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Cutting Geometry and Base-Cone Parameters of Manufacturing Hypoid Gears by Generating-Line Method

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Abstract: Generating-line method, which is based on the generating process of spherical involute curve, is a new processing theory of cutting ideal spherical involute gears. Based on the principle of the new method, this paper expands the traditional geometrical relationship into the cutting geometry of generating-line method, and proposes the base-cone parameters with their formulas to provide necessary parameters for further study. The application examples, such as establishing the coordinate systems and the equation of gear generating line, show the importance of these researches, and the example of calculating geometric parameters illustrate the way to adjust the base-cone parameters. It can be seen from the researches, the cutting geometry contains the tangent relationships between two base cones and a base plane respectively, and the planar conjugated relationship between two generating lines of pinion and gear. The base-cone parameters determine the relationships among base cones, base plane and generating lines. If some of the base-cone parameters need to be adjusted, it can be realized by modify the value of the parameter r_2, ε or Z_0.

Keywords: Hypoid gear, generating-line method, cutting geometry, base-cone parameters.

1. INTRODUCTION

Hypoid gears are widely used to transmit crossed-axis power and motion in vehicles, ships and aircrafts, and could offer higher load capability and axis position flexibility than spiral bevel gears. The basic geometry of hypoid gears were established by Wildhaber [1] and Baxter [2], after decades of development, there are two major processes called face milling and face hobbing for cutting hypoid and spiral bevel gears in the current gear manufacturing industry [3].

In recent years, a large number of developments in the field of manufacturing spiral bevel and hypoid gears have been obtained. Qi Fan developed mathematical models of hypoid gear drives processed by face milling and face hobbing, and researched the tooth contact analysis and the tooth surface error correction [4-6]; in order to improve the load distribution and reduce the transmission error, Vilmos Simon proposed the optimal machine tool setting and tooth modifications of spiral bevel and hypoid gears [7-9]. All of these developments above were built on the traditional methods of face milling and face hobbing. As a result of applying engineering approximation, the tooth profile curves cut by these methods are not ideal spherical involutes, therefore the advantages of using spherical involute profiles, such as transmission ratio constancy and angular displacement insensitivity, are partly lost. Also the calculation and adjustment of machine tool settings are complex, and the interchangeability of the gears cut by these traditional methods is relatively poor [10]. Y.C. Tsai *et al.* [11] and M.J. Al-Daccak *et al.* [12] respectively proposed the modeling of bevel gears by using exact spherical involute profiles, but they did not discuss the feasibility of cutting ideal spherical involute gears.

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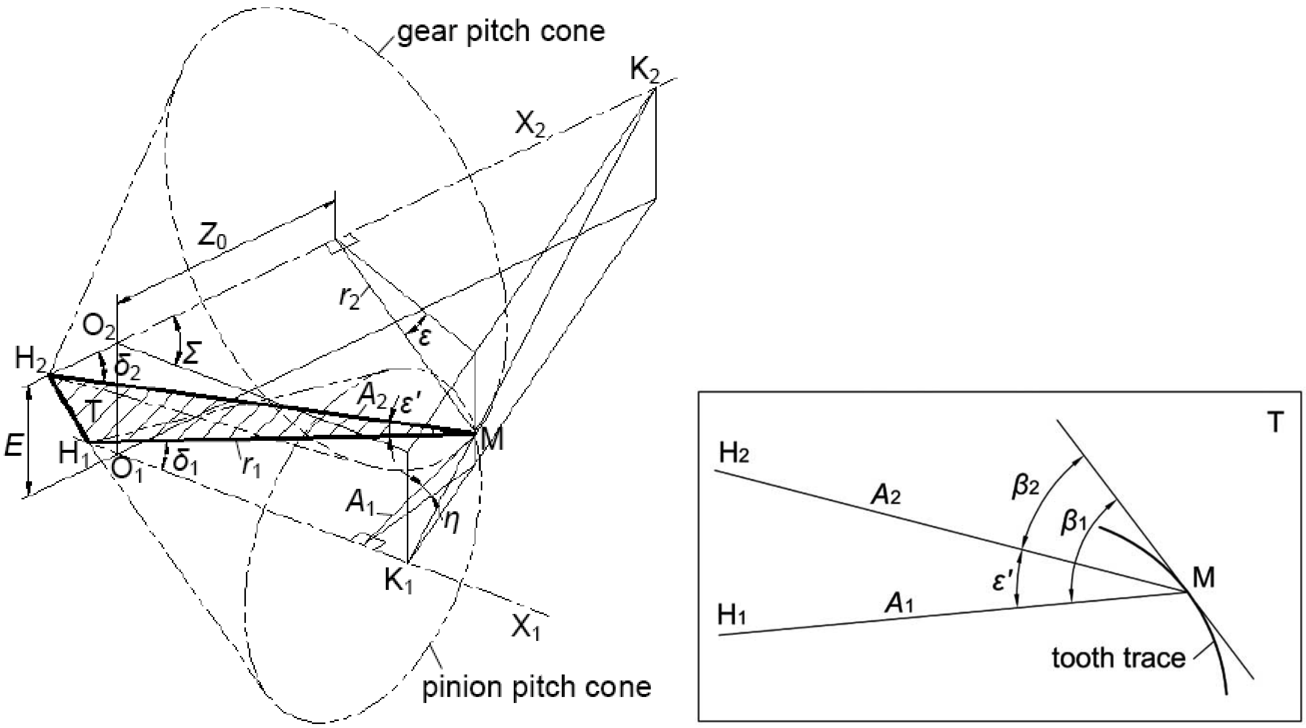
Based on the generating principle of spherical involute and the theory of conjugated tooth surfaces, we have proposed *the generating-line method of cutting spherical involute gears* [13, 14]. This new theory can be used to process ideal spherical involute bevel gears; however the principle of cutting hypoid gears by this method still needs to be researched systematically. Therefore, in order to facilitate the further studies of the shape of generating lines and the cutting motion parameters, this paper develops the cutting geometry and base-cone parameters of manufacturing hypoid gears by this new method, and indicates the important effects of the researches through the application examples.

