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Fuzzy Observer-based Robust Guaranteed Cost Fault-tolerant Design for Uncertain Nonlinear NCS

Jun Wang[1,*], Wei Li[1] and Ling Guan[2]

[1]*College of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou, 730050, China*

[2]*GNGC Gansu Yinguang Chemicals Industria l(G) Co., Ltd. Energetic material Fellow Subsidiary, Baiyin, 730900, China*

Abstract: Through the comparison and analysis of some recent patents on the fault-tolerant designed method, this paper addresses the problem of robust guaranteed cost fault-tolerant for nonlinear networked control systems (NNCS) with network-induced delay and packed dropout. Firstly, we adopt observer and state-feedback control strategy, wherein states are immeasurable. Then, by constructing appropriate Lyapunov-Krasovskii functional, based on T-S fuzzy model, a delay-dependent sufficient condition is deduced for robust guaranteed cost fault-tolerant control. Further, observer-based gain and state-feedback controller gain can be obtained and optimized. Finally, an example is used to illustrate the effectiveness and feasibility of the proposed approach.

Keywords: Fault-tolerant control, Guaranteed cost, Nonlinear Networked Control System, state observer, T-S Fuzzy Model.

1. INTRODUCTION

According to European Patent 2075961 [1] and U.S. Patent 7426189 [2], (Networked Control System) NCS is a control system of full-distributed real-time feedback which connects the sensors, controllers and actuators from different locations through network.

In the past decade, NCS has become a widespread concern in research areas. Although NCS has advantages of a low cost, easy installation and maintenance, system reliability and flexibility, easy to fault diagnosis, etc., network delay and packet dropout have become primary problems, which cannot be avoided due to bandwidth, irregular flow and unreliable transmission. Not only does it cause the system performance degradation, and even lead to system instability [3-5]. In addition, compared with traditional systems, NCS is larger in scale and more complicated in structure, therefore all kinds of uncertainty and fault-induced factors have greatly increased. As a result more and more researchers have started to focus on the fault-tolerant control of NCS [6-8], hoping to get better performances and higher safety and reliability [9, 10].

In the practical, the nonlinear properties more or less exist in NCS. Due to its complexity, there are fewer studies on the fault tolerant. The preceding fault-tolerant control researches on NNCS are discussed by adopting state-feedback control strategies [11]. However, not all states can be measured. Therefore, targeted at a class of uncertain NNCS and based on observer of fuzzy state, this paper studies the control problem of the robust fault-tolerant guaranteed cost while considering the influences of time-delay and packet dropout under the condition of actuator failures.

*Address correspondence to this author at the College of Electrical and information engineering,Lanzhou University of Technology, Lanzhou, 730050, China; Tel: 0931-2976020; Fax: 0931-2973506;
E-mail: wangj31901@163.com

2. SYSTEM DESCRIPTION

Typical uncertain NNCS described by T-S fuzzy model is shown in Fig. (**1**):

In Fig. (**1**), τ_{ca} and $!_{sc}$ are the time-varying network-induced delays from controller to actuator and from sensor to controller, respectively.

Assumption 1: It is assumed that the sensor is clock-driven, while the controller and actuator are event-driven.

Assumption 2: The sampled data is transmitted with a single packet. The packets reach the controller and actuator by their original transmitting sequence if they are not lost.

Assumption 3: Data packet dropouts are used as a special kind of time delays.

Therefore, we can get according to [12, 13]

$$\tau_{sc1} \le \tau_1(t) \le (\delta_{1\mathrm{M}} + 1)T + \tau_{sc2}$$

$$\tau_{ca1} \le \tau_2(t) \le (\delta_{2\mathrm{M}} + 1)T + \tau_{ca2}$$

Define $\tau_{s2} = (\delta_{1\mathrm{M}} + 1)T + \tau_{sc2}$, $\tau_{s1} = \tau_{sc1}$

