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Modeling and Analysis of the Queue Dynamics in the Nigerian Voting System

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Abstract: Protracted voters' waiting time is widely accepted to be a major impediment to voters' turnout at elections. This paper presents a queueing simulation-optimization based methodology for reducing voter waiting time at the polls. In many places, insufficient ballot materials and poll presiding officials, in combination with lengthy voting time and high voters' traffic, have caused long lines and disenfranchised voters who left without voting. Although the underlying simulation model employs a blend of queueing theory, discrete-event simulation, and optimization, the procedure offers a simplistic methodology, to be used by the typically nontechnical election official, without getting him involved in the intricacies and complexities involved in the modeling process. This paper focuses on methods to mitigate voters' waiting time at the polls and reduce the number of disenfranchised voters. Our simulation results can be used in planning a cost-effective election process that will produce expeditious elections.

AMS MSC (2010): 60K25; 68M20; 90B22; 90B15; 37M05; 93C65; 91B12.

Keywords: Discrete-event simulation modeling, stochastic simulation optimization, resource allocation, voting systems, queueing models, stochastic processes, stochastic modeling.

1. INTRODUCTION

Waiting in lines is a part of our everyday life. Waiting in lines may be due to overcrowding, overfilling or due to congestion. Any time there is more customer demand for a service than can be provided, a waiting line forms. We wait in lines at the movie theater, at the bank for a teller, at a grocery store. Waiting time depends on the number of people waiting before you, the number of servers serving the lines, and the amount of service time for each individual customer. Customers can be either humans or an object such as customer orders to be processed, a machine waiting for repair. Mathematical analytical methods of analyzing the relationship between congestion and delay caused by it can be modeled using Queuing analysis. Queuing theory provides tools needed for analysis of systems of congestion. Mathematically, systems of congestion appear in many diverse and complicated ways and can vary in extent and complexity.

During the local and national elections, many of us exercise our civic right to vote. More likely than not, we also exercise the limitations of our patience by standing in line waiting for our turn at the poll centers. As shown by [1], waiting times to cast votes are directly correlated to voter's turnout. As queueing time increases, voters are more likely to leave without casting their ballots due to impatience and other time commitments. Hence, it is very important to develop voting systems that result in voters waiting the least amount of time possible. Ideally, we would provide a sufficient quantity of ballot materials and poll officials such that voters would never have to wait to cast ballots. However, owing to the cost implications of procuring new voting equipment and hiring new electoral officials, election boards are limited in their ability to procure additional equipment [2]. Equally important, voting systems should provide equity, which means that we should not design systems that favor some voting groups (defined by geography, ethnic, religious, or voting preference) by having shorter lines in some voting locations than others. Such inequities have been a concern in recent elections [3]. While the addition of more voting materials (and polling booths) and personnel would certainly shorten those lines, the local election official in charge of allocating machines to the voting centers may be severely limited due to political and economic constraints. Also, the theoretical determination of the number of electoral presiding officials (and ballot materials), or servers, required in the queueing system that comprises a polling station is typically beyond the technical capabilities of the local election officials [4]. This paper discusses the development of a simulation based methodology for determining voting material requirements at the polls. This procedure frees the election officials of technical modeling complexities, while providing him with the tools he requires for justifying the acquisition of additional personnel (and ballot materials) and allocating the ones he/she has. This methodology provides an excellent example of how mathematical tools of simulation, queueing theory and optimization can be made available to and used by the layman, provided the data requirements and procedures are formulated in his language [5].

Obtaining an optimal scheme for allocating voting officials and materials is challenging and difficult to attain

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for several reasons. (1) Voters arrive at polling stations following a random non-stationary process. The arrival process is usually characterized by surges during the morning, noon and evening times due to work schedules and other time commitments [6]. Also, it is difficult to estimate the expected voters' turnout prior to the Election Day because it depends on many uncontrollable variables (for example, weather and composition of the voting ballot). (2) Non-steady-state queue dynamics during the Election Day. In Nigeria, the polls are open for 10 hours (8am – 6pm). The morning hours (8am – 12noon) are reserved for the voters accreditation exercise while the rest (12 noon – 6pm) of the Election Day is meant for the actual voting exercise. Given the limited amount of time that voting centers are open, the voting queues may still be in a transient state by the close of the day. More often than not, the average arrival rate is greater than the average service rate, which imply queue "explosion" in conventional queueing theory. Such non-stationary arrivals and non-steady-state queues infract the fundamental assumptions of traditional queueing theory. (3) The actual voting settings involve considerable computational complexities. There are hundreds of polling stations across the country and thousands of ballot materials and personnel to be allocated and the input variables are stochastic. The result is either a large-scale non-linear stochastic optimization problem using traditional analytical approximations that may not be valid or a complicated simulation model. Thus, both building model and developing solution methods are challenging endeavors. We model the voting process using a simulation-optimization technique that allows us to employ non-stationary arrivals and non-steady-state queues [2, 6-9]. We allocate voting resources to polling stations using SIMUL8 OPTQUEST optimization tool. The objective in our resource allocation is to improve service and provide voter equity across the polling stations in Nigeria. Our case study is one of the recent major elections conducted in Nigeria.

