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The Weighted Least Squares Method Affects the Economic Rotation Age of Loblolly Pine - Two Planting Density Scenarios

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Abstract: Managers are typically interested in using equations that provide accurate volumes for large, merchantable trees while modelers are often more interested in equations that meet statistical assumptions. Therefore, modelers often use weighted (WLS) rather than ordinary least squares (OLS) to fit individual tree volume equations since the random error variance is not constant. Since OLS and WLS produce different parameter estimates, perhaps the method chosen will impact the predicted optimum economic rotation age. To address this concern, individual tree volumes were obtained from plots established in an unthinned loblolly pine plantation in southeastern Arkansas. Parameters of the combined-variable function were then estimated using OLS and WLS for the same dataset. Stand-level projections and predicted diameter distributions for two planting densities (748 and 2,692 seedlings per hectare) were then obtained from a growth and yield model (PTAEDA 3.1) and individual tree volumes were predicted using the two parameter estimation approaches. When conducting economic analyses, we found the optimum rotation age between OLS and WLS can differ by as much as 11 yrs (i.e. 10 yr clearcut *vs* 21 yr clearcut).

Keywords: Models, ordinary least squares, *Pinus taeda* L., plantations.

INTRODUCTION

When developing equations to be used by foresters, the statistical method of weighted least squares (WLS) is commonly used to account for non-constant variance. Some examples include understory vegetation across a range of stand densities [1] or individual tree volumes as a function of diameter and height [2-4]. WLS models are typically used because parameter estimates have lower standard errors than estimates generated using ordinary least squares (OLS). From a statistical perspective, a low standard error is preferred.

From a theoretical aspect, WLS is always used to estimate parameters. OLS (where the weight is constant for all observations) is simply a special case of WLS. When someone decides to use OLS, they assume the error covariance-variance structure is $\sigma^2\mathbf{I}$ (i.e. the variance is assumed constant). Conceptually, a weight is applied to all observations, but when the weight is constant for all data points, in practice it can be ignored.

When estimating parameters using OLS, equal weight is applied to all observations because the random error variance is assumed constant across the domain of the regressor(s). From a statistical theory perspective, if this assumption is assumed but is not true, using OLS to estimate parameters produces unbiased parameter estimates but the estimates are inefficient [5, pgs. 96-97). Inefficiency results because all observations should not equally influence the parameter estimates [6; pg. 279]. When the variance is non-constant, from a statistical theory perspective, WLS should be used to obtain more efficient parameter estimates. In this context, the word "efficient" refers to the fact that across repeated random samplings of the same size, WLS parameter estimates will, on average, be closer to the population parameter than the OLS parameter estimates [7; pgs. 366-368, pg. 417, 8]. It should be understood though that for any one observed dataset from the population, the OLS estimate for any parameter may in fact be closer to the parameter value than the WLS estimate.

Parameter Estimation Equations

When estimating parameters of a linear regression model using OLS (variance is assumed constant) and matrices, the following equation is used:

$$\underline{\hat{\beta}} = \left(\mathrm{X}^{\mathrm{T}}\mathrm{X}\right)^{-1}\mathrm{X}^{\mathrm{T}}\underline{\mathrm{y}} \tag{1}$$

where:

$\underline{\hat{\beta}}$ -- are parameter estimates (k x 1), where k is the number of regressors in the model plus one for the intercept,

$\underline{\mathrm{y}}$ -- are observed values of the dependent variable (n x 1), where n is the number of observations, and

X -- regressor matrix (n x k) consisting of a column of 1s and the observed values of the regressor(s).

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Equation (2) will produce parameter estimates of linear regression models when the variance is non-constant and known:

$$\underline{\hat{\beta}} = \left(X^T V^{-1} X\right)^{-1} X^T V^{-1} \underline{y} \tag{2}$$

where:

V -- is the diagonal random error covariance-variance matrix consisting of the true weights along the diagonals and 0s on the off-diagonals since the random errors are assumed to be uncorrelated across observations, and all other variables as previously defined.