2. BASIC GEOMETRY

The axodes of hypoid gears are two tangent revolving hyperboloids. In order to simplify the designing and manufacturing, axodes are usually replaced by a pair of pitch cones [15]. As shown in Fig. (**1a**), the angle between pinion axis X_1 and gear axis X_2 is shaft angle Σ, and the length of common perpendicular O_1O_2 is the pinion offset E; two pitch cones are in tangency at point M which is usually the center point of the tooth surfaces, and the common tangent plane, which is also called pitch plane, is T; at point M, pitch cone distances of pinion and gear are A_1 and A_2, and pitch radius are r_1 and r_2; pitch angles are δ_1 and δ_2, and the distance from point O_2 to the center of gear pitch circle is Z_0; the pitch cone vertexes of pinion and gear are H_1 and H_2, and the angle between H_1M and H_2M is ε'; the line K_1K_2, which crosses the point M and is vertical to the T plane, intersects X_1 and X_2 at points K_1 and K_2; in the vertical plane of axis X_1, the angle between projections of X_2 and K_1K_2 is η; in the vertical plane of axis X_2, the angle between projections of X_1 and K_1K_2 is ε. As shown in Fig. (**1b**), in the pitch plane T, the helix angles at the point M of pinion and gear are β_1 and β_2, and this paper calls β_1 and β_2 pitch helix angles. In addition, the number of teeth of pinion and gear are N_1 and N_2.



a b

Fig. (1). Basic geometry of hypoid gears.

The position of point M can be determined by given the parameters r_2, ε and Z_0, if we know the parameters N_1, N_2, Σ, E and the position of point M, all of other parameters mentioned above can also be determined uniquely [2]. Hypoid gears can be designed and manufactured by traditional methods according to this basic geometry. But for the new generating-line method in this paper, this geometry needs to be expanded according to the new cutting principle.

3. CUTTING GEOMETRY OF GENERATING-LINE METHOD

For a bevel gear drive, as shown in Fig. (**2**), pitch plane T and two pitch cones are in tangency alone line OM, while base plane Q and two base cones are in tangency alone line OU_1 and OU_2, and the angle α between plane T and plane Q is the pressure angle on the back cone. When the motion of