There are a number of spectacular points that make this paper outstanding. First, we have incorporated in the model, the behavior of an arbitrary arriving voter. The behavior of an arriving voter plays a critical role in determining the performance of a given voting system. In this paper (and in the simulation model), we considered the balking and reneging attributes of the prospective voter. Second, we have incorporated retrials in the arrival process. Retrials occur when voters (customers) return to the system (after a variable length of time) since they left the system earlier (due to impatience) without getting the required service. Third, data analysis results have revealed that there are a variable number of voters waiting at the polling center prior to the start of the accreditation procedure on the Election Day. These voters arrived before 8am. Thus, we have incorporated in the simulation model, a preloaded queue at the startup of a simulation run.

Moreover, for any given resource allocation policy and expected voters turnout, the model can predict system throughput, average waiting time, average queue length, and the number of prospective voters who were still waiting in line by the end of the Election Day. Finally, we developed a stochastic simulation optimization model to evaluate the efficiency and equity parameters of the given resource allocation and our proposed resource allocation methodology.

2. LITERATURE REVIEW

Only a handful of papers apply mathematical models to solving congestion and resource-allocation problems plaguing the voting processes across the globe. At the moment, the only papers of which we are aware that apply probability models and queueing techniques to voting systems are [2, 5-10]. Refs. [2, 9] simulated the voting process using a simulation model that allows them to employ non-stationary arrivals and non-steady-state queues. They used a greedy improvement heuristic to allocate voting machines to precincts. The objective in their machine allocation is to provide voter equity across precincts. Refs. [6, 8] use simulation to model the voting exercise. Using simulation allows for the implementation of the realistic complication in the voting process such as voting-machine failures and non-stationary voter arrivals. Ref. [7] suggest using queueing theory to ascertain the effectiveness of a given machine-allocation policy in terms of voters' waiting times and to improve on such allocation policies. Ref. [7] use simple analytical queueing models to predict average waiting times for voters. Allen and Bernshteyn then suggest an optimization model that uses a minimax objective function to allocate voting machines. Specifically, they suggest allocating machines to minimize the maximum expected voter waiting time across all precincts. The minimax objective is designed to promote voter equity as we discussed above, but there are many other objectives that could be considered. However, Ref. [7] do not consider complicating issues such as non-stationary voter arrivals, machine failures, and specific differences in voting-time requirements due to differences in ballot lengths. Furthermore, they propose only simple greedy-heuristic solution methods for their models, which can produce significantly suboptimal policies. There are several simpler methods used to allocate voting machines to precincts that have been used in previous elections. An intuitive and simple method of allocating voting machines used by many election boards is to allocate machines in proportion to the expected number of voters at each precinct [6]. This method ignores any direct models of queueing effects and differences between precincts. At least one county in Ohio, USA used a utilization equalization allocation policy in the 2008 presidential election to allocate voting machines. This method enforces voter equity by equalizing the utilization of voting machines rather than equalizing waiting times of voters. Moreover, the utilization rate is obtained by traditional queueing theory, which assumes stationary arrivals and steady-state operating conditions. [5] shows that excessive voter waiting time is widely accepted to be a deterrent to voter turnout at elections. He describes a procedure for reducing voter waiting time at the polls. While the underlying model employs a joint use of queueing theory and simulation, the procedure is a cookbook-like methodology, to be used by the typically nontechnical election official, making complex modeling details transparent to him. Grant's paper describes important aspects of model development and provides an illustration of the use of the methodology of queues [2, 6-9, 11-13].

3. COMPARATIVE ANALYSIS OF ANALYTIC MODELS AND SIMULATION MODELS

Voters do not arrive according to a stationary arrival process and this is one of the most difficult problems analysts have to deal with in modeling voting queues. Furthermore, a steady state may not be reached owing to the limitation of the length of an Election Day and the fact that queues are assumed to begin empty. The balking and reneging characteristics of an arriving voter also contribute to the complexities in the modeling process.

Strong simplifying assumptions (such as stationary arrivals, steady-state queues, and patient voters) about the voting system are required when analytical queuing models are applied. Analytic models enable us to gain insights and obtain system performance indicators such as expected waiting times very quickly without developing complicated simulation models. Closed-form queueing-models in conjunction with optimization models can be used to determine optimal policy decisions. The solution methods described in this paper rely on simulation-optimization search techniques.

