# Decomposition of Target Coverage Graph in Wireless Sensor Network

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**Abstract:** In order to reduce the algorithm complexity of the target coverage problem in Wireless Sensor Network, three kinds of approaches are designed to decrease the network scale, that is, one is deleting the redundant sensor nodes, another is removing the redundant targets, and the third is decomposing the target coverage graph into multiple independent sub graphs. This paper proves that the three approaches are correct, and presents the Construct Independent Sub Graph Algorithm(CISGA) to divide target coverage graph. Experiment results show that CISGA can reduce 30% of network scale and greatly reduce the algorithm complexity of target coverage problem.

**Keywords**: Decomposition, redundant node, redundant target, independent sub graph, network scale.

## 1. OVERVIEW

Wireless sensor network is composed of a large number of sensor nodes deployed in monitoring area [1, 2], through wireless communication connection to construct a self-organizing network system with multiple hops [3, 4]. It can collaboratively perceive [5, 6], collect, and process the perceived object information in network coverage area [7, 8], and sent to observer [9, 10].

Target coverage is an important problem of wireless sensor network (WSN) [11], under the premise of completing the monitor task, this paper studies how to save energy as much as possible to override lifetime with maximize goals [12]. The main way to override lifetime with maximize goals, is to reasonably configure the state of each node in network, balance the network energy consumption and effective coverage rate, improve the network energy efficiency [13]. Reference [14, 15] gives the definition of problem to cover life cycle with maximize goals.

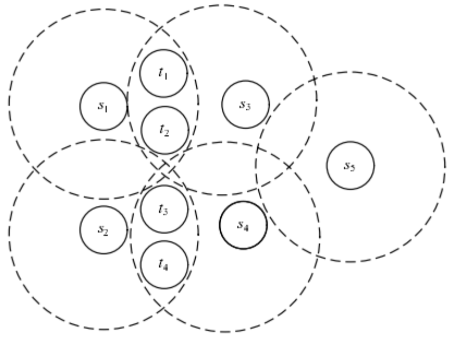
Definition 1. In a sensor network, a Target Coverage Problem (TCP) is under the condition to guarantee that the goals are continuous monitored, schedule the activities of the nodes, to maximize the lifetime of network coverage

TCP is a NP complete problem (14-15). With the increase of target and sensor nodes, wireless sensor network's scale increases rapidly, the difficulty of TCP is also growing exponentially. In the wireless sensor, reducing the network scale is an important way to reduce the algorithm complexity. But there is no related algorithm research such as deleting redundant nodes, deleting redundant targets, and decomposing the targets coverage diagram of wireless sensor network into multiple independent sub graphs. This paper adopts the method of decreasing the network scale to reduce the algorithm complexity. First of all, the method of deleting the redundant sensor nodes is put forward. Second, the method to delete redundant targets is provided. Finally, the target coverage figure is decomposed into multiple independent sub graphs, and the Construct Independent Sub Graph Algorithm (CISGA) is also described.

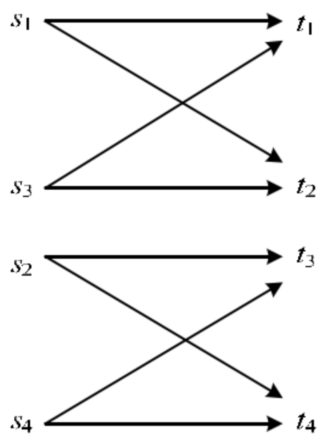
## 2. PROBLEM DESCRIPTION

In the two-dimensional plane, m targets with a random distribution are required to be continuously monitored. To complete the detection task, n sensor nodes are randomly delivery within the target area. Assume that the effective monitoring radius of each node is $R_s$, the communication radius is $R_c$, and $R_C \geq 2R_S$; and the base station can obtain the coordinates of all sensor nodes and targets, and can calculate the coverage relationship of sensor nodes to target coverage.