Then $\tau_{s1} \le \tau_1(t) \le \tau_{s2}$,

similarly have $\tau_{c1} \le \tau_2(t) \le \tau_{c2}$

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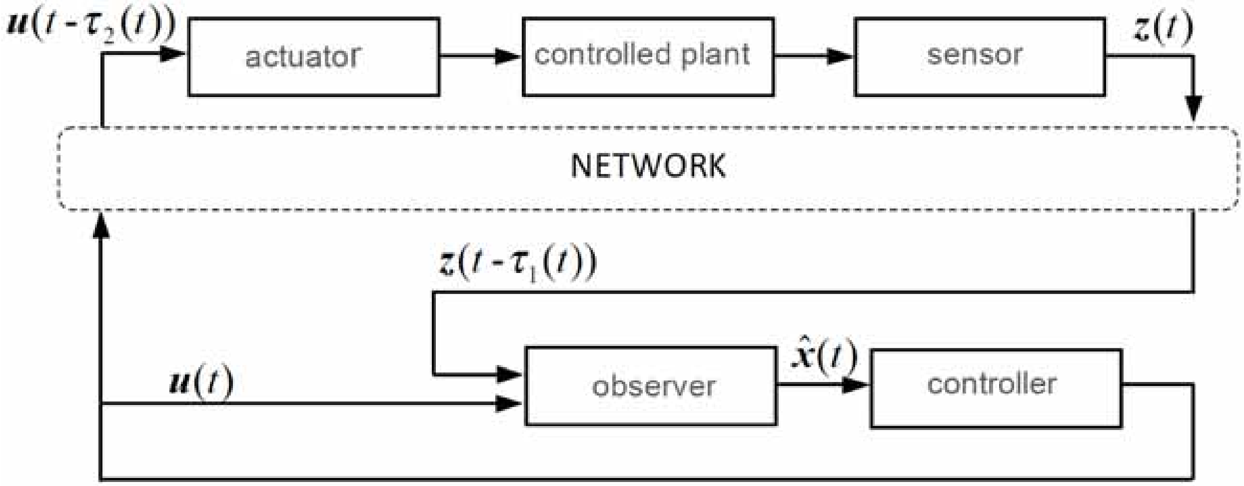

Fig. (1). Framework of networked control systems Based on state observer.

where $\tau_1(t)$ and $\tau_2(t)$ are total time-varying delays respectively of controller-to-actuator and sensor-to- controller including network-induced delays and data packet dropouts. τ_{sc1} , τ_{sc2} and τ_{ca1} , τ_{ca2} are positive constant representing the lower and upper delay bound of corresponding transmission channel, respectively. δ_{1M} and δ_{2M} are the maximum number of data packet dropouts of corresponding transmission channel, respectively.

According to Japanese Patent JP 06035508 [14], consider a NNCS with parameter uncertainties represented by T-S fuzzy model as follows:

Fuzzy rule i: if θ_1(t) is F_{i1} and if θ_2(t) is F_{i2} and … and θ_n(t) is Fin, then

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_i + \Delta\boldsymbol{A}_i)\boldsymbol{x}(t) + (\boldsymbol{B}_i + \Delta\boldsymbol{B}_i)\boldsymbol{u}(t-\tau_2(t)) \quad (1)$$

$$\boldsymbol{z}(t) = \boldsymbol{C}_i\boldsymbol{x}(t-\tau_1(t)) \qquad (i=1,2,...,N) \quad (2)$$

where i is the number of if-then rules; F_{ij} (j=1,2,…,n) and θ(t)=[θ_1(t), θ_2(t),…,θ_n(t)]T are fuzzy sets and premise variables, respectively; $\boldsymbol{x}(t)\in R^n$, $\boldsymbol{u}(t)\in R^m$ and $\boldsymbol{z}(t)\in R^l$ are state vector, control input vector and output control vector, respectively; $\boldsymbol{A}_i$ $\boldsymbol{B}_i$ and $\boldsymbol{C}_i$ are constant matrices with appropriate dimensions, respectively; $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ are time-varying unknown matrices with appropriate dimensions, respectively, which stand for uncertainty of structure in the system model and can be described as

$$[\Delta\boldsymbol{A}_i, \quad \Delta\boldsymbol{B}_i] = \boldsymbol{D}\boldsymbol{F}(t)[\boldsymbol{E}_{ai}, \quad \boldsymbol{E}_{bi}]$$

where $\boldsymbol{D}$ 、 $\boldsymbol{E}_{ai}$ and $\boldsymbol{E}_{bi}$ are known constant matrices with appropriate dimensions; $\boldsymbol{F}(t)$ is an unknown matrix function with Lebesgue measurable elements satisfying the inequality $\boldsymbol{F}^T(t)\boldsymbol{F}(t)\le\boldsymbol{I}$.

For any given $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$, by using a singleton fuzzifier, product inference and centre-average defuzzifier, the local models can be integrated into a global nonlinear model:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{N}\mu_i(\theta(t))[(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i)\boldsymbol{x}(t)+(\boldsymbol{B}_i+\Delta\boldsymbol{B}_i)\boldsymbol{u}(t-\tau_2(t))] \quad (3)$$

$$\boldsymbol{z}(t) = \sum_{i=1}^{N}\mu_i(\theta(t))\boldsymbol{C}_i\boldsymbol{x}(t-\tau_1(t)) \quad (4)$$

where $\mu_i(\theta(t)) = \dfrac{a_i(\theta(t))}{\sum_{i=1}^{N}a_i(\theta(t))}$, and $\mu_i(\theta(t))$ is the weight ratio of each fuzzy rule satisfying $\mu_i(\theta(t))\ge 0 \quad (i=1,2,...,N)$ and $\sum_{i=1}^{N}\mu_i(\theta(t))=1$; $a_i(\theta(t)) = \prod_{j=1}^{n}F_{ij}(\theta_j(t))$, and $F_{ij}(\theta_j(t))$ is grade of the membership of θj(t) in fuzzy set Fij, and it is assumed that $a_i(\theta(t))\ge 0 (i=1,2,...,N)$ and $\sum_{i=1}^{N}a_i(\theta(t))>0$.

