

The Method of Design Sine Grating Based on Optimal PWM

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Abstract: In the structured light 3d optical measurement field, the sinusoidal grating design is one of the research focus. The traditional focus methods of generating sine pattern can't be widely accepted because of many well known defects. While the another traditional binary pattern generated from Ronchi grating which need defocusing processing, its application has also been a certain limit because of its projected pattern contrast ratio is not high. In this paper, we have proposed a designing sine grating method with the spatial modulation theory based on Optimal PWM in electronic engineering. Compared with Ronchi grating, this method of generating pattern have better fringe contrast, and more simple defocusing requirements, so this method have good practical economic value.

Keywords: 3d profile measurement, Sine grating, PWM, The Fourier transform.

1. INTRODUCTION

Non-contact optical three-dimensional contour measurement technology [1] is widely used in CAD/CAM, reverse engineering, rapid prototyping and virtual reality, etc, its main research methods are moire profilometry, phase measuring profilometry, Fourier transform profilometry and spatial phase detection, etc. The outline of the measuring methods need to project sinusoidal pattern to the surface of the object to be tested, and there are two ways of generating sine pattern [2], one is the traditional focusing method of generating sine patterns (FSP) [3], namely, the projector directly focus project out the sinusoidal fringe pattern. Although this approach has been widely used at present, there are some obvious defects on FSP: 1. The projector nonlinear gamma problem. 2. the camera and the projector must be precise synchronization. 3. the camera exposure time demanding. Although the TI [4] (Texas Instruments) can solve the above problems, its high equipment prevent its widely application, the other one is DBP(defocusing binary patterns), namely the method of getting sinusoidal pattern by defocusing binary pattern. It doesn't have a projector's nonlinear gamma problem, does not need complex devices to generate sinusoidal pattern, and doesn't exist the no synchronization problems and the demanding of exposure time between FSP cameras and projectors. With the help of general commercial projector, DBP can project sinusoidal pattern, but its key to success is to design a good binary pattern [5] to make the fringe contrast best. At present, the

common adoption of DBP is Ronchi grating method [6], but it has the problem of pattern projected to the measured object have low contrast, and the problem of requiring high defocusing degree.

This paper first presents the principle and method of Pulse Width Modulation(PWM) in electronic engineering, on this basis, further put forward one design method of sine grating by using optimal space width modulation [7], then compare the pattern's frequency spectrum and filtering performance between this method and the traditional Ronchi grating, finally indicate this design is better than Ronchi grating both on grating fringe contrast and defocusing requirement.

2. DESIGN OF SINE GRATING

Sampling control theory has an important conclusion: equal narrow impulses with different shapes have basically the same effect when they are add on inertial link. PWM control technology [7] is based on the theory of the conclusion, and we have designed two kinds of sinusoidal grating method based on the above PWM control technology.

The method of using space dimension optimal width modulation [8] design sine grating, its guiding ideology is that making area in every pattern's interval equal area in the corresponding sine grating pattern's interval. As shown in Fig. (1), $[0, X]$ (X is sinusoidal grating period), the period is divided into equal length N (N is even) interval, and each interval width is $X = X / N$. With the origin as a starting point for interval number, numbers are $[1, 2, \dots, N]$, the intersections between x axis and each interval are:

