Harmonic Frequency Estimation Based on Modified-MUSIC Algorithm in Power System

Xiangwen Sun* and Ligong Sun

Electrical. Eng. College, Henan University of Science & Technology, Luoyang, 471039, China

Abstract: This paper presents a new harmonics frequency estimation method. Unlike the conventional harmonic frequency estimation method (fast Fourier transform), the new algorithm is based on spectrum analysis techniques often used to estimate the direction of angle; the most popular is the multiple signal classification (MUSIC) algorithm. The drawbacks of MUSIC algorithm are concluded. Improved-MUSIC approximation algorithm is introduced and compared with FFT based on algorithm for harmonic frequency estimation. Theoretical analysis and simulations show this algorithm is a super-resolution algorithm with small data length.

Keywords: Harmonic frequency estimation, modified-MUSIC algorithm, Eigen decomposition.

1. INTRODUCTION

With the rapid development of power electronics technology, they are increasingly being installed in power grids, which makes grid voltage and current waveform distortion more problematic. If the harmonic components exceed the specification, it will not only result in the decline of efficiency of energy production, transmission and utilization, but can also cause relay protection device malfunction and electric energy metering confusion. Furthermore, harmonic will not only cause serious interference to the communication and electronic equipment, but will also influence the electric power system and the electric equipment operational safety, reliability, stability and economy seriously. The accurate, real-time detection of the harmonic frequency is the prerequisite for harmonic control. Therefore, the rapid estimation method for harmonic frequency has become the scholar's research goal recently, and many results have been achieved. Such as fast Fourier transform (FFT), short-time Fourier transform (STFT), Prony method [1], Pisarenko's harmonic decomposition [2], Singular Value Decomposition (SVD) [3-5], wavelet transforms, Adaline artificial neural network, Multiple signal classification (MUSIC) algorithm [6], Estimating signal parameters via rotational invariance techniques (ESPRIT) algorithm [7] etc.

It is satisfactory that the estimation accuracy of FFT and STFT, requires a very low computational effort, for stationary signals where properties do not change with time. For non-stationary signals, these methods can not track the signal's dynamic change and suffer from spectral leakage and picket fence effects.

The Prony method assumes that the original data can be fitted by a linear combination of multiple exponential terms. On this basis, the frequency, amplitude and phase of exponential term can be estimated by using the least square method estimates. The drawback of Prony method [1] is that it is computationally expensive because solving a nonlinear least squares problem is necessary. Besides, it is sensitive to noise

SVD, Adaline artificial neural network and other methods mentioned above can improve accuracy of harmonic frequency estimation, but they have a few shortcomings. SVD is more mathematically complicated than FFT, this limits SVD from working in online systems. Adaline artificial neural network has been used to track time-varying harmonics which are polluted by white noise. One of the drawbacks is the tracking error which is relatively great due to the non-stationary nature of the signal, i.e., sudden changes in frequency, phase, and amplitude of the fundamental and harmonic components.

In recent years, with the development of spectrum analysis techniques, Pisarenko's harmonic decomposition, multiple signal classification (MUSIC) algorithm, estimating signal parameters via rotational invariance techniques (ESPRIT) algorithm have been used to estimate the harmonics in power grid.

MUSIC algorithm is one of the most popular subspace techniques for estimating the directions of arrival (DOA) of multiple signals. MUSIC algorithm searches for directions of the steering vectors, which are orthogonal to the noise subspace. The frequencies of the multiple incident signals can be estimated by spectral peak searching. Using the orthogonality of signal subspace and noise subspace, MUSIC algorithm could resist noise more efficiently.

The variance of MUSIC algorithm approaches Cramér–Rao bounds. So, it is a super-resolution approach. The utmost difficulty of this algorithm in engineering practices is matrix decomposition which is a huge computational burden and isn't suitable for real-time applications.