It is assumed that all of the state variables are unmeasurable for NNCS, but it is observable. According to parallel distributed compensation (PDC) technique, the fuzzy dynamic output feedback controller is observer-based of full-dimension which shares the same premise parts as the fuzzy system, and has the following form

$$\dot{\hat{\boldsymbol{x}}}(t) = \sum_{i=1}^{N}\mu_i(\theta(t))[(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i)\hat{\boldsymbol{x}}(t)+(\boldsymbol{B}_i+\Delta\boldsymbol{B}_i)\boldsymbol{u}(t)+ \\ \boldsymbol{I}_i(\boldsymbol{C}_i\boldsymbol{x}(t-\tau_1(t))-\boldsymbol{C}_i\hat{\boldsymbol{x}}(t)] \quad (5)$$

$$\boldsymbol{u}(t) = \sum_{i=1}^{N}\mu_i(\theta(t))\boldsymbol{K}_i\hat{\boldsymbol{x}}(t) \quad (6)$$

where $\hat{\boldsymbol{x}}(t)\in R^n$ is the estimation of state vector $\boldsymbol{x}(t)$; $\boldsymbol{K}_j$ is the control gain matrix for the jth controller rule, $\boldsymbol{I}_i$ is the observer gain matrix for the ith observer rule.

Substituting (6) into (4) and (5) yields the close-loop NNCS as follow

$$\dot{\hat{\boldsymbol{x}}}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\theta(t))\mu_j(\theta(t))[(\tilde{\boldsymbol{A}}_i+\tilde{\boldsymbol{B}}_i\boldsymbol{K}_j- \\ \boldsymbol{I}_i\boldsymbol{C}_i)\hat{\boldsymbol{x}}(t)+\boldsymbol{I}_i\boldsymbol{C}_i\boldsymbol{x}(t-\tau_1(t)] \quad (7)$$

$$\dot{x}(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\theta(t))\mu_j(\theta(t))[\tilde{\boldsymbol{A}}_i\boldsymbol{x}(t)+\tilde{\boldsymbol{B}}_i\boldsymbol{K}_j\hat{x}(t-\tau_2(t))] \tag{8}$$

where $\tilde{\boldsymbol{A}}_i=\boldsymbol{A}_i+\Delta\boldsymbol{A}$ 、 $\tilde{\boldsymbol{B}}_i=\boldsymbol{B}_i+\Delta\boldsymbol{B}$

Defining the estimation error $\boldsymbol{e}(t)=\hat{\boldsymbol{x}}(t)-\boldsymbol{x}(t)$, we get

$$\begin{aligned}\dot{\boldsymbol{e}}(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\theta(t))\mu_j(\theta(t))[(\tilde{\boldsymbol{A}}_i+\tilde{\boldsymbol{B}}_i\boldsymbol{K}_j-\boldsymbol{I}_i\boldsymbol{C}_i)\boldsymbol{e}(t)+\\ (\tilde{\boldsymbol{B}}_i\boldsymbol{K}_j-\boldsymbol{I}_i\boldsymbol{C}_i)\boldsymbol{x}(t)+\boldsymbol{I}_i\boldsymbol{C}_i\boldsymbol{x}(t-\tau_1(t))-\\ \tilde{\boldsymbol{B}}_i\boldsymbol{K}_j\boldsymbol{x}(t-\tau_2(t))-\tilde{\boldsymbol{B}}_i\boldsymbol{K}_j\boldsymbol{e}(t-\tau_2(t))]\end{aligned} \tag{9}$$

Lemma 1[15]: Given constant matrices S,H and E of appropriate dimensions and with matrix S symmetric, then

$$\boldsymbol{S}+\boldsymbol{H}\boldsymbol{F}(t)\boldsymbol{E}+\boldsymbol{E}^T\boldsymbol{F}^T(t)\boldsymbol{H}^T<0$$

for all F(t) satisfying $\boldsymbol{F}^T(t)\boldsymbol{F}(t)\le\boldsymbol{I}$,if and only if there exists a scalar $\varepsilon>0$ such that

$$\boldsymbol{S}+\varepsilon\boldsymbol{H}\boldsymbol{H}^T+\varepsilon^{-1}\boldsymbol{E}^T\boldsymbol{E}<0$$