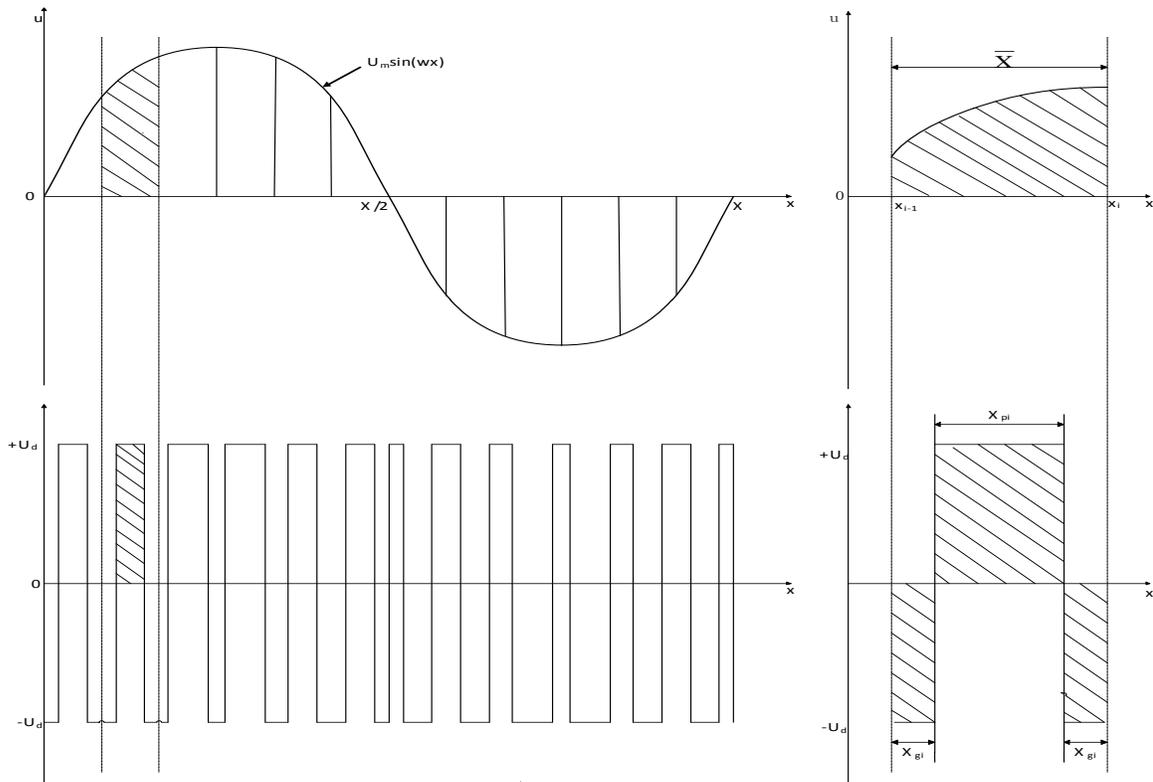


Fig. (1). Schematic of Optimal PWM grating.

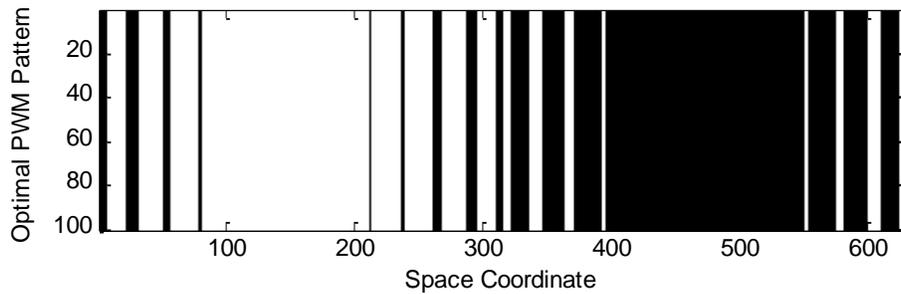


Fig. (2). Optimal PWM Pattern.

$x_0, x_1, x_2, \dots, x_N$ $x_i = X \cdot i$, and the $0 \leq j \leq 2N - 1$ interval can be expressed as: $[x_{j-1}, x_j]$. Set the reference sine grating patterns for $u(x) = U_m \sin(\omega x)$, the corresponding period is $\bar{X} = 2\pi / \omega$, here setting the value of the grating of concave and convex chamfer as $-U_d, +U_d$, respectively. X_{gi} and X_{pi} (X_{gi} and X_{pi} is the corresponding concave and convex chamfer part) is arranged in turn in each interval, and $X = X_{pi} + 2X_{gi}$.

According to the principle of equal area, the area in i -th interval equal to the area in the corresponding sine wave grating, namely,

$U_d X_{pi} + 2(-U_d) X_{gi} = \int_{x_{i-1}}^{x_i} U_m \sin(\omega x) dx$, thus we can obtain:

$$X_{gi} = \frac{X}{4} - \frac{U_m}{4\omega \cdot U_d} [\cos(\omega x_{i-1}) - \cos(\omega x_i)] \quad (1)$$

$$X_{pi} = X - 2X_{gi} = \frac{1}{2}X + \frac{U_m}{2\omega \cdot U_d} [\cos(\omega x_{i-1}) - \cos(\omega x_i)] \quad (2)$$

Within the range $[1, N/2]$, values X_{pi}, X_{gi} is corresponding to $+U_d, -U_d$. According to the symmetry of sine wave is easy to obtain that values X_{pi}, X_{gi} in the interval $[N/2 + 1, N]$ equal to the values in the first half of the cycle, but the corresponding position's amplitude values are $-U_d, +U_d$. Set Values X_{pi}, X_{gi} as grating interval, we can get the required sine grating pattern. When $N = 16, U_m = 1$, according to the above X_{pi}, X_{gi} values, the pattern of the design grating can be shown in Fig. (2).