*Address correspondence to this author at the Xiyuan Road, Luoyang, Postcard: 471039, China; Tel: 86+15670318675;
E-mail: sun_xiangwen@163.com

The paper is organized as follows. In Section 2, we first describe data model and state the problem. In Section 3, a modified MUSIC algorithm will be proposed based on a general MUSIC algorithm. In Section 4, several simulated signals are tested to prove the superior performance of the modified algorithm.

2. THE PRINCPLE OF MUSIC ALGORITHM

Hypothesis of the harmonic signal in power system can be described as follows:

$$x(n)=\sum_{i=1}^{M}A_i\cos(2\pi f_i n+\varphi_i)+e(n) \tag{1}$$

Where A_i, f_i, φ_i are the amplitude, normalized frequency,

Initial phase, respectively; $e(n)$ represents the corrupting additive zero-mean noise, and M is the order of harmonic.

Convert $x(n)$ from (1) to complex frequency signal as formula (2) for the mathematical simplicity.

$$x(n)=\sum_{i=1}^{M}A_i e^{j(2\omega_i n+\varphi_i)}+e(n) \tag{2}$$

Where $\omega=2\pi f$, $j=\sqrt{-1}$

Simultaneously, considering harmonics in power system can be regarded as the complex cosine signal; covariance function of $x(n)$ can be written as:

$$r(k)=\sum_{i=1}^{M}a_i e^{j\omega_i k}+P_n\delta(k) \tag{3}$$

Where $a_i=A_i^2$, P_n is the variance of $e(n)$.

The $P\times P$ array covariance matrix can be written as:

$$R=\begin{bmatrix} r(0) & r(1) & \cdots & r(P-1)\\ r^*(-1) & r(0) & \cdots & r(P-2)\\ \vdots & \vdots & \vdots & \vdots\\ r^*(P-1) & r^*(P-2) & \cdots & r(0)\end{bmatrix} \tag{4}$$

According to the hypothesis condition (1) and the properties of the Toeplitz matrix, we can get

$$rank(R)=M \tag{5}$$

According to the matrix theory, the Eigen-decomposition of the covariance matrix R can be written as:

$$R=\sum_{k=1}^{P}\lambda_k e_k e_k{}^H \tag{6}$$

Where λ_k and e_k are the Eigenvalues and corresponding

Eigenvectors, $(\cdot)^H$ is the Hermitian transpose operator.

Let the eigenvalues be sorted in no ascending order, then, the matrices

$$V_S=\left[V_1,V_2,\cdots,V_M\right], V_N=\left[V_{M+1},V_{M+2},\cdots,V_P\right] \tag{7}$$

Contain the signal-space and noise-subspace eigenvectors, respectively.

In practical situations, the exact array covariance matrix R is unavailable and its sample estimate is,

$$\hat{R}\triangleq\frac{1}{K}\sum_{k=1}^{P}x(\mathrm{n})\,\mathrm{x}(\mathrm{n})^H \tag{8}$$

The Eigen decomposition of the sample covariance matrix

$$\hat{R}=\hat{V}_S\hat{\Lambda}_S\hat{V}_S{}^H+\hat{V}_N\hat{\Lambda}_N\hat{V}_N{}^H \tag{9}$$

Where the sample eigenvalues are again sorted in ascending order $(\lambda_1\geq\lambda_2\cdots\geq\lambda_P)$ and the matrices $\hat{V}_S=\left[\hat{V}_1,\hat{V}_2,\cdots,\hat{V}_M\right],\hat{V}_N=\left[\hat{V}_{M+1},\hat{V}_{M+2},\cdots,\hat{V}_P\right]$ contain in their columns are the signal-subspace and noise-subspace eigenvectors of $\hat{R}$ respectively. Correspondingly, the diagonal matrices $\hat{\Lambda}_S=diag\{\lambda_1,\cdots\lambda_M\}$ and $\hat{\Lambda}_N=diag\{\lambda_{M+1},\cdots\lambda_P\}$ are built from the signal-subspace and noise-subspace eigenvalues of $\hat{R}$, respectively.