Lemma 2 (matrix separation lemma [16]). If

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ & a_{22} & a_{23} & a_{24}\\ & & a_{33} & a_{34}\\ * & & & a_{44}\end{bmatrix}<0,\begin{bmatrix} b_{11} & b_{12} & b_{13}\\ & b_{22} & b_{23}\\ * & & b_{33}\end{bmatrix}<0,\begin{bmatrix} c_{11} & c_{12}\\ & c_{22}\end{bmatrix}<0,$$

then

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0\\ & a_{22} & a_{23} & a_{24} & 0\\ & & a_{33} & a_{34} & 0\\ & * & & a_{44} & 0\\ & & & & 0\end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ & b_{11} & 0 & b_{12} & b_{13}\\ & & 0 & 0 & 0\\ & & & b_{22} & b_{23}\\ & & & & b_{33}\end{bmatrix}+\begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0\\ & c_{22} & 0 & 0 & 0\\ & & 0 & 0 & 0\\ & * & & 0 & 0\\ & & & & 0\end{bmatrix}<0$$

3. MAIN RESULTS

3.1. Robust Guaranteed Cost Fault-tolerant Analysis of NNCS with Actuator Failures Fore-casting

Considering the possible actuator failures, we can introduce a switching matrix $\boldsymbol{L}$ as follow

$\boldsymbol{L}$=diag{l1,l2,…ln},

where $l_i=\begin{cases}1, & \text{the ith actuator normal}\\ 0, & \text{the ith actuator failure}\end{cases}$

$\boldsymbol{L}\in\Omega$, and Ω denotes the set of all possible actuator failures switching matrices except $\boldsymbol{L}$=0.

When the switching matrix $\boldsymbol{L}$ is laid between the feedback matrix $\boldsymbol{K}_j$ and input matrix $\tilde{\boldsymbol{B}}_i$, we get the following uncertain nonlinear networked closed-loop fault system (UNNCFS)

$$\begin{aligned}\dot{\boldsymbol{x}}(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\theta(t))\mu_j(\theta(t))[\tilde{\boldsymbol{A}}_i\boldsymbol{x}(t)+\\ \tilde{\boldsymbol{B}}_i\boldsymbol{L}\boldsymbol{K}_j\boldsymbol{x}(t-\tau_2(t))+\tilde{\boldsymbol{B}}_i\boldsymbol{L}\boldsymbol{K}_j\boldsymbol{e}(t-\tau_2(t))]\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{\boldsymbol{e}}(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i(\theta(t))\mu_j(\theta(t))[(\tilde{\boldsymbol{A}}_i+\tilde{\boldsymbol{B}}_i\boldsymbol{L}\boldsymbol{K}_j-\boldsymbol{I}_i\boldsymbol{C}_i)\boldsymbol{e}(t)+\\ (\tilde{\boldsymbol{B}}_i\boldsymbol{L}\boldsymbol{K}_j-\boldsymbol{I}_i\boldsymbol{C}_i)\boldsymbol{x}(t)+\boldsymbol{I}_i\boldsymbol{C}_i\boldsymbol{x}(t-\tau_1(t))-\\ \tilde{\boldsymbol{B}}_i\boldsymbol{L}\boldsymbol{K}_j\boldsymbol{x}(t-\tau_2(t)-\tilde{\boldsymbol{B}}_i\boldsymbol{L}\boldsymbol{K}_j\boldsymbol{e}(t-\tau_2(t)]\end{aligned} \tag{11}$$

According to the UNNCFS, we define the guaranteed cost function

$$J=\int_0^{\infty}[x^T(t)\boldsymbol{Q}x(t)+u^T(t)\boldsymbol{R}u(t)]dt \tag{12}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are given positive definite symmetric matrices.

In our study, based on observer, the aim of robust fault-tolerant guaranteed cost control is to seek the control gain matrix $\boldsymbol{K}_j$ and the observer gain matrix $\boldsymbol{I}_i$ such that the UNNCFS (10)-(11) is asymptotically stable and the guaranteed cost function J satisfies $J\le J^*$, and then J^* is said to upper boundary of guaranteed cost function.

Theorem 1: For the UNNCFS described by (10)-(11) and the guaranteed cost function J, given positive constants τ_{sc1}, τ_{sc2}, τ_{ca1}, τ_{ca2}, δ_{1M}, δ_{2M} and d, if there exit matrices, $\bar{\boldsymbol{Q}}=\bar{\boldsymbol{Q}}^T>0$, $\boldsymbol{R}=\boldsymbol{R}^T>0$, $\bar{\boldsymbol{R}}_i=\bar{\boldsymbol{R}}_i^T>0$, $\bar{\boldsymbol{Z}}_i=\bar{\boldsymbol{Z}}_i^T>0$ $(i=1,2)$, $\bar{\boldsymbol{Q}}_{ij}=\bar{\boldsymbol{Q}}_{ij}^T>0$ (i=1,2,j=1,2), , and $\boldsymbol{X}$,$\boldsymbol{Y}_{1j}$,$\boldsymbol{Y}_{2i}$ $(i,j=1,2,\dots,N)$ with appropriate dimensions, satisfying the following LMIs