Concerns About Weighted Least Squares

If V is known and the variance is non-constant, equation (2) will absolutely produce more efficient parameter estimates relative to OLS. However, in practice, V must be estimated and therefore equation (3) is used to estimate parameters when using WLS:

$$\underline{\hat{\beta}} = \left(X^T \hat{V}^{-1} X\right)^{-1} X^T \hat{V}^{-1} \underline{y} \tag{3}$$

where:

$\hat{V}$ -- is the diagonal covariance-variance matrix consisting of predicted weights along the diagonals and 0s on the off-diagonals since the random errors are assumed to be uncorrelated across observations, and all other variables as previously defined.

If V is poorly estimated, then the WLS parameter estimates may in fact be less efficient than OLS [6; pg. 280]. This is a widely known problem of using weighted least squares. However, problems often not addressed are the biological, economic, and hence, management implications of using WLS to estimate parameters.

For example, when using weighted least squares to model understory vegetation [1] state:

"The intercept and absolute value of the slope are smaller than would have resulted from an unweighted least squares (i.e. OLS) analysis because of the much lower weights given stands with open canopies."

Despite this concern, they did not address the potential management impacts of the smaller intercept and slope. Hence, they never addressed which method, OLS or WLS (which applied less weight to observations from open canopies) would be more suitable for managers.

When compared to OLS, WLS assigns less weight to observations where the error variance is greater. From a management standpoint, WLS usually applies more weight to observations that have the least practical importance. For instance, when estimating individual tree volume, larger trees often have greater variances. Therefore, more weight is given to the smaller trees (Fig. **1**). However, the smaller trees (with their smaller variances) are either not commercially usable or have less value than the larger trees. It seems logical that resource managers would prefer at least equal accuracy in volume estimates of the larger trees.

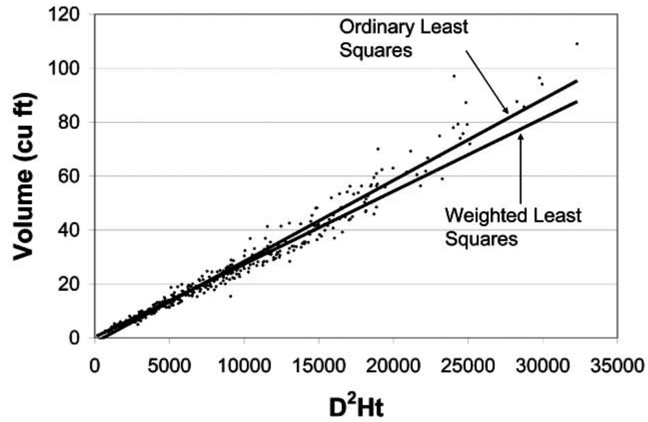

Fig. (1). Plot of all 431 individual tree total cubic foot outside-bark volume observations plotted over D^2Ht (D = in. and Ht = ft). Predictions of total cubic foot volume when using ordinary least squares [equation 4] and weighted least squares [equation 5] are also presented.

For understory vegetation, lower variances are observed at higher stand densities and thus WLS applies more weight to these observations, but often high stand densities provide low amounts of forage to wildlife and domestic animals and therefore provide little in the way of stocking. Thus, when predictions more applicable to management are of greater interest than probability values associated with various parameter estimates, perhaps OLS should be used to estimate parameters. In the presence of non-constant variance, OLS provides a compromise between management considerations and sampling theory.

The method of regression analyses (i.e. using OLS or WLS) can have a substantial impact on volume estimates and ultimately economic projections [8]. However, to date, there have been no reports that examine whether using OLS or WLS can impact the optimum economic rotation age. In this paper, we wanted to determine how much of a difference the OLS method will make in the "optimum" economic rotation age (as determined by the Bare Land Value - BLV).

MATERIALS AND METHODOLOGY

WLS and OLS regression analyses were used to estimate parameters of the combined-variable volume equation [9; pg. 8] for individual trees in a loblolly pine (*Pinus taeda* L.) plantation. The WLS and OLS equations were then used separately to estimate volumes of trees projected using a growth model to determine the economic impact of using the two alternative parameter estimation procedures.

Data

Volume equations were fit using data obtained from permanent plots in an unthinned loblolly pine plantation located near Monticello, Arkansas (http://www.afrc.uamont.edu/growthyield/montthinprun/index.html). The stand was planted in 1958 at a spacing of 2.44 m by 2.44 m using seedlings obtained from a state nursery located in Arkansas. Genetic stock was of a local seed source. Five plots were originally established in 1984 and measured at ages 27, 30, 35, and 45 yr (see Table **1** for a summary of tree characteristics). Individual tree outside-bark total volumes were obtained using the Grosenbaugh height accumulation method [10] – thus volumes were obtained without harvesting the trees. By this method, volumes are estimated from measurements of heights to the occurrence of diameters in a diminishing arithmetic progression up the stem. Volume is computed using these data and published coefficients.