Sensor network graph is presented as $G(S,T,E,W)$. Among them, S is a randomly distributed sensor node set, $S$=$\{s_1,s_2,...s_m\}$, T is a random distribution of m target sets, $T$=$\{t_1,t_2,...t_m\}$ ; $E$ is the position relationship set in $e_{ij}$ when $e_{ij}$=1 , $E=\{e_{ij} \mid i \in \{1,2,...,n\}, j \in \{1,2,...,m\}\}$ , $e_{ij}$ represents the position relationship between node $s_i$ and $t_j$ , $e_{ij}$=1. Only when the Euclidean distance between the target $t_j$ and node $s_i$ is less than or equal to the coverage radius $R$ , $d(s_i,t_j) \leq R$; otherwise, $e_{ij}$=0 ; $W$=$\{w_1,w_2,......w_n\}$is initial energy set of sensor nodes, $w_i$ represents the initial energy of sensor node $s_i$ , $w_i$ represents the maximum number of energy units that the nodes can work.




(a) Sensor network




(**b**) Coverage relationship

**Fig. (1).** Sensor network and its target coverage network decomposition.

## 3. PROBLEM ANALYSIS

### 3.1. The Deletion of Redundancy Nodes

With the increase of sensor node and target, the computational complexity of target coverage problem gets rapid growth. To decrease the scale of the problem is the best way to reduce the computational complexity. For this reason, this paper designs three kinds of measures to decrease the size of the target coverage problem.

The decomposition of sensor network and its target coverage network is shown in Fig. (**1**). As shown in Fig. (**1a**) there is a sensor network with five sensor nodes $\{s_1, s_2, s_3, s_4, s_5\}$ and four goals $\{t_1, t_2, t_3, t_4\}$. The coverage area of each node is a circle with the node as the round center; the circle radius is the node's perception radius. Fig. (**1b**) describes the coverage relationship between the node and the target. If $T(s_i)$ represents the goal set covered by $(s_i)$, in Fig. (**1b**), $T(s_1) = \{t_1, t_2\}$, $T(s_2) = \{t_3, t_4\}$, $T(s_3) = \{t_1, t_2\}$, $T(s_4) = \{t_3, t_4\}$, $T(s_5) = \phi$. In Fig. (**1b**) $s_5$ does not cover any goal, in order to prove if it can be deleted, the deletion theorem of redundancy nodes will be proved as follows.

Definition 2. Under the condition of without considering node connection, redundant nodes do not cover any target sensor nodes.

Theorem 1. If a node does not cover any target, then the node may be deleted.

Proof: from definition, under the condition of without considering of the connection of sensor nodes, the target coverage problem is scheduling the nodes which can cover the targets, and have no any influence with the nodes which do not cover any targets. Therefore, deleting the redundant nodes does not make the target coverage problem any change.

The following content exports the rules of deleting redundant nodes.

Rule 1. Redundant nodes may be deleted. As shown in Fig. (**1**), deleting node $s_5$ can reduce the number of nodes in wireless sensor networks. In particular, in wireless sensor network with a dense deployment of nodes, there is a large number of redundant nodes. Deleting the redundant nodes can significantly reduce the number of nodes, so as to reduce the computational complexity of target coverage problem.

### 3.2. Deletion of Redundant Targets

As shown in Fig. (**1**), when $t_2$ is covered, $t_1$ must also be overwritten, and vice versa. When $t_4$ is covered, then $t_3$ must also be overwritten, and vice versa. That is to say, if there are two targets of $t_i$ and $t_j$, when $t_i$ coverage set equals to $t_j$ coverage set, deleting one of the goals can reduce the number of targets in wireless sensor network, but the maximum life cycle for the target coverage problem has no any influence.

If $C_i$ represents the coverage set of target $t_i$, $C_i$ is the set of nodes which can cover targets $t_i$. It is shown in Fig. (**1**), $C_1=\{s_1,s_3\}$, $C_2=\{s_1,s_3\}$, $C_3=\{s_2,s_4\}$, $C_4=\{s_2,s_4\}$。

Theorem 2. If $C_i = C_k$, when $t_i$ is covered, $t^*$ is also be covered.

Proof: when $t_i$ node is covered, then $\exists s_u \in C_i$, $s_u$ also covers $t_i$. Because $C_i = C_k$, so $s_u \in C_i$. That is to say, $s_u$ also covers $t_k$.