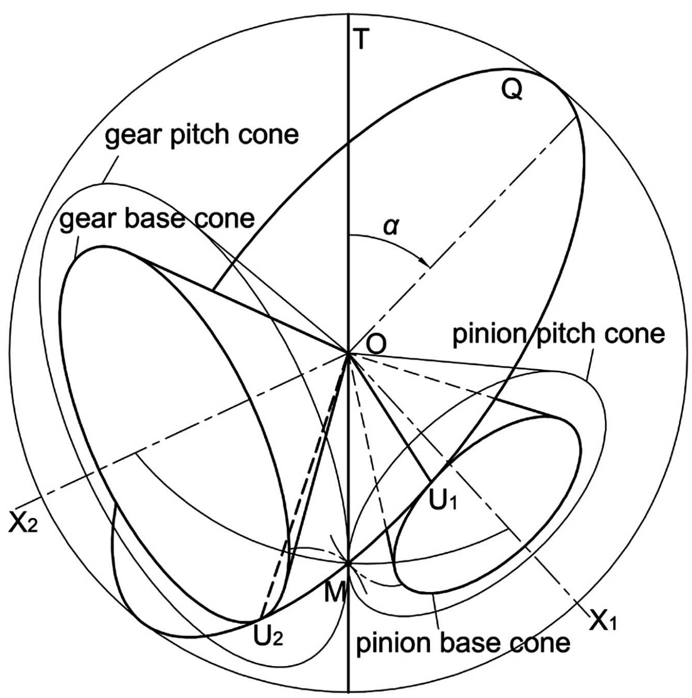


Fig. (2). Schematic of a bevel gear drive.

this pair of pitch cones is pure-rolling in tangency with the pitch plane T, the motions of two base cones are also pure-rolling in tangency with the base plane Q along their own tangent lines respectively, and a pair of ideal spherical involutes with respect to the base cones could be formed by the traces of point M which is fixed on and rolling with the plane Q.

As shown in Fig. (**3**), M′C′ and M″C″ are two spherical involutes on the toe and on the heel of gear respectively; if there is a curve between M′ and M″ in plane Q, a spherical involute profile tooth surface will be formed between toe and heel of the gear. According to this theory, generating-line method takes *generating line* M′M″ as the cutting edge, which removes excess material with the appropriate relative motion between tool and gear blanks. In theory, using the same generating line as the cutting edge could process a pair of bevel gears which are conjugated in line contact.

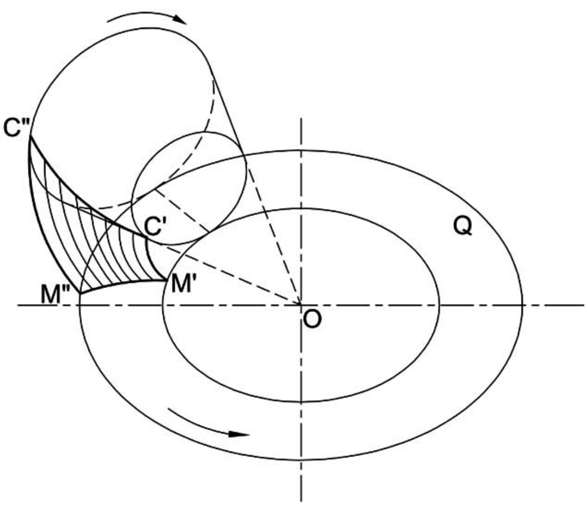


Fig. (3). Principle of generating-line method.

As can be seen from the description above, determining the position of base plane Q and the equation of generating line is the key point of the generating-line method. Therefore, this new method could also be used to cut hypoid gears if these two elements can be determined properly. However, traditional methods of designing and manufacturing hypoid gears do not have the concepts of base cone and base plane, so this paper proposes the following method to establish the base cones and base plane of hypoid gears.

As shown in Fig. (**4**), the final position of rotating pitch plane T about H_2M at an angle α could be defined as plane Q which is the base plane of a hypoid gear drive; the axis X_1 and plane Q intersect at point V, and the tangent relationship between plane Q and two base cones can be used to determine base cones of the pinion and the gear; the tangent lines are VU_1 and H_2U_2, and the base angles of pinion and gear are δ_{b1} and δ_{b2}.

As the base cone vertexes of pinion and gear are not coincident, the generating lines of pinion and gear can not be the same one. Therefore as shown in Fig. (**5**), the generating planes Q_1 and Q_2 are set up in the base plane Q. When the gears are being processed or driving, the motion of pinion base cone and the generating plane Q_1 are pure-rolling in tangency along VU_1, while the motion of gear base cone and the generating plane Q_2 are pure-rolling in tangency along H_2U_2. Based on the principles of gear connection, it can be proved that if the following two conditions are met, a pair of hypoid gears cut by generating-line method will be conjugated in point contact.