At the time of study establishment (at age 27 yr), there were 156 trees. A total of 431 individual tree volume observations were obtained across all measurement ages. Site index (base age 25 yr) was determined to be near 18.9 m.

Table 1. Summary Statistics of Observations from Trees Used to Estimate Parameters of Equations [4] and [5], Number of Trees at the Initial Measurement Period at Age 27 yr was 156. Where: n is the Number of Individual Tree Observations, Min is the Minimum, Mean is the Arithmetic Mean, and Max is the Maximum Value

	n	Min	Mean	Max
dbh (cm)	431	7.9	26.7	46.7
Ht (m)	431	5.9	19.7	29.1
Volume (m^3)	431	0.03	0.68	3.09

Model Development and Parameter Estimation

The combined-variable function can be expressed as:

$$Vol_i = \beta_0 + \beta_1 D_i^2 Ht_i + \varepsilon_i \tag{4}$$

where:

Vol_i -- individual tree total cubic foot outside-bark volume,

D_i -- individual tree outside-bark diameter at breast-height (in.),

Ht_i -- individual total tree height (ft),

ε_i -- random error for an individual tree, assumed to follow a Gaussian distribution (0, σ_j^2), where σ_j^2 is the variance of Vol at a particular D^2Ht, and

β_0, β_1 -- parameters to be estimated.

When using OLS to estimate parameters, σ_j^2 is assumed constant across all observations. With WLS, σ_j^2 is assumed to vary systematically with changes in the regressor, denoted by the letter *j*. To estimate parameters using WLS, a variance stabilizing transformation can be used to obtain a constant variance across the domain of the regressor [5; pgs. 226-228]. After the variance stabilizing transformation, OLS can be used to estimate parameters. It is assumed that the variance of total cubic foot volume can be described by a power function of D^2Ht to the second degree (i.e. 2, 3, 4, 9, pgs. 21-24, [11]), or:

$$Var(Vol) \propto \sigma^2 (D^2Ht)^2$$

Therefore, when dividing both sides of equation (4) by D^2Ht, the variance of volume can be made constant:

$$Var\left(\frac{Vol}{D^2Ht}\right) = \frac{1}{\left(D^2Ht\right)^2} Var(Vol) = \frac{1}{\left(D^2Ht\right)^2}\sigma^2\left(D^2Ht\right)^2 = \sigma^2$$

In this case, $\frac{1}{\left(D^2Ht\right)}$ is the variance stabilizing transformation. To obtain WLS parameter estimates, equation (4) was modified by dividing both sides by D^2Ht producing:

$$\frac{Vol_i}{D_i^2Ht_i} = \beta_0 \frac{1}{\left(D_i^2Ht_i\right)} + \beta_1 + \varepsilon_i \tag{5}$$

where:

All variables as previously defined.

For each parameter estimation method, Proc MODEL of the SAS Institute [12] was used to obtain parameter estimates.

Economic Analyses

To determine the potential economic impacts of using WLS to estimate tree volumes, a distance-dependent individual tree model was used to estimate stand development [13]. Output from PTAEDA 3.1 (e.g. number of trees by diameter class and average height of a particular diameter class for a certain age) was used to conduct economic analyses based on the OLS and WLS estimated combined-variable individual tree volume equations. Input variables included a site in the Coastal Plain physiographic region with a site index of 21.3 m (base age 25 yr); site preparation was chop, burn, and bed, with a first-yr herbaceous weed control treatment. The drainage class selected was well-drained and no fertilization treatments were conducted. This regeneration scenario is representative of commonly used scenarios but results will vary depending upon input variables. Two planting densities were simulated: 748 seedlings per hectare (SPH) with 4.9 m rows (2.7 m within rows) and 2,692 SPH using 3.0 m rows (1.2 m within rows).