Steering vector corresponding to the harmonics frequency is given by

$$a(\omega)=\begin{bmatrix} 1 & e^{-j\omega} & \cdots & e^{-j(p-1)\omega}\end{bmatrix} \tag{10}$$

Based on the orthogonal characteristics of the noise subspace and steering vector, we can get

$$f(\omega)=a^H(\omega)V_nV_n^H a(\omega)=0 \tag{11}$$

Define pseudo power spectrum P_{MUSIC} as follows:

$$P_{MUSIC}=\frac{1}{f(\omega)}\quad \omega=1,2,\cdots\mathrm{M} \tag{12}$$

Then, the harmonics frequency can be estimated from minima the $f(\omega)$ by searching over ω with a fine grid. That means every spectral point (11) has to be computed. Therefore, general MUSIC algorithm has quite high computational complexity.

3. MODIFIED MUSIC ALGORITHM

According to (10), it is obvious that $f(\omega)$ is periodic in ω with the period 2π . Therefore, $f(\omega)$ can be expressed using its Fourier series expansion as:

$$\Delta\omega = \frac{2\pi}{2M-1} \tag{13}$$

Where $F_m = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(\omega)e^{-jm\omega}d\omega$ (14)

Truncating (13) to $2M-1$ points

$$f(\omega) ; \sum_{m=-M}^{M} F_m e^{jm\omega} = \sum_{m=-M}^{M} F_m z^m = P(\mathrm{z}) \tag{15}$$

Where

$\mathrm{z} = e^{j\omega}$

That means the harmonic frequency can be achieved by solving the roots of polynomial $P(z)=0$ in the unit circle.

The polynomial's coefficients F_m can be achieved by the DFT of $f(\omega)$,

$$\hat{F}_m = \frac{1}{2\pi}\sum_{l=-(M-1)}^{M-1} f(l\Delta\omega)e^{-jml\Delta\omega} \tag{16}$$

Where $\Delta\omega = \frac{2\pi}{2M-1}$

It is obvious that the value of M has an important effect on the truncation errors. From the viewpoint of reducing truncation errors, M has to be sufficiently high. On the other hand, the computational complexity of finding the roots may be high. So, let us change $P(z)$'s order from 2(M-1) to 2(j+1) by zero-padding ($\mathrm{j} \gg M$)

$$P(\mathrm{z}) = \sum_{m=-j+1}^{j-1} F_m e^{jm\omega} = \sum_{m=-(M-1)}^{M-1} F_m e^{jm\omega} = \sum_{m=-(M-1)}^{M-1} F_m z^m \tag{17}$$

Therefore, the approximate estimation of harmonics

frequency can be obtained by solving the root of the equation P(z)=0 which locates at the unit circle.

According to (9), this can be seen through direct calculation as:

$$\sigma_n^{2m} R^{-m} = V_s \Lambda_s^m V_s^H + V_n V_n^H \tag{18}$$

Where $\Lambda_{\mathrm{s}}=\mathrm{diag}[\frac{\sigma_{\mathrm{n}}^2}{\lambda_1},\frac{\sigma_{\mathrm{n}}^2}{\lambda_2},\mathrm{L}\frac{\sigma_{\mathrm{n}}^2}{\lambda_{\mathrm{M}}}]$

Take $\frac{\sigma_{\mathrm{n}}^2}{\lambda_{\mathrm{i}}}$<1 account into (18), it can be seen through direct calculation as:

$$\lim_{m\to\infty} \sigma_n^{2m} R^{-m} = V_N V_N^H \tag{19}$$

It means that we can get $\mathrm{V_N V_N^H}$ through (19), the Eigen decomposition procedure of $\hat{R}$ can be avoided.

Based on the analysis above, a Modified-MUSIC algorithm is proposed as below:

Step 1: Calculate $V_N V_N^H$ through (19)

Step 2: Construct $f(\omega)$ through (11)

Step 3: Construct $P(\mathrm{z})$ by FFT

Step 4: Get the root of the equation of $P(\mathrm{z})=0$ which locates at the unit circle.