$$\begin{bmatrix} \bar{\boldsymbol{D}}_{ij}^1 & \bar{\boldsymbol{H}}_1 & \varepsilon_1^{-1}\bar{\boldsymbol{E}}_1^T & \bar{\boldsymbol{H}}_2 & \varepsilon_2^{-1}\bar{\boldsymbol{E}}_2^T\\ & -\varepsilon_1^{-1}\boldsymbol{I} & 0 & 0 & 0\\ & & -\varepsilon_1^{-1}\boldsymbol{I} & 0 & 0\\ & * & & -\varepsilon_2^{-1}\boldsymbol{I} & 0\\ & & & & -\varepsilon_2^{-1}\boldsymbol{I}\end{bmatrix}<0 \tag{13}$$

$$\begin{bmatrix} 0 & 0 & -\boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T & -\boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T\tau_{c2} & -\boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T\tau_{s2}\\ & 0 & \boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T & \boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T\tau_{c2} & \boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T\tau_{s2}\\ & & -\boldsymbol{Y}_{2i}\boldsymbol{C}_i-\boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T & -\boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T\tau_{c2} & -\boldsymbol{C}_i^T\boldsymbol{Y}_{2i}^T\tau_{s2}\\ & * & & 0 & 0\\ & & & & 0\end{bmatrix}<0 \tag{14}$$

Where

$$\bar{\boldsymbol{D}}_{ij}^1=\begin{bmatrix} \bar{\boldsymbol{D}}_{1ij}^1 & \bar{\boldsymbol{D}}_{2ij}^1\\ * & \boldsymbol{D}_{3ij}^1\end{bmatrix}<0$$

$$\bar{\boldsymbol{D}}_{1ij}^1=\begin{bmatrix} \bar{\ddot{\boldsymbol{O}}}_{11} & \bar{\boldsymbol{R}}_1 & \bar{\ddot{\boldsymbol{O}}}_{13} & \bar{\ddot{\boldsymbol{O}}}_{14} & 0 & \bar{\ddot{\boldsymbol{O}}}_{16}\\ & -\bar{\boldsymbol{R}}_1 & 0 & 0 & 0 & 0\\ & & -\bar{\boldsymbol{Z}}_1 & \bar{\ddot{\boldsymbol{O}}}_{34} & 0 & 0\\ & & & \bar{\ddot{\boldsymbol{O}}}_{44} & \bar{\boldsymbol{R}}_2 & \bar{\ddot{\boldsymbol{O}}}_{46}\\ & * & & & -\bar{\boldsymbol{R}}_2 & 0\\ & & & & & -\bar{\boldsymbol{Z}}_2\end{bmatrix}$$

$$\breve{O}_{11} = A_i X^T + X A_i^T + \bar{Q}_{11} + \bar{Q}_{21} - \bar{R}_1 - \bar{Z}_1 + \bar{Q}$$

$$\breve{O}_{13} = B_i L Y_{1j} + \bar{Z}_1 \quad \breve{O}_{14} = d Y_{1j}^T L^T B_i^T$$

$$\breve{O}_{16} = B_i L Y_{1j} \quad \breve{O}_{34} = -d Y_{1j}^T L^T B_i^T$$

$$\breve{O}_{44} = d A_i X^T + d X A_i^T + d B_i L Y_{1j} + d Y_{1j}^T L^T B_i^T - \bar{R}_2 - \bar{Z}_2 + \bar{Q}_{12} + \bar{Q}_{22}$$

$$\breve{O}_{46} = -d B_i L Y_{1j} + \bar{Z}_2$$

$$\bar{D}_{2ij}^1 = \begin{bmatrix} XA_i^T\tau_{c2} & XA_i^T\tau_{s2} & \breve{O}^T\tau_{c2} & \breve{O}^T\tau_{s2} & Y_j^T \\ 0 & 0 & 0 & 0 & 0 \\ \breve{O}^T\tau_{c2} & \breve{O}^T\tau_{s2} & -\breve{O}^T\tau_{c2} & -\breve{O}^T\tau_{s2} & 0 \\ 0 & 0 & (XA_i^T + \breve{O}^T)\tau_{c2} & (XA_i^T + \breve{O}^T)\tau_{s2} & Y_j^T \\ 0 & 0 & 0 & 0 & 0 \\ \breve{O}^T\tau_{c2} & \breve{O}^T\tau_{s2} & -\breve{O}^T\tau_{c2} & -\breve{O}^T\tau_{s2} & 0 \end{bmatrix}$$

$$\breve{O} = B_i L Y_{1j}$$

$$D_{3ij}^1 = diag[0 \quad 0 \quad 0 \quad 0 \quad -R^{-1}]$$

$$\bar{H}_1^T = [D^T \quad 0 \ \cdots \ 0 \quad \tau_{c2} D^T \quad \tau_{s2} D^T \quad 0 \quad 0 \quad 0]_{1\times 11}$$