To obtain merchantable estimates of individual tree volume, we combined the OLS and WLS estimated individual tree total cubic-foot volume equations with a merchantable volume ratio equation presented by [14]. The following inside-bark equation was used:

$$Vol_{Merch} = Vol\ e^{-0.91505\left[\frac{d^{4.93352}}{D^{4.60614}}\right]} \tag{6}$$

where:

d -- specified upper stem outside-bark diameter limit (in.),

Vol_{Merch} -- inside bark cubic foot volume to *d*, and all other variables as previously defined.

Equations (4) and (5) were used to estimate total cubic foot volume while equation (6) was used to estimate merchantable volumes to upper stem diameters. For this analysis, we assume all trees that meet minimum D specifications were merchantable. Minimum D for pulpwood, chip-n-saw and sawtimber were 10.2, 22.9 and 30.5 cm, respectively. For pulpwood, an upper stem outside-bark diameter limit of 5.1 cm was employed; for chip-n-saw, the upper stem outside-bark diameter limit was 10.2 cm; and for sawtimber, the upper stem outside-bark diameter limit was 20.3 cm. For chip-n-saw and sawtimber size trees, all volume above either a 10.2 cm or an 20.3 cm top, respectively, was classified as pulpwood (to a 5.1 cm top). It should be noted in growth and yield modeling that due to the costs and time involved, it is common to utilize volume equations developed using data from vastly different populations than the data used in fitting tree or stand development equations.

For the economic analyses, an interest rate of 6 percent was used and rotation length varied from 10 to 35 yr. For this analysis, we assumed costs of $315.99 per hectare for site preparation and $103.17 per hectare for herbaceous weed control. Establishment costs were obtained from [15] for the Southern Coastal Plain. It was assumed that the diameter of all planted seedlings was 5 mm (at root-collar). Seedling cost was set at $45 per thousand and each seedling would cost $0.10 to plant by hand. The annual management cost was $4.94 per hectare and the annual tax rate was $4.94 per hectare. Since timber is commonly bought and sold using green weight rather than volume, we converted the estimates of merchantable volume (inside bark) to green weight assuming each cubic foot weighs 65 lbs. Since equation (6) was presented in English units [14], all economic analyses were initially calculated on a per acre basis and then converted to a per hectare value.

As part of a sensitivity analysis, four sets of stumpage values were examined. The base-case included region-wide stumpage values similar to that for November 2009 (http://www.tmart-south.com/tmart/prices.html); $7.75 per ton of pulpwood, $15 per ton of chip-n-saw, and $27 per ton of sawtimber. The second set assumed reasonable prices ($10 - $19.50 - $35) and the third set involved optimistic prices ($20 - $30 - $44). The fourth set ($7.75 - $19.50 - $44) checks to see the impacts on rotation age when sawtimber is more than five times that for pulpwood. For example, in some locations there might be a short distance to the nearest sawmill but a long distance to the nearest pulpmill.

RESULTS AND DISCUSSION

The Total Sum of Squares for the error term was lower for the OLS fitted equation (Table **2**). This is likely a result of WLS under-predicting volumes of larger trees (Fig. **1**) since the WLS method applied less weight to trees with large volumes. [8] showed the variance stabilizing transformation ($\frac{1}{(D^2Ht)}$) helped to stabilize the variance.

Predictions of total cubic meter volume per hectare were also affected by the parameter estimation techniques (Fig. **2**). The predicted difference was greater when seedlings were planted 1.2 m apart (2,692 SPH). With the wider-spacing (748 SPH) the volume difference between the OLS and WLS methods was less drastic. At early ages, the WLS volume estimates were greater, at age 25 they were nearly equal, and at older ages the OLS volume estimates were greater. This cross-over is also observed in the wide-spacing BLV plots (Fig. **3**).

Since the D^2Ht values for the 748 SPH case are generally within the range of 5,000 to 9,000 (Fig. **1**), the particular parameter estimation procedure had no effect on the optimum economic rotation age. For both estimation methods the optimum rotation was essentially the same for all four sets of revenues (Fig. **3**).