From theorem 2, it shows that if the coverage sets of two targets are equal, another goal will be covered, another goal must also be covered. Therefore, here is given the definition of redundant targets.

Definition 3. when two or more target coverage set are equal, the target with minimum subscript is a necessary target, other goals are redundant. Redundant targets can be deleted.

Suppose in a certain period of time $b_i$, in order to ensure the targets set $T$ is completely covered, the optimal active node set is $S_1$. To prove after deleting redundant targets below, in order to ensure the target set $T'$ is completely covered, the optimal active node set S1 doesn't change.

Theorem 3. In order to ensure the full coverage of the target, removing redundant targets does not reduce the active nodes in active node set.

Proof: if $T_1=\{t_i,t_{i+1},...,t_j\}$ is a target set with same coverage set, the active node $s_k(s_k \in S_1)$ is the only coverage target set $T_1$.

When deleting the redundant targets in $T_1$, it is necessary to keep the essential target in $T_1$, in order to guarantee the necessary target coverage, $S^*$ must keep active state. That is to say, deleting redundant targets will not reduce the active nodes in the active node set. As a result, this leads to theorem 4.

Theorem 4. If redundant targets are deleted, the maximum life cycle of target coverage problem keeps invariant.

Proof: Suppose the optimal time series of the maximum life cycle in target coverage problem are $<S_1,T,b_1>$, $<S_2,T,b_2>$,…, $<S_k,T,b_k>$. Thereinto, $T$ is the initial target set; $b_k$ represents the activity time; $S_k$ is the activity node set of $b_k$.

From theorem 3, it is known that removing the redundant nodes in target set, doesn't change the active node set at this moment. After deleting redundant nodes in $T$, the active node set will not reduce, of course it also won't increase. In this way, the optimal time series of the maximum life cycle of target coverage problem will not change. The theorem is proven.

Below the deleting rules of redundant targets are derived.

Rule 2. when $C_i = C_k$, a necessary goal is reserved, the redundant targets are deleted. In particular, in the wireless sensor networks with dense distribution of target, deleting redundant nodes can significantly reduce the target number, thus reduce the algorithm complexity of target coverage problem.

### 3.3. The Decomposition of Target Coverage Diagram

Each target coverage network can be expressed in a target coverage diagram. For example, Fig. (**1a**) can be represented as Fig. (**1b**). To facilitate understanding, the decomposition of target coverage network is described as the decomposition of target coverage diagram. In target coverage diagram, if a subgraph has no public nodes and public sides with other parts, then the subgraph can be separated as an independent subgraph.

It is shown in Fig. (**1b**), the targt coverage diagram is divided into two independent network, one subgraphy is $G(S_1,T_1,E_1,W_1)$, thereinto, $S_1=\{s_1,s_2\}$ ; $T_1=\{t_1,t_2\}$ ; $E_1=\{e_{11},e_{12},e_{31},e_{32}\}$. Another subgraphy is $G(S_2, T_2, E_2, W_2)$, thereinto, $S_2=\{s_2,s_4\}$ ; $T_2=\{t_3,t_4\}$ ; $E_1=\{e_{23},e_{24},e_{43},e_{44}\}$.

There is a given target coverage diagram $G(S,T,E,W)$. $G(S,T,E,W)$ can be divided into two independent sub networks $G(S_1,T_1,E_1,W_1)$ and $G(S_2,T_2,E_2,W_2)$, only when $S_1 \cup S_2 = S$, $T_1 \cup T_2 = T$, $E_1 \cup E_2 = E$, $W_1 \cup W_2 = W$, $S_1 \cap S_2 = \phi$, $T_1 \cap T_2 = \phi$, $E_1 \cap E_2 = \phi$, $W_1 \cap W_2 = \phi$, and so on, the subgraph can be decomposed into smaller independent subgraph, so far until it cannot be decompose again.

Here a kind of construct independent subgraph algorithm is proposed as below.

CISGA(Graph $G(S,T,E,W)$)

Step 1 delete the redundant nodes.

Step 2 delete the redudant target, and delete the related side with redundant nodes.

Step 3 initialize $k=1$.

