). Dynamic response curves of welding speed to backside width of weld pool with robust controller based on state observer response with disturbance response without disturbance.

then

$$J = \sum_{k=0}^{\infty} [x(k)^T Sx(k) + u(k)^T Ru(k)] = \sum_{k=0}^{\infty} x(k)^T [S + K^T RK]x(k) \tag{20}$$

$$\leq -\sum_{k=0}^{\infty} [x(k+1)^T Px(k+1) - x(k)^T Px(k)] = x(0)^T Px(0)$$

apparently, the performance index is smaller than $x(0)^T Px(0)$ for any k .

Moreover (19) is equivalent to

$$x(k)^T \{ [(G + \Delta G) + (H + \Delta H)K]^T P [(G + \Delta G) + (H + \Delta H)K] - P \} x(k) \leq x(k)^T [S + K^T RK] x(k)$$

so (18) holds.

Theorem 3 System (3) is quadratically stable via state feedback if given constant $\varepsilon \in \mathfrak{R}^+$ there exist matrices $0 < Q^T = Q \in \mathfrak{R}^{n \times n}$ and $Y \in \mathfrak{R}^{m \times n}$ satisfying

$$(21)$$

and the guaranteed cost controller is given by

$$u(k) = Kx(k), K = YQ^{-1} \tag{22}$$

Proof By lemma 2, (21) is equivalent to

$$Y^T RY + QSQ - Q + (1 + \varepsilon)(GQ + HY)^T Q^{-1} (GQ + HY) + (1 + \varepsilon^{-1})\alpha^2 \left[\begin{pmatrix} G & H_q \\ 0 & A_p \end{pmatrix} \begin{pmatrix} A_p & 0 \\ 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \right]^T \left[\begin{pmatrix} G & H_q \\ 0 & A_p \end{pmatrix} \begin{pmatrix} A_p & 0 \\ 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \right] \leq 0 \tag{23}$$

let $K = YQ^{-1}$, $P = Q^{-1}$, then (23) is equivalent to

$$(1 + \varepsilon)(G + HK)^T P(G + HK) + (1 + \varepsilon^{-1})(\Delta G + \Delta HK)^T P(\Delta G + \Delta HK) - P + S + K^T RK \leq 0 \tag{24}$$

according to fact 1, it can be concluded that (24) holds if the following inequality holds.

$$(G + HK)^T P(G + HK) + 2(G + HK)^T P(\Delta G + \Delta HK) + (\Delta G + \Delta HK)^T P(\Delta G + \Delta HK) - P + S + K^T RK \leq 0 \tag{25}$$

let $V_k = x(k)^T Px(k)$, then

$$\Delta V_k = x(k)^T [(G + HK)^T P(G + HK) + 2(G + HK)^T P(\Delta G + \Delta HK) + (\Delta G + \Delta HK)^T P(\Delta G + \Delta HK) - P]x(k) \leq -x(k)^T [S + K^T RK]x(k)$$

so, system (3) is quadratically stable under the control of the guaranteed cost controller (22). The sampled-data systems as shown in Fig. (1) is certainly quadratically stable, too.

3.3. Design of the Optimal Guaranteed Cost Controller

In this section, we shall exploit the parametrized representation of guaranteed cost controllers for the system as shown in Fig. (1) to present a design procedure for the optimal guaranteed cost controller which minimizes the guaranteed cost of the closed-loop uncertain system.