However, for the 2,692 SPH case, the age for the optimum BLV differed for some price scenarios. The biggest difference (i.e. 11 yr) was observed for the third set of revenues ($20 - $30 - $44) where optimum rotation ages were 10 yr (WLS) and 21 yr (OLS). At age 10 yr (where all merchantable trees have D^2Ht values smaller than 5,000) the WLS volume estimate was 83% greater than the OLS estimate (Fig. **2**). At age 21 yr, the volume estimates for OLS and WLS were more similar. At that age, about 18% of merchantable trees have D^2Ht between 5,000 and 9,000, and these larger trees make up 35 to 40% of the total volume estimate (for both methods). However, due to the high pulpwood revenues, and the large yield at age 10 yr for the WLS method, the increase in yield from ages 10 to 21 yr is not great enough to offset the loss in revenue due to discounting (Fig. **3**). This occurs despite the additional 159 merchantable cubic meters at age 21 yr (or 90% more than the merchantable volume at age 10 yr).

Since the OLS volume per hectare at age 10 yr is substantially less than the WLS method, the growth per hectare from ages 10 to 21 yr is enough to offset the loss in revenue due to discounting and therefore the optimum rotation age occurs later. For OLS, the increase in merchantable cubic meter per hectare volume from age 10 to 21 yr is 182 cubic meters (i.e. 185% more). For the WLS method, there is a slight peak in BLV at about age 17. This peak results because of greater volumes in the chip-n-saw product class which has greater revenue relative to the

Table 2. Parameter Estimates, Standard Errors, and Sum of Squares Error (SSE) when the Model is Fitted Using Ordinary Least squares and Weighted Least Squares (Number of Volume Observations (n) = 431 and Number of Trees Equals 156). Statistics are Calculated Using English Units

Estimation Method	$\hat{\beta}_0$	Std. error	$\hat{\beta}_1$	Std. Error	Total SSE
Ordinary least squares	-1.68903	0.2757	0.003003	0.000026	4884.3
Weighted least squares	0.284409	0.0442	0.002703	0.000019	6542.8

pulpwood class. However, the peak is still relatively small compared to the peak at age 10 because of the large amount of predicted pulpwood volume at age 10 and the impact of discounting revenues for another 7 yrs (17 yrs rather than 10 yrs).

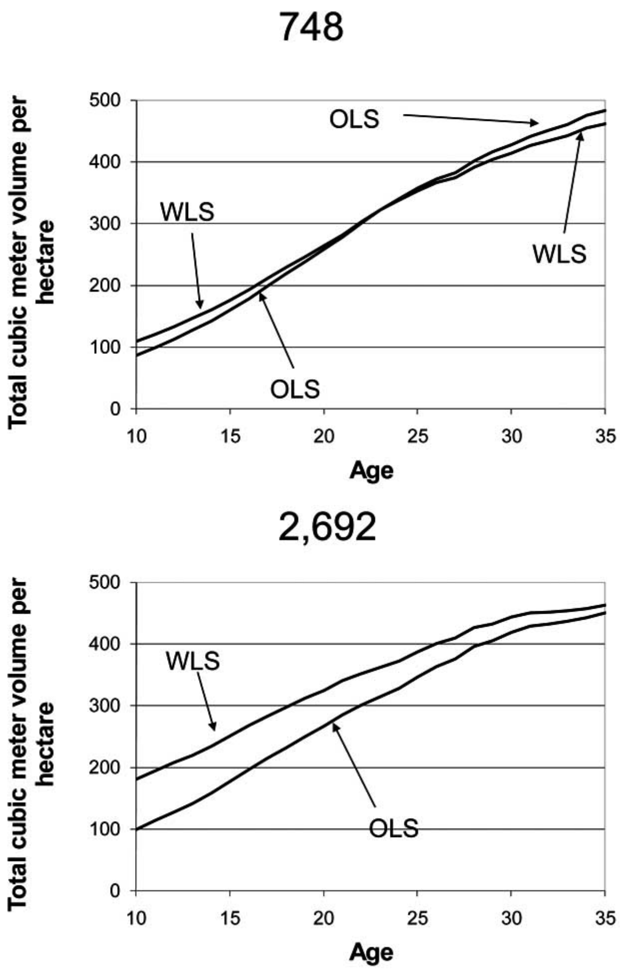

Fig. (2). Predicted total cubic meter outside-bark volume per hectare for two planting densities (748 and 2,692) as estimated using PTAEDA 3.1. Parameters of the combined-variable volume equation were estimated using weighted least squares (WLS) and ordinary least squares (OLS). Volumes were predicted in cubic feet per acre and then converted to cubic meters per hectare using a multiplier of 0.06997.