Step 4 while $E \neq \varnothing$ do

{

$S_k=\varnothing, T_k=\varnothing, E_k=\varnothing, W_k=\varnothing$;

There is $e_{ij}$, $e_{ij} \in E$;

//from target $t_j$ to start depth searching to the subgraph $G_k$

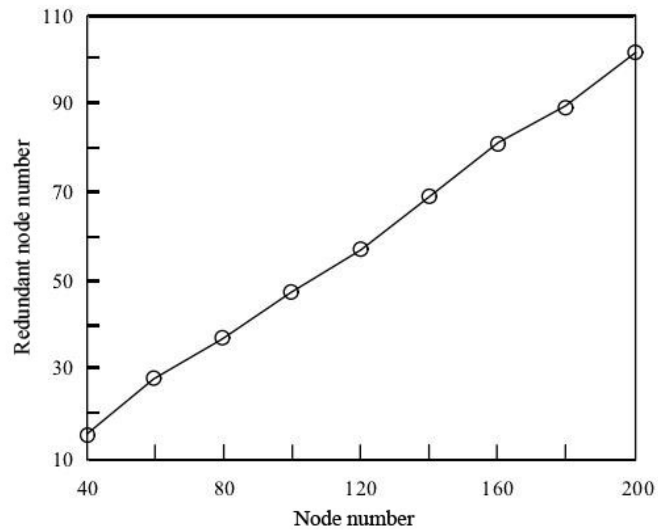



**Fig. (2).** The number of redundant nodes in wireless sensor network with different nodes.

BFS(int $k$, Vnode $t_j$, Graph $G$,Graph $G_k$)

//from diagram G delete the elements in independent sub graph $G_k$

$S=S-S_K$, $W=W-W_K$, $T=T-T_K$, $E=E-E_K$, $k=k+1$ ;

}

Step 5 return $G(S_1,T_1,E_1,W_1)$, $G(S_2,T_2,E_2,W_2),\ldots$

A kind of depth priority algorithm to search independent sub graph is proposed below, hereinto, $v$ is the vertex set, $v=S\cup T$; $G_K$ is the independent subgraph including target $v$.

BFS(int $k$, Vnode $v$, Graph $G$, graph $G_K$)

{

// from vertex $v$ to start the depth priority traverse, to search the independent sub graph $G_K$

Step 1 set the visiting flag of $v$ as True; visit node $v$.

Step 2 if $v$ is node, then $v$ will be add into node set $S_k$; otherwise $v$ is add into target set $T_k$.

Step 3 for ($w$=the first adjacent node of $v$ in graph $G$; $w$; $w$=the next adjacent node of $v$ (relative to $w$)

{

If $w$ is not visited, then

{

Add side $e_{vw}$ into $E_k$;

DFS(int $k$, Vnode $w$,Graph $G$, Graph $G_k$)

}

}

Step 4 return back the subgraph $G_k$.

}

Theorem 5 CISGA algorithm doesn't change the maximum life cycle of target coverage problem.

Proof: CISGA algorithm only separates the subnets with independent structure from the whole. It does not increase and decrease side; it also does not increase or decrease any nodes. That is to say, it doesn't change the structure of target coverage diagram, also does not change the target coverage problem, thus will not change the maximum life cycle of target coverage problem.

## 4. ALGORITHM SIMULATION

Matlab is adopted as simulation software in Windows XP, in 500m X 500m rectangular area, two-dimensional coordinates of targets, nodes, and the initial energy are randomly produced. Assume that the sensor nodes are coverage all-around, with covering radius $R_s=40$ meters.

Experiment 1. Randomly place 25 targets in monitoring area, then randomly place 40, 60, 80, 100, 120, 120, 160, 180, 200 nodes. Comparing the number of redundant nodes, the means of multiple experiments are taken as the experimental results; the experimental results are shown in Fig. (**2**).

It is shown from Fig. (**2**), there exists redundant nodes in wireless sensor network. Deleting the redundant nodes can significantly reduce the size of the target coverage problem. When node distribution is denser, in the non-target area there are more redundant nodes, the effect of deleting the redundant nodes to reduce the scale of algorithm is more obvious. The experimental results show that deleting the redundant nodes can greatly reduce the size of the target coverage problem.