Step 5: Search the local peaks of $f(\omega)$ at the location near the root which is obtained in Step 4

4. SIMULATION EXPERIMENT

In the section, we present some simulation results to illustrate the performance capability of proposed algorithms. We compare the quality of harmonic frequency estimated using our method with the FFT algorithms.

Case 1:

The simulated power harmonic signal is expressed as:

$$x(t) = \cos(2\pi\times 50t + \pi/4) + 0.3\cos(2\pi\times 100t + \pi/5) + 0.2\cos(2\pi\times 150t + \pi/6) \tag{20}$$

In this case, the sampling frequency is chosen as 1000Hz, 80 sampling point.

According to Fig (**1**), FFT and the algorithm, we proposed that the extra estimation of harmonic frequency obtained under the circumstance of inter-harmonics, does not exist.

Case 2:

In this case, the sampling frequency is chosen as 1000Hz, 80 sampling point

$$x(t) = 0.1\cos(2\pi\times 45t + \pi/4) + 0.05\cos(2\pi\times 50t + \pi/5) + 5\cos(2\pi\times 70t + \pi/6) . \tag{21}$$

It is well known that the frequency resolution of FFT mainly depends on the sampling frequency and the amount of data. In case 2 Sampling frequency is 1000 Hz, $f_s/N = 12.5Hz$, according to Fig. (**2**) the harmonics of 45Hz and 50Hz can not be identified by using FFT algorithm. Under the same conditions, the algorithm we proposed identifies every harmonic accurately.

Case 3:

The simulated power harmonic signal is expressed as:

$$x(t) = 2\cos(2\pi\times 35t + \pi/4) + 0.5\cos(2\pi\times 50t + \pi/5) + 0.2\cos(2\pi\times 70t + \pi/6) \tag{22}$$

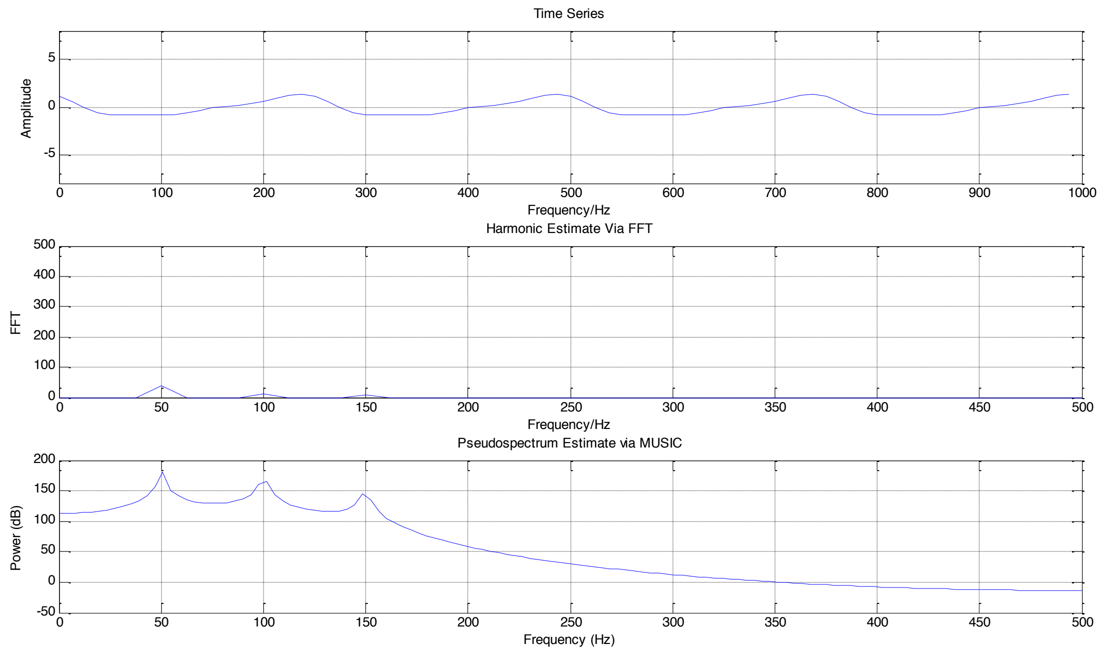

Fig. (1). Simulation result of Case 1.