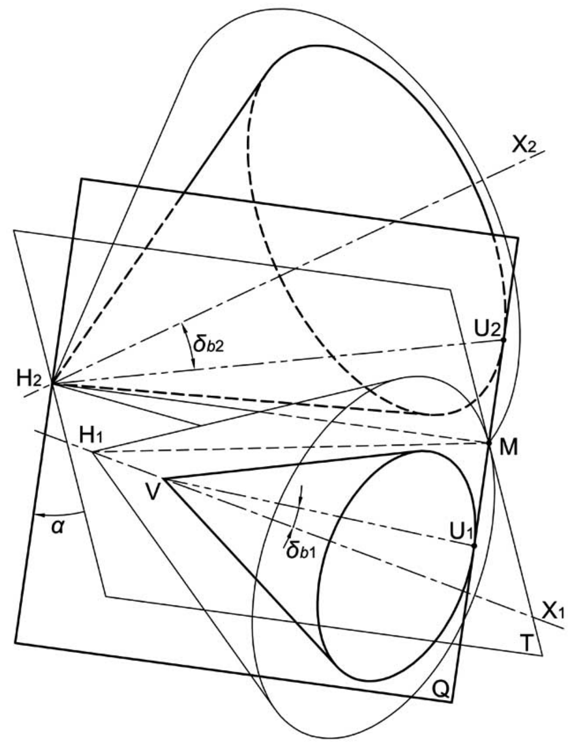


Fig. (4). Geometry of cutting hypoid gears.

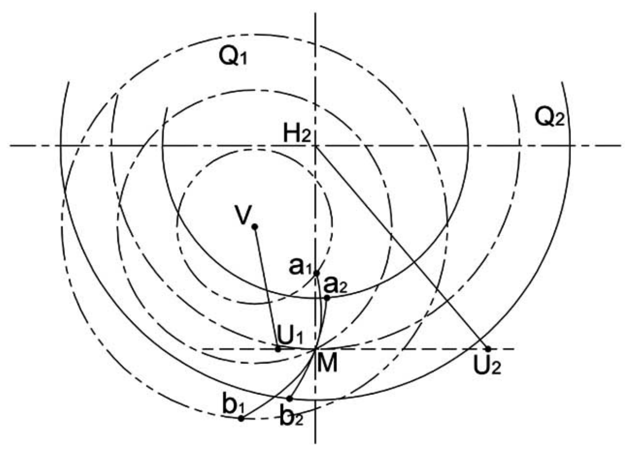


Fig. (5). Relationship of generating lines.

Condition 1: The rotation speeds of generating plane Q_1 and Q_2 meet the transmission ratio of the gears.

Condition 2: As the pinion generating line a_1b_1 rotates with Q_1 and the gear generating line a_2b_2 rotates with Q_2, a_1b_1 and a_2b_2 is a pair of planar conjugated curves, while point M was one of their contact points, and there is no curvature interference between the two generating lines.

Because the shape of generating lines have not been determined, similar to the concepts of the convex and concave of spiral bevel gears, this paper defines the left and