According to (18) and (19), it can be derived that if the following optimization problem has a solution, then the controller $u(k) = Kx(k)$ is the optimal guaranteed cost controller. As a result, we obtain the following theorem.

$$\begin{cases} \min \gamma \\ s.t. x(0)^T Px(0) \leq \gamma \\ \text{Inequality} \\ \left[\begin{array}{ccc|c} QSQ - Q & (GQ + HY)^T \begin{pmatrix} G & H_q \\ 0 & A_p \end{pmatrix} \begin{pmatrix} A_p & 0 \\ 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \\ (GQ + HY) & -(1 + \varepsilon)^{-1}Q & 0 & 0 \\ \begin{pmatrix} G & H_q \\ 0 & A_p \end{pmatrix} \begin{pmatrix} A_p & 0 \\ 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} & 0 & -(1 + \varepsilon^{-1})\alpha^2 Q & 0 \\ Y & 0 & 0 & -R^{-1} \end{array} \right]^T Y^T \leq 0 \end{cases}$$

Theorem 4 If the following optimization problem

$$\begin{cases} \min \gamma \\ s.t. \begin{pmatrix} \gamma & x(0)^T \\ x(0) & Q \end{pmatrix} \geq 0 \\ \left[\begin{array}{ccc|c} QSQ - Q & (GQ + HY)^T \begin{pmatrix} G & H_q \\ 0 & A_p \end{pmatrix} \begin{pmatrix} A_p & 0 \\ 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} \\ (GQ + HY) & -(1 + \varepsilon)^{-1}Q & 0 & 0 \\ \begin{pmatrix} G & H_q \\ 0 & A_p \end{pmatrix} \begin{pmatrix} A_p & 0 \\ 0 & A_p \end{pmatrix} \begin{pmatrix} QT \\ BY \end{pmatrix} & 0 & -(1 + \varepsilon^{-1})\alpha^2 Q & 0 \\ Y & 0 & 0 & -R^{-1} \end{array} \right]^T Y^T \leq 0 \end{cases} \tag{26}$$



Fig. (2). Weld workpiece of welding speed to backside width of weld pool with robust controller in case of varied heat sink.

has a solution $\gamma \in \mathfrak{R}^+$, $0 < Q^T = Q \in \mathfrak{R}^{n \times n}$, $Y \in \mathfrak{R}^{m \times n}$

Then, control law of the form (22) is the optimal state feedback guaranteed cost controller which ensures the minimization of the guaranteed cost for the uncertain system (1).

Proof The proof of this theorem can be given by combining theorem 2 with 3.

4. APPLICATION IN GAS TUNGSTEN ARC ELDTING SYSTEM

The gas tungsten arc welding dynamic process is a typical control system with uncertainties as well as time-delay . In this system the input is welding speeding and the output is Width of weld pool backside.

4.1. Mathematics Model and Controller

By using the sensing system of weld pool's image, the weld shape parameters can be got so that some varying data of the backside width of weld pool to the pulse duty ratio can be obtained. The correlative experimental condition is as follows:

- Welding current: Base value 35 A
- Peak value 135 A
- Welding speed: 0-30pulses 16cm/min
- 31~50 pulses 13cm/min
- Pulse duty ratio: 45%

The other experimental conditions are shown in Table 1.

Table 1. Experimental Conditions of Pulsed GTAW

Parameter Name	Parameter Number	Parameter Name	Parameter Number
Welding material	Q235B	Electro-arc length, l/mm	3.0
Pulse frequency f/Hz	1	Diameter of tungsten d/mm	3.0
Base value of current I_b/A	50	Taper of tungsten Pole $\theta/(^\circ)$	30

Flow of argon q_v $/(L \cdot \text{min}^{-1})$	8.0	Size of welding workpiece V/mm^3	250×100×2
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With the square method the transfer function of the backside width of weld pool to the pulse duty ratio is identified as

$$G(s) = \frac{3.4718}{5.0021s^2 + 4.0402s + 1.0} e^{-2t} \tag{27}$$

while the welding process is in steady state, its mathematics model in the state space is

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -0.194 & -0.8971 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= [-0.5486 \ 0] x(t) \end{aligned} \tag{28}$$

By discretizing Eq. (28), the discrete-time nominal system model can be obtained as follows

$$x(k+1) = \begin{bmatrix} 0.71228 & 0.4116 \\ 0 & 0.6329 \end{bmatrix} x(k) + \begin{bmatrix} 0.0116 \\ 0.0699 \end{bmatrix} u(k) \tag{29}$$

Because the state variables in welding process are not all measured physically, the necessary observer with 1-dimension is designed as

$$z(k+1) = -0.1z(k) - 2.176y(k) + 0.3743u(k) \tag{30}$$

While the robust controller is

$$u(k) = - \begin{bmatrix} 0.8226 & 0.5399 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{31}$$

4.2. Simulation and Experiment Results

Fig. (1) shows the simulation curves of the states of the butt GTAW system under the controller (31) with disturbance (random signal) and without disturbance respectively, which describes the satisfied function of the robust controller.