For the 2,692 SPH, the optimum age is 21 yr for WLS and 26 yr for OLS for revenue sets one ($7.75 - $15 - $27) and two ($10 - $19.50 - $35). For the fourth set ($7.75 - $19.50 - $44) the optimum rotation is 26 yr for both WLS and OLS. These results, including those of the third set, demonstrate that in some cases, the use of OLS can lead to different optimum economic rotation ages relative to WLS.

This conclusion is very interesting. Could it be that a growth and yield model that suggests intensive management can reduce rotation ages does so merely because of the individual tree volume parameter estimation method? For the third set of revenues ($20 - $30 - $44), if a high planting density is established, the WLS method implies a plantation should be managed exclusively for just pulpwood (rotation age 10 yr), while the OLS method indicates a plantation should be managed to produce both pulpwood and chip-n-saw products (rotation age 21 yr).

Rather than examining a particular optimum age, Fig. (**3**) shows there are really optimum peak rotation ages for three sets of revenues (excluding the third set), basically where the BLV trajectories level off for some period of time. For the wider-spacing, the peaks essentially occur at the same time. However, for the closer-spacing, for three sets of revenues (excluding the third set) the WLS peaks occur at earlier ages, although for the fourth set of revenues the maximum BLV occurs at the same age (26 yr). Differences in the timing of the peaks likely result from greater volume prediction at younger ages when using WLS (because of a relatively large percentage of trees with D^2Ht values smaller than 5,000) and since pulpwood revenue is relatively close to sawtimber revenue for revenue sets one, two, and three. For the fourth set, the sawtimber revenue is substantially greater than the pulpwood revenue reducing the impact that the greater WLS volume predictions at younger ages have on determining the optimum economic rotation age – thus delaying harvest to obtain larger sawtimber trees is economically beneficial regardless of the parameter estimation method.

More intensive management scenarios could result in WLS having a greater economic impact for lower planting densities since values of D^2Ht will be greater and may exceed the range of 9,000 (Fig. **1**). Different upper stem outside-bark diameter limits, varying interest rates, and different regeneration costs will also alter the economic impacts of using the WLS method.

Using WLS or OLS can have a large impact on volume estimates and this can affect the predicted optimum economic rotation age. Across a landscape, the difference in stand-level optimum economic rotation age could have a substantial impact on harvest schedules. Of course, in reality, the "true" model is not known and therefore both OLS and WLS merely provide parameter estimates. Which model to use will typically depend on user preference. Some managers might decide to select the model that best accounted for the combination of biological, economic, and statistical considerations.

CONCLUSIONS

From a statistician's viewpoint, when estimating parameters, it is absolutely correct to use WLS to reduce the impact of observations with larger variances. However, sometimes biological and economic penalties are associated with WLS. These penalties might alter the recommended rotation age. Allowing statistical considerations to override economic considerations might seem appropriate from an academic point of view, but it might be questioned by a forestry consultant. This paper demonstrates that, under some cases, economic rotation ages can differ substantially between models based on biological and economic considerations and those based only on statistical theory. As a result, the predicted optimum management scheme can differ substantially. For the "optimistic" set (third set) of stumpage prices, the close spacing BLV when using WLS nearly exceeds the wide spacing BLV (Fig. **3**); this could have serious implications in terms of what is considered the