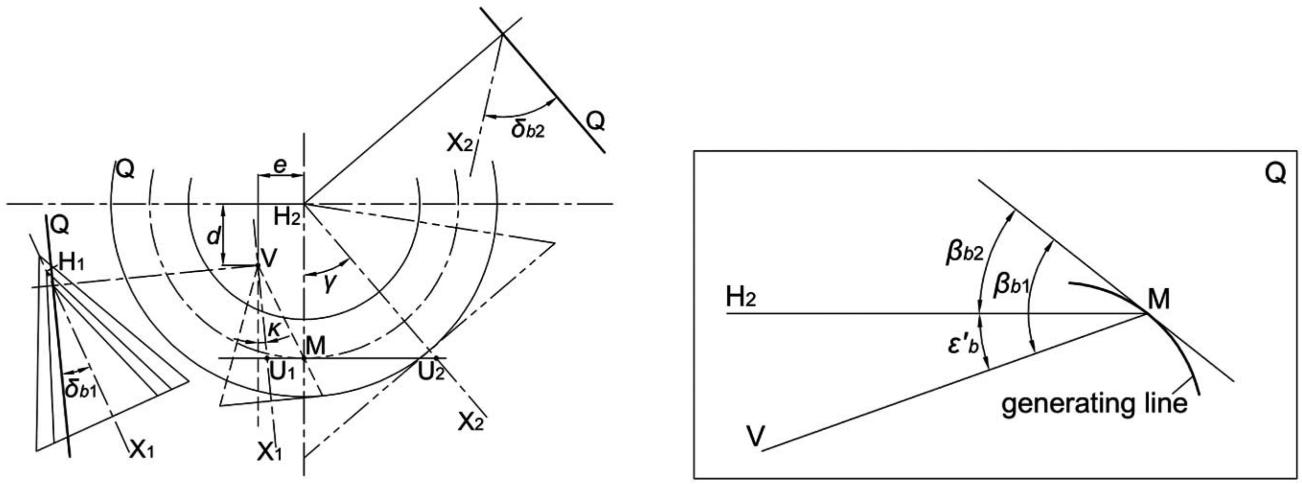


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Fig. (6). Schematic of base-cone parameters.

right sides of tooth surfaces of hypoid gears as follows: observing a gear tooth alone the tooth trace from the heel to the toe, putting the topland above and the root below, and then two sides of the tooth in sight are called the left side and right side respectively. Therefore, the situation shown in Fig. (**4**) is the geometry of cutting left sides of tooth surfaces of hypoid gears which the pinion is offset below from the center of the gear, and this paper mainly researches the situation of cutting left sides of tooth surfaces, if the angle α is rotating to the opposite direction, the cutting geometry of right sides of tooth surfaces can be easily obtained.

4. BASE-CONE PARAMETERS

In order to facilitate the further studies of the parameters of the machine tool and the blade, it is necessary to determine the geometrical relationships among base cones, base plane and generating lines. On base of the cutting geometry as shown in Fig. (**6**), this paper defines *the base-cone parameters* as below to determine those geometrical relationships.

On the basis of designing, assume the pitch-cone pressure angle α_n, which indicates the angle between normal direction of the tooth surface and the pitch plane T at point M, is known. Then the angle α between pitch plane T and base plan Q can be determined by

$$\alpha = \sin^{-1} \frac{\sin \alpha_n}{\sqrt{1 - \cos^2 \alpha_n \sin^2 \beta_2}} \tag{1}$$

The base cone angles of pinion and gear are determined, respectively, by

$$\delta_{b1} = \sin^{-1}\left(\cos\alpha \sin\delta_1 + \sin\alpha \cos\delta_1 \sin\varepsilon'\right) \tag{2}$$

$$\delta_{b2} = \tan^{-1} \sqrt{\frac{\sin^2 \delta_2}{\tan^2 \alpha + \cos^2 \delta_2}} \tag{3}$$

As shown in Fig. (**6a**), in the base plane Q, d denotes the distance from the projection of point V on the line H_2M to point H_2, provided d is positive if the projective point lies between M and H_2; otherwise, d is negative. Furthermore, e denotes the distance from point V to line H_2M; κ denotes the angle between VU_1 and H_2M; and γ denotes the angle between H_2U_2 and H_2M. These parameters are determined, respectively, by

$$d = A_2 - \frac{A_1 \tan\delta_1 \cos\varepsilon'}{\tan\alpha \sin\varepsilon' + \tan\delta_1} \tag{4}$$

$$e = \frac{A_1 \sin\delta_1 \sin\varepsilon'}{\cos\alpha \sin\delta_1 + \sin\alpha \cos\delta_1 \sin\varepsilon'} \tag{5}$$

$$\kappa = \tan^{-1}\left(\frac{\sin\varepsilon' \cos\alpha - \sin\alpha \tan\delta_1}{\cos\varepsilon'}\right) \tag{6}$$

$$\gamma = \cos^{-1} \frac{\cos\delta_2}{\cos\delta_{b2}} \tag{7}$$

As shown in Fig. (**6b**), in the base plane Q, ε'_b denotes the angle between VM and H_2M; β_{b1} denotes the angle between the tangential direction of generating lines at point M and VM, and β_{b2} denotes the angle between the tangential direction of generating lines at point M and H_2M, so this paper calls β_{b1} and β_{b2} base helix angles. These parameters are determined, respectively, by

$$\varepsilon'_b = \tan^{-1} \frac{e}{A_2 - d} \tag{8}$$

$$\beta_{b2} = \tan^{-1}\left(\cos\alpha \tan\beta_2\right) \tag{9}$$

$$\beta_{b1} = \beta_{b2} + \varepsilon_b \tag{10}$$

5. APPLICATION EXAMPLES

Based on the researches of the cutting geometry and the base-cone parameters, many further researches can be done, such as establishing the coordinate systems to build the tooth