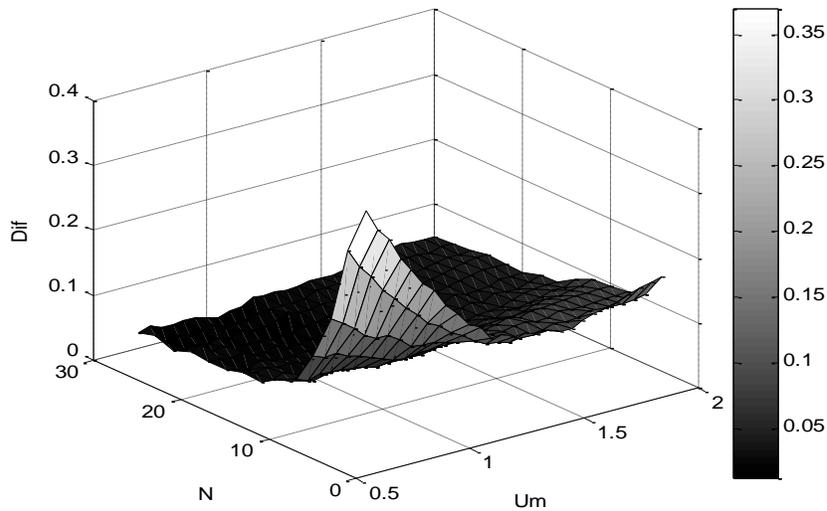


Fig. (3). Dif relationship diagram based on N and Um.

3. THE PARAMETERS DESIGN OF THE GRATING AND SPECTRUM ANALYSIS

The grating pattern must do defocusing to get sinusoidal pattern with continuous variation of gray level, namely binary grating pattern through a circular aperture of defocused imaging projection optical system.

The optical transfer function of projection system is:

$$H(f_x, f_y) = \frac{J_1(2\pi\beta\sqrt{f_x^2 + f_y^2} / 2f_0)}{\pi\beta\sqrt{f_x^2 + f_y^2} / 2f_0} \tag{3}$$

In the formula, J_1 is a first order Bessel function, and β is the degree of defocusing.

As a result, the sine grating spectrum after filtering is:

$$Y(f_x, f_y) = G_r(f_x, f_y)H(f_x, f_y) \tag{4}$$

In the above formula, $G_r(f_x, f_y)$ is kinds of grating spectrum components, and $Y(f_x, f_y)$ is the pattern spectrum after filtering. The defocusing operation is similar to pass binary pattern through a gaussian low-pass filter, and the filtering the sinusoidal fringe is $y(x)$.

The filtering grating stripe $y(x)$ is related to the modulation wave $u(x) = U_m \sin(\omega x)$. Whether the grating’s design is good or not up in two indicators, one is that low harmonics of the binary stripe before filtering occupy small proportion, the other one is that the light and shade contrast of filtering stripes must be high.

The former indicator can be described as $Dif = \frac{\sum_{i=2}^{10} G_i}{G_1}$, in the formula, G_i represent the amplitude of binary stripes in

the i -th frequency component, and G_1 is the fundamental component, the latter indicator can be described as

$$K = \frac{y_{max} - y_{min}}{y_{max} + y_{min}}, y_{max}, y_{min}$$

are the maximum, minimum of the filtering stripe’s amplitude respectively.

Both indicators are related to the two parameters U_m, N of the modulation wave, Where U_m is the modulation depth, N is the carrier ratio, and N is the carrier ratio.