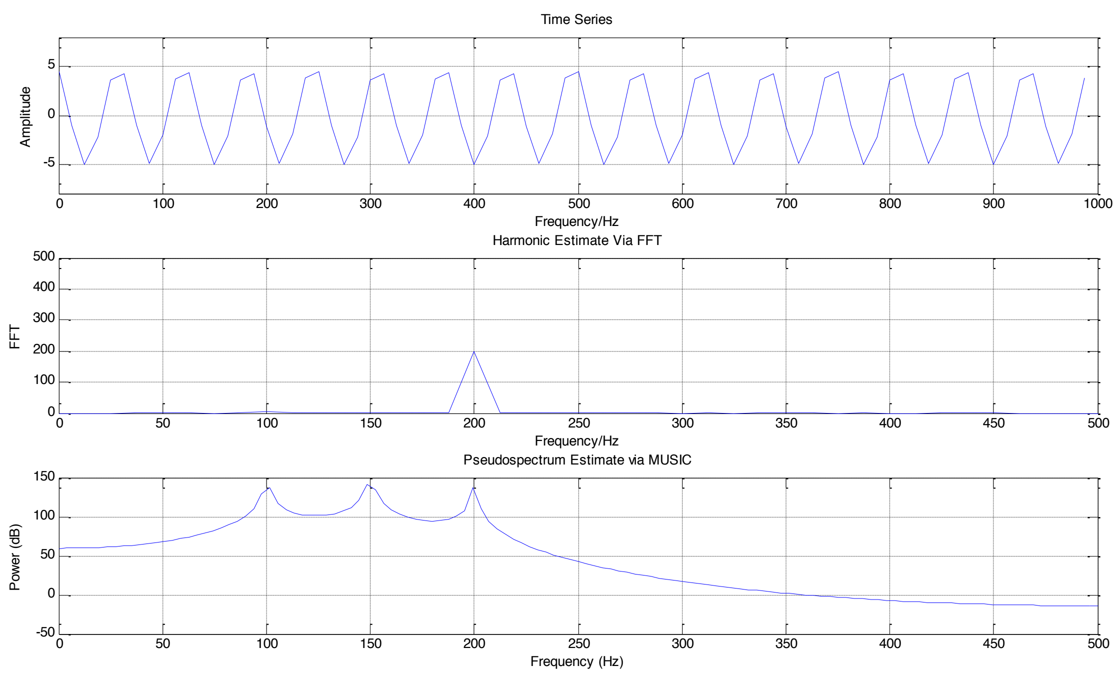

Fig. (2). Simulation result of Case 2.

In this case, the sampling frequency is chosen as 1000Hz, 80 sampling point.

According to Fig. (**3**), FFT algorithm will leave out the harmonics with less amplitude than the others. While, the algorithm we proposed can identify every harmonic under the same condition.

CONCLUSION

MUSIC algorithm is the most effective method for frequency estimation, time delay estimation and direction of arrival estimation etc., with arbitrarily high resolution in theory. The utmost difficulty of this algorithm in engineering practices is matrix decomposition, which is a huge computa-