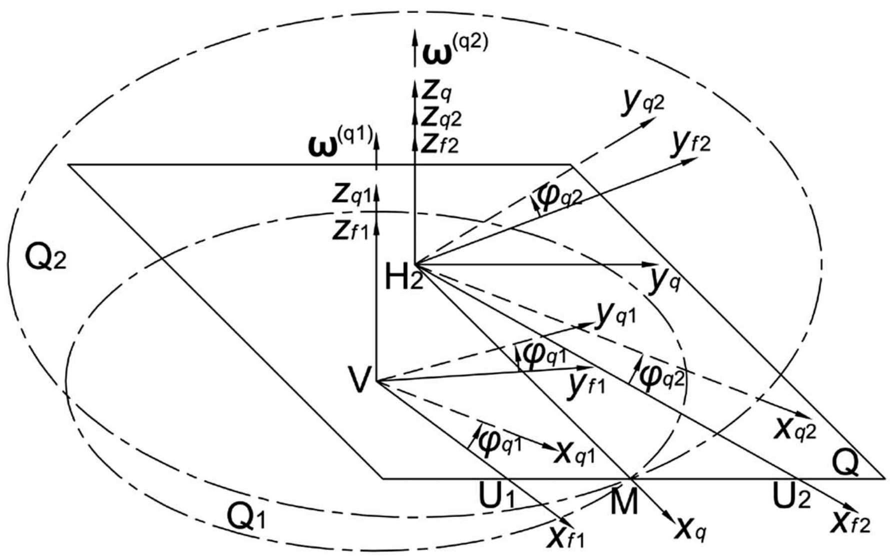

Fig. (7). Coordinate systems S_q, S_{f1}, S_{f2}, S_{q1}, S_{q2}.

surfaces model and analyze the situation of meshing, establishing the equation of gear generating line to study the shape of generating lines and the influence of shape errors, and so on.

5.1. Coordinate Systems

As shown in Fig. (**7**) and Fig. (**8**), the coordinate systems S_q, S_{f1}, S_{f2}, S_{q1}, S_{q2}, S_1 and S_2 are established as given below.

Fixed system $S_q(x_q, y_q, z_q)$ is fixed with the initial position of plane Q and attached to the machine house.

Auxiliary fixed systems $S_{f1}(x_{f1}, y_{f1}, z_{f1})$ and $S_{f2}(x_{f2}, y_{f2}, z_{f2})$ are fixed with the initial positions of generating planes Q_1 and Q_2 respectively.

Moving systems $S_{q1}(x_{q1}, y_{q1}, z_{q1})$ and $S_{q2}(x_{q2}, y_{q2}, z_{q2})$ are fixed with generating planes Q_1 and Q_2 respectively, and simulated motions of cutting edges. S_{q1} and S_{q2} are supposed to rotate about z_{q1} and z_{q2} respectively with the angular velocity $\boldsymbol{\omega}^{(q1)}$ and $\boldsymbol{\omega}^{(q2)}$, and the rotation angles starting from the initial positions are φ_{q1} and φ_{q2}.

Moving systems $S_1(x_1, y_1, z_1)$ and $S_2(x_2, y_2, z_2)$ are fixed with base cones and simulated motions of gear blanks. The initial position of S_1 is the position of rotating S_{f1} about y_{f1} at an angle δ_{b1}, and the initial position of S_2 is the position of rotating S_{f2} about y_{f2} at an angle δ_{b2}. S_1 and S_2 are supposed to rotate about x_1 and x_2 respectively with the angular velocity $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$, and the rotation angles starting from the initial positions are φ_1 and φ_2.

Then, on the basis of the cutting geometry and base-cone parameters proposed above, the transformation matrixes can be determined as below.

$$\mathbf{M}_{q\_f1} = \begin{bmatrix} \cos\kappa & -\sin\kappa & 0 & d \\ \sin\kappa & \cos\kappa & 0 & -e \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