First of all, we do analysis of the carrier ratio N , the larger the N means the larger the number of single cycles needed to characterize the fringe, and the more resource intensive, so we limit the Even N within 30. If D if is smaller, the correspond K may be not smaller, so we guarantee D if is in the lower range of values to find the low range of values corresponding to the maximum value K , and the maximum value of K that is the optimum parameters corresponding to the parameter.

From Figs. (3 and 4), D if minimum is 0.0133, We set the value of D if within the scope of $[0.0133, 0.0133 + 0.005]$, this range can ensure that the value of low-order harmonics locating in the lower value, the corresponding K values are $[0.9065 0.8921 0.9091 0.7974 0.9109 0.9385 0.6595 0.7510 0.8416 0.6446 0.8604 0.9151 0.9044]$. We select the maximum value of K is 0.9385, the corresponding U_m and K is 1.1 and 24. Fig. (2) is a binary pattern when the above two parameters are in the optimal case, its spectral analysis is shown in Fig. (5), we used the Ronchi grating spectrum in Fig. (6) for comparison.

Table 1 lists two grating frequency components. DBP grating must take defocusing processing operations, harmonic components within 10 times are likely to affect the filtering effect, so Table 1 lists front 10 frequency component amplitudes corresponding to Figs. (5 and 6).

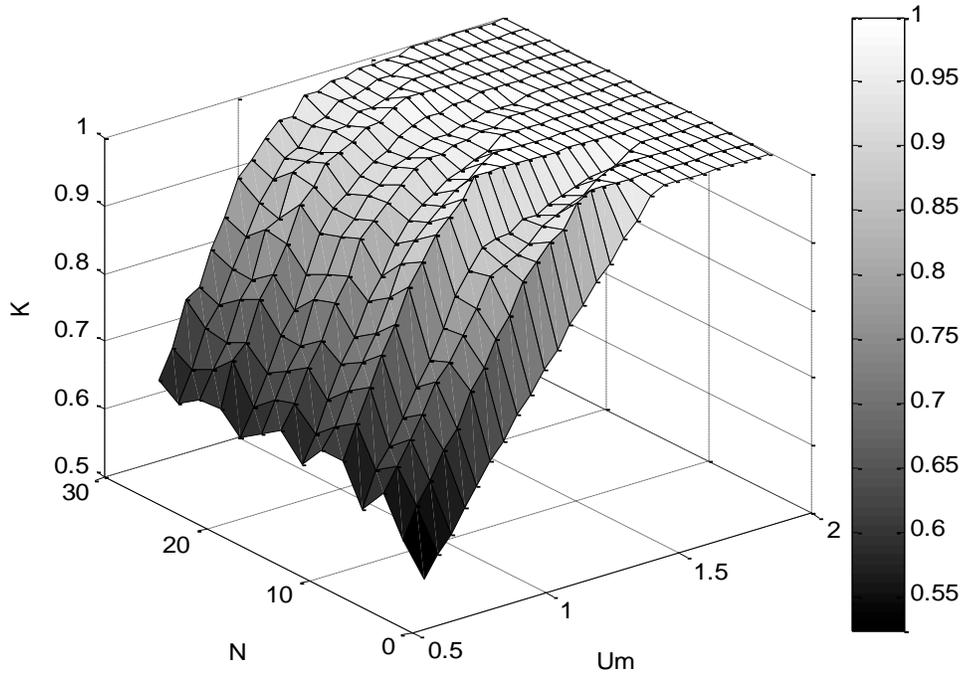


Fig. (4). K relationship diagram based on N and f_c .

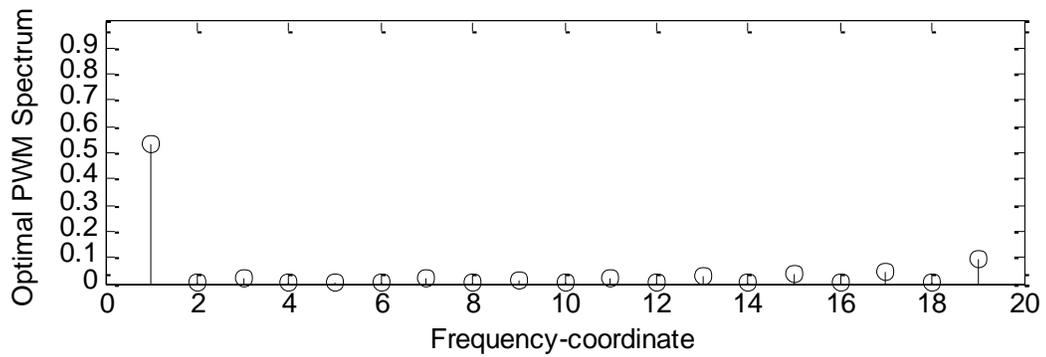


Fig. (5). The sinusoidal gratings's spectrum based on Optimal PWM.