$$\bar{E}_1 = [E_{ai} X^T \quad 0 \quad E_{bi} L Y_{1j} \quad 0 \quad 0 \quad E_{bi} L Y_{1j} \quad 0 \ \cdots \ 0]_{1\times 11}$$

$$\bar{H}_2^T = [0 \quad 0 \quad 0 \quad dD^T \quad 0 \cdots \ 0 \quad \tau_{c2} D^T \quad \tau_{s2} D^T \quad 0]_{1\times 11}$$

$$\bar{E}_2 = [E_{bi} L Y_{1j} \quad 0 \quad -E_{bi} L Y_{1j} \quad E_{ai} X^T + E_{bi} L Y_{1j} \quad 0 \quad -E_{bi} L Y_{1j} \quad 0 \ \cdots \ 0]_{1\times 11}$$

then existing the observer-based state-feedback control law (5)-(6) for robust guaranteed cost fault-tolerant control such that UNNCFS(10)-(11) with $K_j = Y_{1j} X^{-T}$ and $I_i = d^{-1} X Y_{2i}$ is asymptotically stable, and the guaranteed cost function J satisfies the following boundary.

$$J \le J^* = \begin{bmatrix} x^T(0) \\ e^T(0) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} x(0) \\ e(0) \end{bmatrix} + \sum_{i=1}^{2} \int_{-\alpha_i}^{0} \begin{bmatrix} x^T(s) \\ e^T(s) \end{bmatrix}^T \begin{bmatrix} Q_{i1} & 0 \\ 0 & Q_{i2} \end{bmatrix} \begin{bmatrix} x(s) \\ e(s) \end{bmatrix} ds + \int_{-\tau_{c2}}^{0} \tau_{c2} \int_{\theta}^{0} \begin{bmatrix} \dot{x}^T(s) \\ \dot{e}^T(s) \end{bmatrix}^T \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} \dot{x}(s) \\ \dot{e}(s) \end{bmatrix} ds d\theta + \int_{-\tau_{s2}}^{0} \tau_{s2} \int_{\theta}^{0} \begin{bmatrix} \dot{x}^T(s) \\ \dot{e}^T(s) \end{bmatrix}^T \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} \begin{bmatrix} \dot{x}(s) \\ \dot{e}(s) \end{bmatrix} ds d\theta$$

Proof: the proof is cut off due to space limitation. contact the authors for the detailed proof.

Remark 1: In order to determine the observer gains I_i, we have to set $P_2 = dP_1$ to obtain the LMI conditions. The scalar d should be given prior to solve LMI (13) and (14). How to choose scalar d for optimization is still an open problem. Hence, we determine the scalar d by the trial and error method.

Remark 2: The theorem 1 provides a robust fault-tolerant guaranteed cost condition in form of delay-dependent, which include some of the information of delay from controller to actuator and from sensor to controller. Especially, introducing of lower delay bound, the result is less conservative [17].

3.2. Optimization of Guaranteed Cost

Control gain matrix K_j and observer gain matrix I_i can be solved *via* the above theorem 1, and the guaranteed cost function J satisfies $J \le J^*$, but J^* is not the minimum upper boundary of guaranteed cost, K_j and I_i gain matrices are only the suboptimal solution. Thence, we introduced the following optimization algorithm for optimal guaranteed cost gain matrices which minimize J^*.

$$\min J^* = \vartheta_1 + \vartheta_2 + Trace(M_{11}) + Trace(M_{12}) + Trace(M_{21}) + Trace(M_{22}) + Trace(T_1) + Trace(T_2) + Trace(S_1) + Trace(S_2)$$

s.t. *i*) (13) *ii*) (14)

$$iii) \begin{bmatrix} -\vartheta_1 & x^T(0) \\ * & -P_1^{-1} \end{bmatrix} < 0 \quad iv) \begin{bmatrix} -\vartheta_2 & e^T(0) \\ * & -P_2^{-1} \end{bmatrix} < 0$$

$$v) \begin{bmatrix} -M_{11} & N_{11}^T \\ * & -Q_{11}^{-1} \end{bmatrix} < 0 \quad vi) \begin{bmatrix} -M_{12} & N_{12}^T \\ * & -Q_{12}^{-1} \end{bmatrix} < 0$$

$$vii) \begin{bmatrix} -M_{21} & N_{21}^T \\ * & -Q_{21}^{-1} \end{bmatrix} < 0 \quad viii) \begin{bmatrix} -M_{22} & N_{22}^T \\ * & -Q_{22}^{-1} \end{bmatrix} < 0$$

$$ix) \begin{bmatrix} -W_1 & \tau_{c2} T_1^T \\ * & -\tau_{c2} R_1^{-1} \end{bmatrix} < 0 \quad x) \begin{bmatrix} -W_2 & \tau_{c2} T_2^T \\ * & -\tau_{c2} R_2^{-1} \end{bmatrix} < 0$$