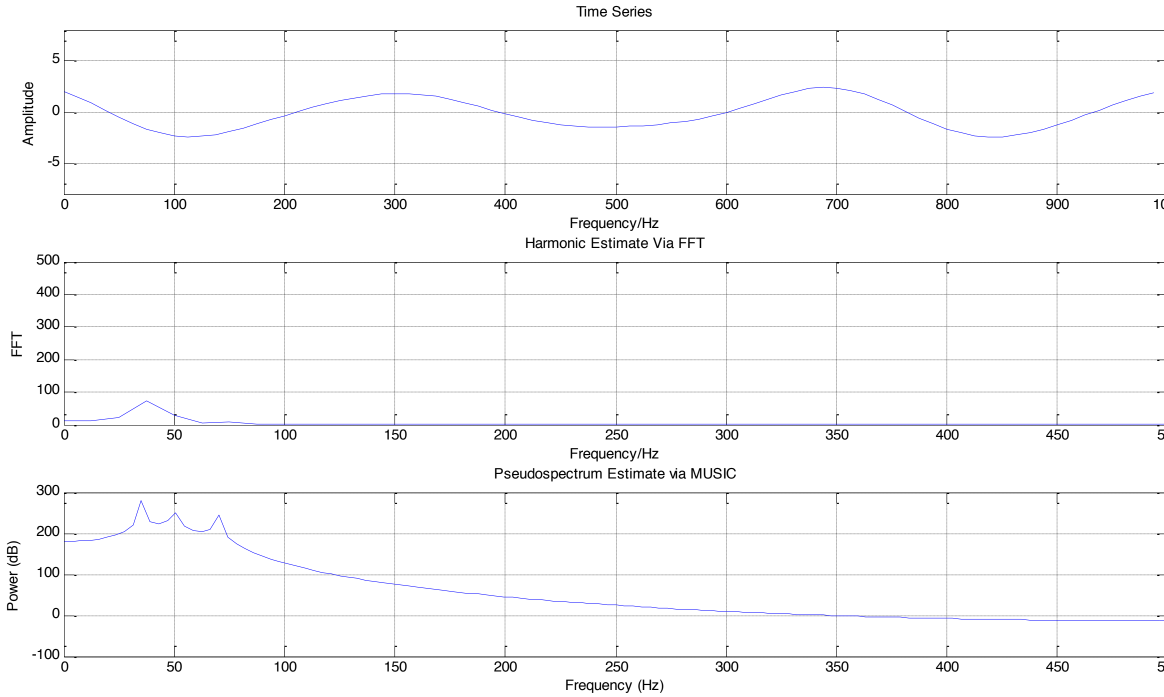

Fig. (3). Simulation result of Case 3.

tional burden. Therefore, MUSIC algorithm isn't suitable for real-time applications.

To address these problems mentioned above, the paper presents a new harmonic estimation algorithm and applies it in the harmonic frequency estimation in a power system. The experimental result indicates that harmonic frequency can be estimated accurately.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

Declared none.

REFERENCES

[1] Q. F. Ding,and H. Z. Cheng, “estimation of harmonic and inter-harmonics based on Prony method”, In: Transactions of China Electrotechnical Society 2005, vol. 20, no. 10, pp. 94-97.

[2] Y. H. Hu, and B.C. Phan, “Frequency estimation error in Pisarenko harmonic decomposition method”, In: Proceeding of the IEEE 1988, vol. 76, no. 1, pp. 82-84.

[3] T. Lobos, and T. Kozina, and H. Koglin, “Power system harmonics estimation using linear least squares method and SVD”, In: IEE Proceedings-Generation, Transmission and Distribution 2001, vol. 148, no. 6, pp. 567-572.

[4] T. Cai, and S. Duan, B. Liu, F. Liu, and C. Chen, “ Real-value MUSIC algorithm for power harmonics and interharmonics estimation”, In: Int J Circuit Theory, vol. 39, pp. 1023-35, 2010.

[5] A. Bracale, and G. Carpinelli, “An ESPRIT and DFT-based new method for the waveform distortion assessment in power”, In: Proc. 20th Electricity, pp. 241-251, 2009.

[6] S. Osowski “SVD technique for estimation of harmonic components in a power system”, In: Proceedings IEE Generation, Transmission and Distribution, vol. 141, pp. 430-4, 1994.

[7] B. Subudhi and A. M. Panda “Parameter estimation techniques applied to power networks.”, In: Proceedings ENCON, pp. 1-5, 2008.

Received: October 16, 2014 Revised: December 23, 2014 Accepted: December 31, 2014