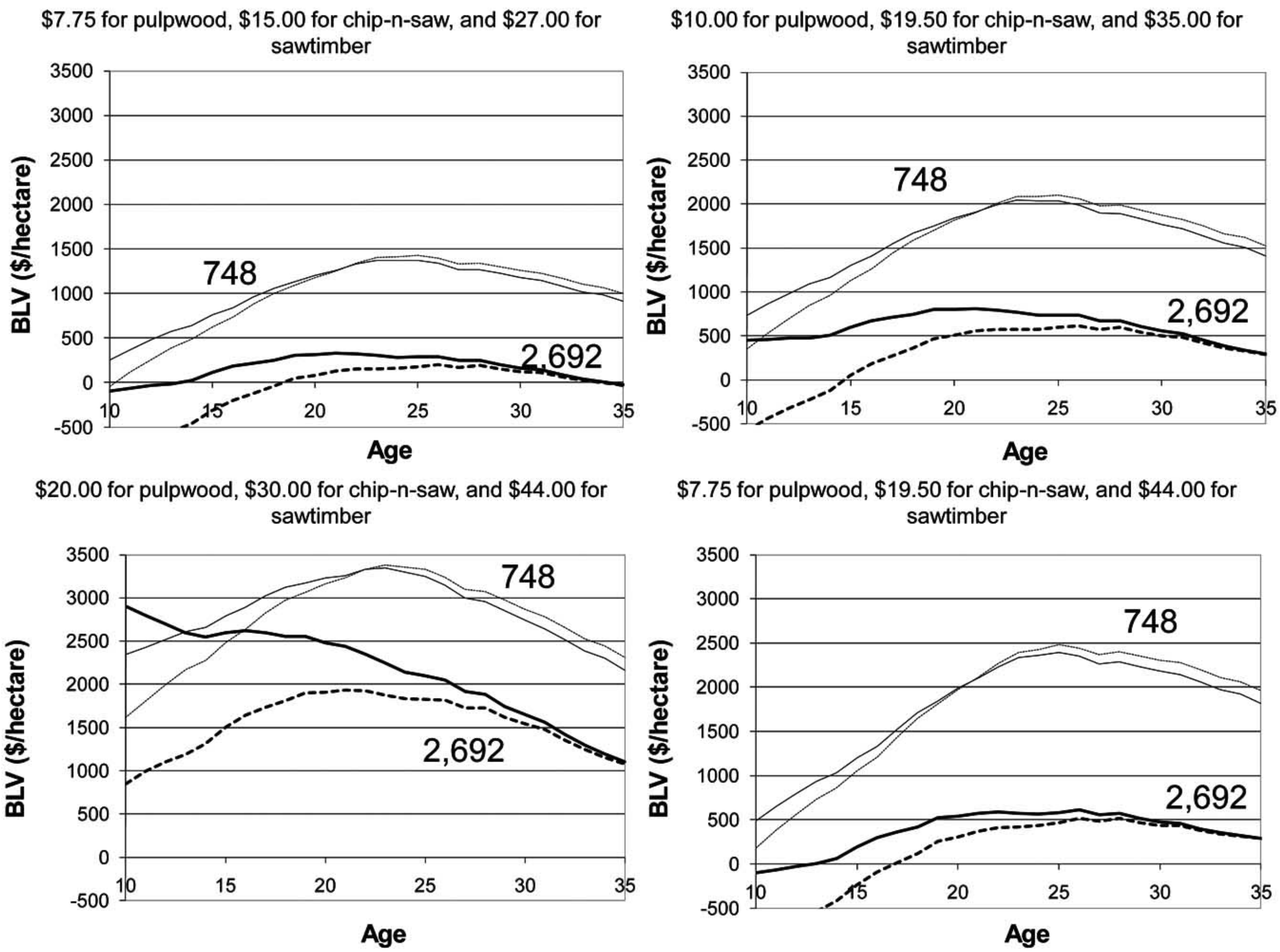

Fig. (3). The effect of stumpage prices and planting density on bare land value (BLV) using PTAEDA 3.1. Parameters of the combined-variable volume equation were estimated using either weighted least squares (solid lines) or ordinary least squares (dashed lines). The numbers 748 and 2,692 correspond to the number of seedlings planted per hectare. BLV values were initially calculated using per acre values and then converted to per hectare values by multiplying by 2.47.

target tree size. Users must remember that growth and yield models do not provide the same predictions [16] because of factors such as sampling error and personal bias about how trees and stands grow but also because of disagreements about statistical methods thought to best estimate parameters; the structure of models can lead to vastly different "optimum" management scenarios.

For instances where non-constant variance exists and prediction is the overriding concern, OLS may be desirable when the economic or biologically most important observations are those with larger variances. However, when one is interested in significance levels of parameter estimates, WLS will be superior (5, pgs. 96-97). For individual tree volume, we know diameter and height influence volume and our greatest concern is the accurate prediction of volume for the merchantable and most valuable trees, not significance levels of the parameter estimates.

ABBREVIATIONS

BLV	=	Bare Land Value
OLS	=	Ordinary Least Squares
SPH	=	Stems Per Hectare
WLS	=	Weighted Least Squares

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