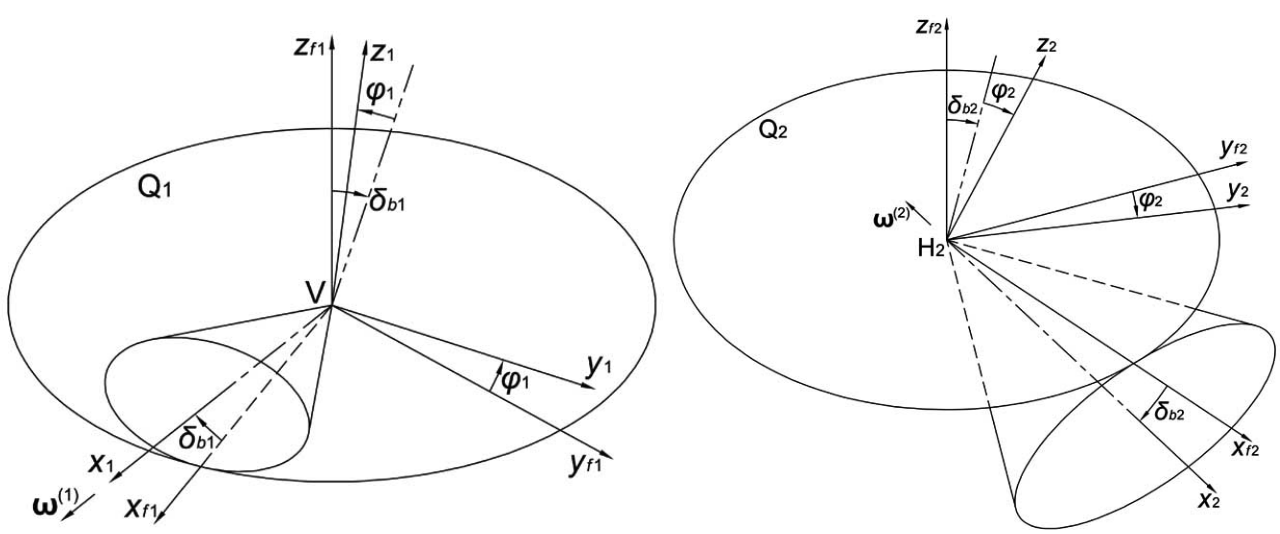

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Fig. (8). Coordinate systems S_1, S_2.

$$\mathbf{M}_{q\_f2} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$\mathbf{M}_{f1\_q1} = \begin{bmatrix} \cos\phi_{q1} & -\sin\phi_{q1} & 0 & 0 \\ \sin\phi_{q1} & \cos\phi_{q1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$\mathbf{M}_{f2\_q2} = \begin{bmatrix} \cos\phi_{q2} & -\sin\phi_{q2} & 0 & 0 \\ \sin\phi_{q2} & \cos\phi_{q2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$\mathbf{M}_{f1\_1} = \begin{bmatrix} \cos\delta_{b1} & -\sin\delta_{b1}\sin\phi_1 & -\sin\delta_{b1}\cos\phi_1 & 0 \\ 0 & \cos\phi_1 & -\sin\phi_1 & 0 \\ \sin\delta_{b1} & \cos\delta_{b1}\sin\phi_1 & \cos\delta_{b1}\cos\phi_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$\mathbf{M}_{f2\_2} = \begin{bmatrix} \cos\delta_{b2} & -\sin\delta_{b2}\sin\phi_2 & \sin\delta_{b2}\cos\phi_2 & 0 \\ 0 & \cos\phi_2 & \sin\phi_2 & 0 \\ -\sin\delta_{b2} & -\cos\delta_{b2}\sin\phi_2 & \cos\delta_{b2}\cos\phi_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

where matrix $\mathbf{M}_{q\_f1}$ denotes the coordinate transformation from system S_{f1} to S_q, and other matrix have the same subscript meaning.

5.2. Equation of Gear Generating-Line

To make the structure and movement of machine tool simple, the shape of generating lines should be simple and easy to form a continuous movement. Therefore the most appropriate shapes are straight line and circular arc. In order to improve processing efficiency, the shape of the gear generating line should be determined preferentially.

If the gear generating line a_2b_2 is a straight line, then its equation in coordinate system S_{q2} should be represented as,

$$\begin{cases} x_{q2c}(u) = x_1 + u\cos\theta \\ y_{q2c}(u) = y_1 + u\sin\theta \\ z_{q2c}(u) = 0 \end{cases} \tag{17}$$

where u is a parameter, x_1, y_1 and θ can be determined by the cutting geometry and the base-cone parameters, namely,

$$\begin{cases} x_1 = A_2\cos\gamma \\ y_1 = -A_2\sin\gamma \end{cases} \tag{18}$$

$$\theta = -\beta_{b2} - \gamma \tag{19}$$

If the gear generating line a_2b_2 is a circular arc, then its equation in coordinate system S_{q2} should be represented as