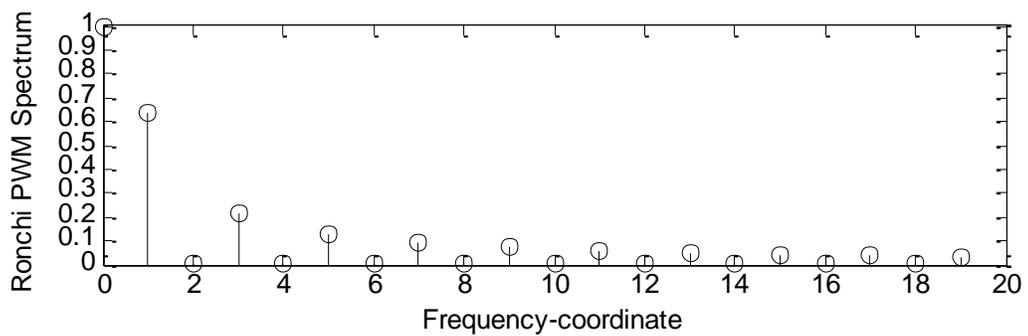


Fig. (6). Ronchi grating's spectrum.

Table 1. Frequency components of two grating pattern.

Grating f	1	2	3	4	5	6	7	8	9	10
Optimal	0.5322	0.0033	0.0218	0.0053	0.0028	0.0063	0.0182	0.0027	0.0146	0.0041
Ronchi	0.6366	0.0016	0.2122	0.0016	0.1273	0.0016	0.0910	0.0016	0.0707	0.0016

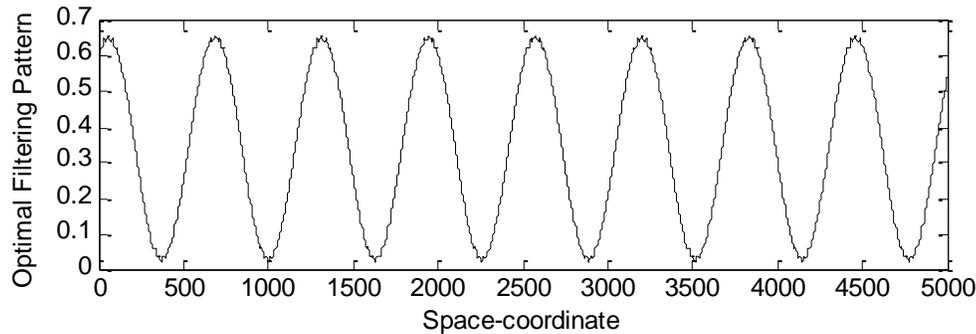


Fig. (7). Optimal PWM pattern after filtering.

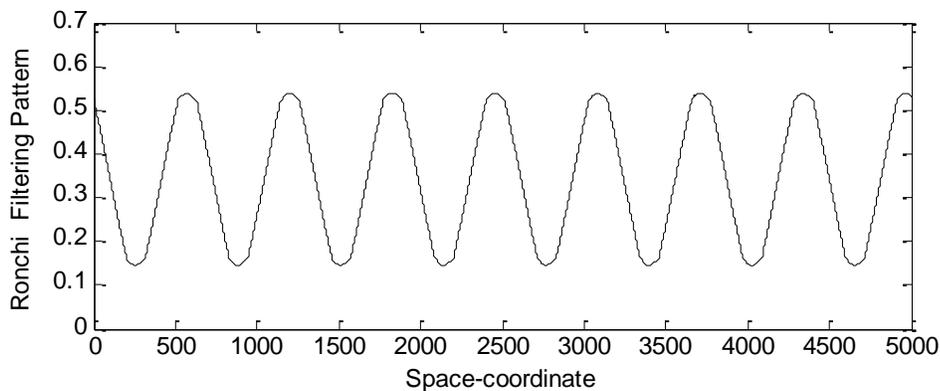


Fig. (8). Ronchi pattern after filtering.