In the welding experiment, the conditions are the same as those in 4.1. Then Fig. (2) shows the welding effect of a weld workpiece when the backside width of weld pool is required to be 5mm. The practical welding error is less than 5%.

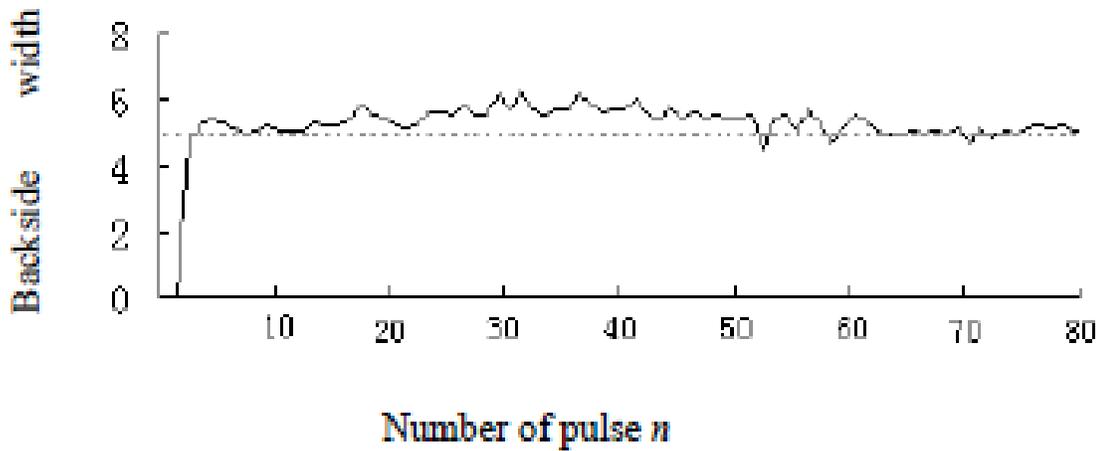


Fig. (3). Dynamic response curves of welding speed to backside width of weld pool with PID controller under the condition of varied heat sink.



Fig. (4). Weld workpiece of welding speed to backside width of weld Pool with PID controller under the condition of varied heat sink.

Under the same experimental conditions, with the utility of self-finding optimality the parameters of PID controller are adjusted. When the backside width of weld pool is 5mm, the parameters can be got as follows

$$K_p = 18.5 \quad T_i = 1.95 \quad T_D = 0.5$$

Then the simulation response curves of the butt GTAW system under the PID controller with disturbance unknown in the practical welding situation and the real welding effect of a weld workpiece are shown in Fig. (3) and Fig. (4) respectively.

From Fig. (3) and Fig. (4) it is not difficult to know that when the heat-sink condition goes so bad that the adjustment function of the PID controller to such a dynamic process with disturbance is not preferable, and the backside width of weld pool behaviors as curves similar to saw-tooth waves during about 24th ~ 52th pulse. The practical welding error is nearly 23%.

5. CONCLUSION

By using the second method of Lyapunov and an LMI approach, this paper studies robust control and robust optimal control for sampled-data systems with structured uncertainty. The guaranteed cost control law and the optimal guaranteed cost control law are derived. These control laws are

designed by solving a class of linear matrix inequalities and a class of dynamic optimization problem with LMIs constraints respectively. In the practical welding process the robust controller has better effect than the classical PID controller under the same experimental conditions.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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