$$xi) \begin{bmatrix} -G_1 & \tau_{s2} S_1^T \\ * & -\tau_{s2} Z_1^{-1} \end{bmatrix} < 0 \quad xii) \begin{bmatrix} -G_2 & \tau_{s2} S_2^T \\ * & -\tau_{s2} Z_2^{-1} \end{bmatrix} < 0$$

where:

$$\int_{-\alpha_1}^{0} x^T(s) x(s) ds = N_{11}^T N_{11}$$

$$\int_{-\alpha_1}^{0} e^T(s) e(s) ds = N_{12}^T N_{12}$$

$$\int_{-\alpha_2}^{0} x^T(s) x(s) ds = N_{21}^T N_{21}$$

$$\int_{-\alpha_2}^{0} e^T(s) e(s) ds = N_{22}^T N_{22}$$

$$\int_{-\tau_{c2}}^{0} \int_{\theta}^{0} \dot{x}^T(s) \dot{x}(s) ds d\theta = T_1^T T_1$$

$$\int_{-\tau_{c2}}^{0} \int_{\theta}^{0} \dot{e}^T(s) \dot{e}(s) ds d\theta = T_2^T T_2$$

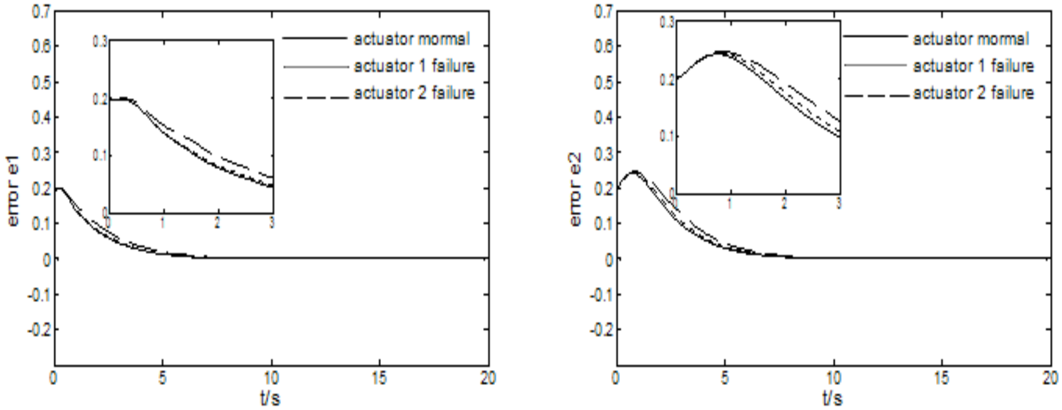

Fig. (2) response curve of the error vector.

The proposed optimization algorithm is a convex problem which is constrained by LMIs, and then it can be solved via the mincx solver of LMI tools.

4. THE CIRCUIT

In this section, we use an example to demonstrate the effectiveness of our main result.Consider the following NNCS with parameter uncertainties which is borrowed from [12]:

The membership function are $M_1(x_2) = \sin^2 x_2$ and $M_2(x_2) = \cos^2 x_2$

Rule 1: if $x2$ is $M1$, then

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_1 + \Delta\boldsymbol{A}_1)\boldsymbol{x}(t) + (\boldsymbol{B}_1 + \Delta\boldsymbol{B}_1)\boldsymbol{u}(t)$$

$$\boldsymbol{z}(t) = \boldsymbol{C}_1\boldsymbol{x}(t)$$

Rule 2: if $x2$ is $M2$, then

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_2 + \Delta\boldsymbol{A}_2)\boldsymbol{x}(t) + (\boldsymbol{B}_2 + \Delta\boldsymbol{B}_2)\boldsymbol{u}(t)$$

$$\boldsymbol{z}(t) = \boldsymbol{C}_2\boldsymbol{x}(t)$$

Where

$$\boldsymbol{A}_1 = \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix} \boldsymbol{A}_2 = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \boldsymbol{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -0.5 \end{bmatrix}$$

$$\boldsymbol{B}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \boldsymbol{C}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -0.5 \end{bmatrix} \boldsymbol{C}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$$

and matrices $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ (i=1,2) satisfy $[\Delta\boldsymbol{A}_i \ , \ \Delta\boldsymbol{B}_i \ ] = \boldsymbol{DF}(t)[\boldsymbol{E}_{ai} \ , \ \boldsymbol{E}_{bi} \ ]$, where

$$\boldsymbol{D} = \begin{bmatrix} 0.31 & 0.1 \\ 0 & 0 \end{bmatrix} \quad \boldsymbol{F}(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}$$

$$\boldsymbol{E}_{ai} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix} \quad \boldsymbol{E}_{bi} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix} \quad (i = 1, 2)$$

In cases of actuator normal and possible failures, the switching matrices L_0=diag (1,1), L_1=diag (0,1) and L_2=diag (1,0) indicate actuator normal and actuator 1,2 failure, respectively.