$$\begin{cases} x_{q2c}(\theta) = x_0 + r\cos(\theta+\mu) \\ y_{q2c}(\theta) = y_0 + r\sin(\theta+\mu) \\ z_{q2c}(\theta) = 0 \end{cases} \tag{20}$$

where θ is a parameter, r is the arc radius of gear generating line, x_0, y_0 and μ can be determined by the cutting geometry and the base-cone parameters, namely

$$\begin{cases} x_0 = A_2\cos\gamma - r\sin(\beta_{b2}+\gamma) \\ y_0 = -A_2\sin\gamma - r\cos(\beta_{b2}+\gamma) \end{cases} \tag{21}$$

$$\mu = 90° - \beta_{b2} - \gamma \tag{22}$$

5.3. Example of Calculating Geometric Parameters

Consulting the data of designing a pair of Gleason hypoid gears, mainly geometrical parameters of cutting hypoid gears by generating-line method can be calculated as follows.

The basic parameters are shown in Table **1**, and these parameters are given by the basic conditions of designing. As shown in Table **2**, the key geometrical parameters r_2, ε and Z_0, which determining the position of point M, are assumed known based on the data of Gleason gears, and so is the average pressure angle α^*. Then, on the basis of traditional formulas, the basic geometrical parameters can be calculated easily, the results are shown in Table **3**. Finally, based on the equations (1) to (10), the results of the base-cone parameters are calculated and shown in Table **4**.

Table 1. The Basic Parameters of Designing

Symbols	Data	Symbols	Data
N_1	11	Σ (deg)	90
N_2	43	E (mm)	34

Table 2. The Key Geometrical Parameters Assumed in Advance

Symbols	Data	Symbols	Data
r_2 (mm)	88.16	Z_0 (mm)	30.56
ε (deg)	20.3	α^* (deg)	19

Table 3. The Results of the Basic Geometrical Parameters

Symbols	Data	Symbols	Data
r_1 (mm)	30.688	A_2 (mm)	92.618
η (deg)	6.375	ε' (deg)	21.237
δ_1 (deg)	16.707	β_1 (deg)	49.981
δ_2 (deg)	72.151	β_2 (deg)	28.744
A_1 (mm)	106.964	α_n (deg)	25.668

Table 4. The Results of the Base-Cone Parameters

Symbols	Data	Symbols	Data
α (deg)	28.728	κ (deg)	10.537
δ_{b1} (deg)	24.762	γ (deg)	56.180
δ_{b2} (deg)	56.585	ε'_b (deg)	23.901
d (mm)	32.611	β_{b1} (deg)	49.588
e (mm)	26.593	β_{b2} (deg)	25.686

During the process of researching the shape of generating lines, the parameters of the gear generating line could influence the shape of the pinion generating line, and we can see from the conditions above, the parameters of the gear generating line are determined by the base-cone parameters. Therefore, when some of the base-cone parameters need to be adjusted to optimize the shape of the pinion generating line, it can be realized by modify the value of the parameter r_2, ε or Z_0. For example, if the value of Z_0 is changed from 30.56 to 30.00, the other parameters should be changed as shown in Table **5**.

Table 5. The Results of Changing the Value of Z_0

Symbols	Data	Symbols	Data
r_1 (mm)	30.194	α (deg)	27.997
η (deg)	6.493	δ_{b1} (deg)	24.896
δ_1 (deg)	16.997	δ_{b2} (deg)	57.033
δ_2 (deg)	71.839	d (mm)	33.767
A_1 (mm)	103.287	e (mm)	26.020
A_2 (mm)	92.782	κ (deg)	10.745
ε' (deg)	21.271	γ (deg)	55.054
β_1 (deg)	48.283	ε'_b (deg)	23.793
β_2 (deg)	27.012	β_{b1} (deg)	48.027
α_n (deg)	25.344	β_{b2} (deg)	24.234

6. CONCLUSIONS

On the basis of the theory of generating-line method, this paper established the cutting geometry and base-cone parameters, and these researches made the new method suitable to manufacture hypoid gears. The cutting geometry contains the tangent relationships between pinion base cone and base plane, and between gear base cone and base plane, respectively, and also contains the planar conjugated relationship between two generating lines of pinion and gear. The base-cone parameters determine the relationships among base cones, base plane and generating lines, and they are necessary to be used to establish the coordinate systems and the equations of generating lines. It can be seen from the example, if some of the base-cone parameters need to be adjusted, it can be realized by modify the value of the parameter r_2, ε or Z_0.

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