By the results shown in Fig. (5) to Fig. (6) and Table 1, for optimal method in Fig. (5), its frequency is mainly on the fundamental wave and harmonic 15, 17 and the greater values, there is almost no amplitude in other low harmonics, By the contrast of Ronchi grating spectrum, it is easy to find that various odd harmonic components are big.

Therefore, the conditions of low-pass filtering for optimal grating is much easier than the Ronchi grating's.

4. PERFORMANCE ANALYSIS OF SINE GRATING AFTER DEFOCUSING

From Figs. (5, 6) and the spectrum in Table 1, the traditional Ronchi grating need higher defocusing demand, namely, it need a narrow band low pass filter and the Optimal PWM grating in the paper, need broadband low pass filter, its filtering requirements is far better than Ronchi grating's.

Harmonic has a great influence on the light intensity of projection. We define the intensity of light and shade

contrast as: $K = \frac{Y(t)_{\max} - Y(t)_{\min}}{Y(t)_{\max} + Y(t)_{\min}}$. According to the filtering

operation in formula 3 and 4, for the optimal sine grating patterns in Fig. (2), we use broadband gaussian filtering to recovery sine wave as shown in Fig. (7), the light and shade contrast

$K = 0.9385$. For grating pattern corresponding to the Ronchi grating spectrum in Fig. (6), we must develop a narrowband gauss filter on it, the recovery sine wave as shown in Fig. (8), and its light and shade contrast $K=0.55$. Therefore, the Optimal PWM sine grating in this article, its fringe pattern to the surface of the object to be tested by the lens has better light and shade contrast than Ronchi grating.

CONCLUSION

This paper has designed a sinusoidal grating based on Optimal PWM in electronic engineering. Applying the Pulse Width Modulation in time dimension to the space dimension,

according to nonuniform spacing interval after modulation to rule grating. After selecting the optimal parameter values, grating's low pass filter conditions are very low, and the pattern contrast of this sinusoidal grating through the lens defocusing is higher than the conventional Ronchi grating's. Therefore, the design of the Optimal PWM sinusoidal grating can improve the precision of three-dimensional contour measurement, it can be widely used in the field of 3D profile measurement.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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REFERENCES

- [1] C. Y. Ren, H. X. Jiang, S. Li, L. Zhang, and H. Liu, "Structure and Spectrum analysis of quasi-Sinusoidal grating in phase measurement profilometry", *J. Heilongjiang Inst. Sci. Tech.*, vol. 20, no. 3, pp. 201-205, Aug 2010.
- [2] S. Y. Lei, and S. Zhang, "Digital sinusoidal fringe pattern generation: Defocusing binary patterns vs. focusing sinusoidal patterns", *Optics Lasers Eng.*, vol. 48, no. 5, pp. 561-569, May 2010.
- [3] P. Cheng, C. Zhang, A. Abdugheni, X. D. Chen, and B. K. Wu, "A new method of making projection grey sinusoidal grating", *Acta Photon. Sin.*, vol. 39, no. 12, pp. 2174-2177, Dec 2010.
- [4] S. Y. Lei, and S. Zhang, "Flexible 3-D shape measurement using projector defocusing", *Opt. Lett.*, vol. 34, no. 20, pp. 3080-3082, Oct 2009.
- [5] B. Zhou, F. Qiao, L. Rao, and Z. Y. Liu, "A design and analysis of sinusoidal gratings based on mean pwm", *J. Central South Univ. (Sci. Tech.)*, vol. 44, no. s1, pp. 461-464, Sep 2013.
- [6] W. Lohry, and S. Zhang, "3D shape measurement with 2D area modulated binary patterns", *Optics Lasers Eng.*, vol. 50, no. 27, pp. 917-921, Sep. 2012.
- [7] X. Li, and W. Chen, *Pulse Width Modulation Technology*. CA: Wuhan of China, pp. 19-22, 1996.
- [8] Y. J. Wang and S. Zhang, "Optimal pulse width modulation for sinusoidal fringe generation with projector defocusing", *Optics Lett.*, vol. 35, no. 24, pp. 4121-4123, 2010.

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