Considering the actual NCS, we assume T=0.05s, choose d=0.4, $\delta_{iM}(i=1,2)=2$, $\tau_{sci}(i=1,2)$ are 0.01s and 0.1s, respectively. $\tau_{cai}(i=1,2)$ are 0.01s and 0.1s, respectively. Then we can solve convex problem (13), (14) by using LMI toolbox to obtain

$$\boldsymbol{K}_1 = \begin{bmatrix} -3.5235 & -0.3810 \\ 1.2765 & -1.8105 \end{bmatrix}, \boldsymbol{K}_2 = \begin{bmatrix} -4.2102 & -0.4680 \\ 1.4688 & -1.9980 \end{bmatrix}$$

$$\boldsymbol{I}_1 = \begin{bmatrix} -1.1745 & 0.1270 \\ 0.4255 & 0.6035 \end{bmatrix}, \boldsymbol{I}_2 = \begin{bmatrix} -1.1695 & 0.1300 \\ 0.4080 & 0.5550 \end{bmatrix}$$

and guaranteed cost function $J \le 32.246$.

Setting the initial conditions of the system $x(0)=[2,2]^T$, $e(0)=[0.2,0.2]^T$, in the cases of L_0, L_1 and L_2, response curve of the error vector $e1$, $e2$ are shown in Fig. (**2**), and zero-input response curve of the state vector $x1$, $x2$ are shown in Fig. (**3**).

It can be seen from Fig. (**2**) that state estimation error quickly approximate to zero, which show that the states of closed-loop NNCS with possible actuator failures can be estimated well via the observer.

Fig. (**3**) can be seen that NNCFS is not only asymptotically stable, but also has good dynamic performance. These indicate that the proposed method makes the NNCS against possible actuator failures have the capability of robust fault-tolerant guaranteed cost.

In addition, in terms of 3.2, we can receive the following optimal robust guaranteed cost fault-tolerant controller and observer gains:

$$\boldsymbol{K}_{opt1} = \begin{bmatrix} -0.7349 & -0.2540 \\ 0.8521 & 1.2074 \end{bmatrix} \quad \boldsymbol{K}_{opt2} = \begin{bmatrix} -0.8339 & -0.2650 \\ 0.8116 & 1.1210 \end{bmatrix}$$

the upper boundary of guaranteed cost function J^*=28.363 .

5. CURRENT AND FUTURE DEVELOPMENTS

In this paper, based on T-S fuzzy model, we studies the control problem of the robust guaranteed cost fault-tolerant

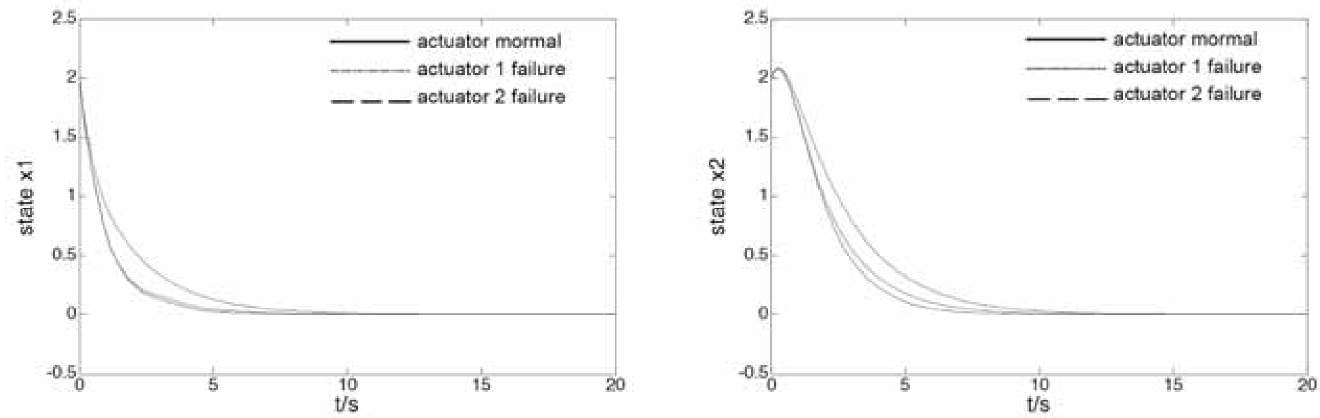

Fig. (3) response curve of the state vector.

on uncertain NNCS which includes time-delay and packet dropout under the condition of possible actuator failures. By constructing appropriate Lyapunov-Krasovskii functional, adopting the Jensen inequality and matrix separation technologies, this paper deduces a delay-dependent sufficient condition for robust guaranteed cost fault-tolerant. Further, we give the approach of solving for optimal controller and observer gains via optimizing. Finally, an example is used to illustrate the effectiveness and feasibility of proposed approach. In the future, in order to save the limited communication resources and improve the safety reliablity of NCS, we will research the active-passive hybrid fault-tolerant control under the event-trigger communication mechanism against the actuator failure.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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