Experiment 2. Randomly place 100 nodes in monitoring area, and then randomly place 25, 55, 85, 115, 135, 135, 205, 235, 265, 295, 295 targets. Compare the number of redundant targets, take the mean of multiple experiments as the experimental result, the experimental result is shown in Fig. (**3**).

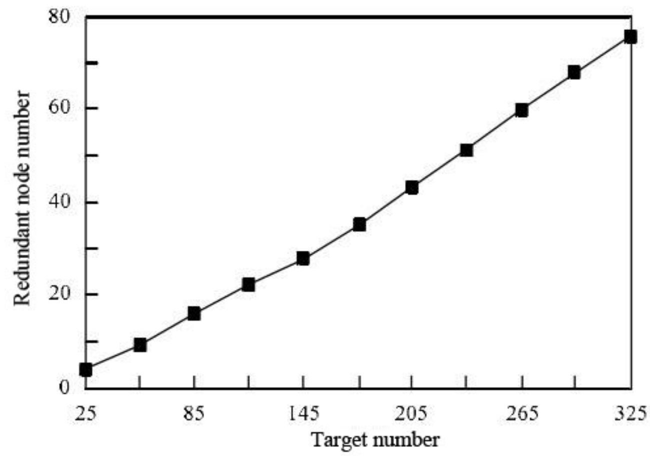



**Fig. (3).** The number of redundant targets for different targets in wireless sensor network.

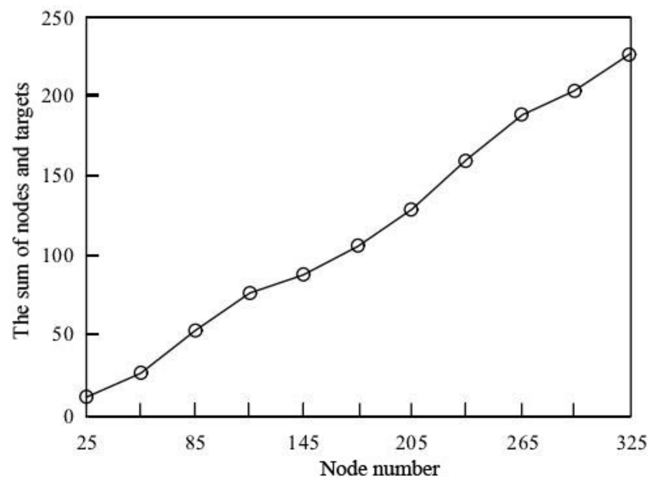



**Fig. (4).** The scale of the largest independent subnet in the different nodes network.

Fig. (**3**). shows that there exists redundant targets in the wireless sensor network. Deleting redundant targets can greatly reduce the size of the target coverage problem. When the target distribution is denser, there are more targets in the node coverage area, thus there are more redundant targets.

The experimental results show that deleting redundant targets can significantly reduce the size of the target coverage problem.

Experiment 3. Randomly place 25 targets in monitoring area, and then randomly place 25, 55, 85, 115, 135, 135, 205, 235, 265, 295, 295 targets. Compare the largest scale of independent subnet, take the mean of multiple experiments as the experimental results, the experimental results are shown in Fig. (**4**).

Fig. (**4**) shows CISGA algorithm can significantly reduce the scale of the target coverage diagram. With the increase of nodes, the effect that CISGA algorithm reduces the network size is more obvious.

## CONCLUSION

In order to reduce the scale of the target coverage problem, this paper designs the strategies of deleting redundant nodes, redundant targets, and decomposing independent subgraph, and puts forward an algorithm to construct independent subnet. The simulation experiments show that CISGA algorithm can reduce 30% of the network scale, significantly reduce the complexity of the target coverage problem. The algorithm has low complexity, high efficiency and good scalability.

At present, in wireless sensor network, the target coverage and connectivity [16], the target coverage based on the directional antenna nodes, the target coverage based on the mobile nodes [17], the target coverage of three-dimensional space, multiple target associated coverage problem [18], and so on, all of them are NP-complete problems. Studying the network decomposition algorithm of these questions is the next step of work content.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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