Because of the short time frame of an actual Election Day, analytical results for the queue dynamics and resource-allocation problem require transient queueing analysis with non-stationary arrivals. Obtaining transient information is generally considered much more complicated in comparison to a steady state analysis [14]. Equally important is the fact that the expected turnout on the Election Day cannot be greater than the number of registered voters; thus, violating the simplifying assumption of an infinite calling population in analytic queueing modeling. Such limitations of the current analytical results on transient queues weaken the advantages of analytical models, which become more difficult to implement and needs more computational time to obtain results. Thus, it is natural to turn to stochastic simulation, with its lesser reliance on simplifying assumptions that might render the model questionable in terms of validity. However, we then need to apply proper statistical design and analysis methods in order to deal with uncertainty in the output, and to enable valid and precise conclusions [2, 5-10].

4. THE SIMULATION-OPTIMIZATION MODEL

In this section of the paper, we describe the details of the simulation model, the input parameters and the assumptions made in the process of model development. We also provide details of the optimization modeling process. The simulation model is developed using SIMUL8 while the optimization model is implemented using OPTQUEST and Visual Logic Programming Language.

4.1. Description of the Simulation Model

We will describe the simulation model as depicted in Figs. (**1**, **2**) below from the viewpoint of an arbitrary arriving voter.

The time period of the Election Day is partitioned into two main time/activity sessions. The voters' accreditation exercise is done during the morning session (8am – 12noon) while the actual voting exercise is conducted between 12noon and 6pm. The output (in terms of voters) of the accreditation exercise forms the input of the actual voting exercise. The accreditation process is modeled as a tandem queueing system with single or multiple servers at each stage and finite buffers in-between servers. The queue in front of the first server is assumed to be of infinite capacity. Potential voters must go through all 3 phases of service to get accredited. Results from data analysis (also by inspection) have shown that voters upon arrival tend to balk if the queue is long. In addition, voters who have experienced a prolonged waiting time in queue leave the system without casting their vote. This is called reneging. A substantial number of the balked and reneged voters return to the system after a variable amount of time. The above mentioned voters' behavioral attributes (balking and reneging) have adverse negative impact on the performance of the Nigerian voting system. If a voter is certified during the accreditation process, then he/she can participate in the actual voting

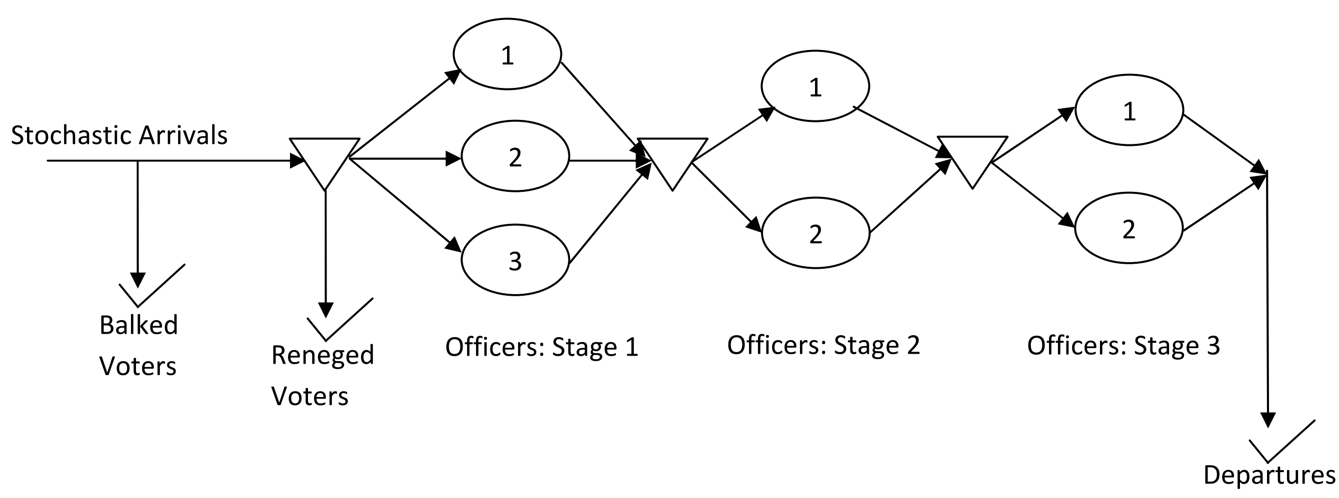

Fig. (1). The Voters' Accreditation.

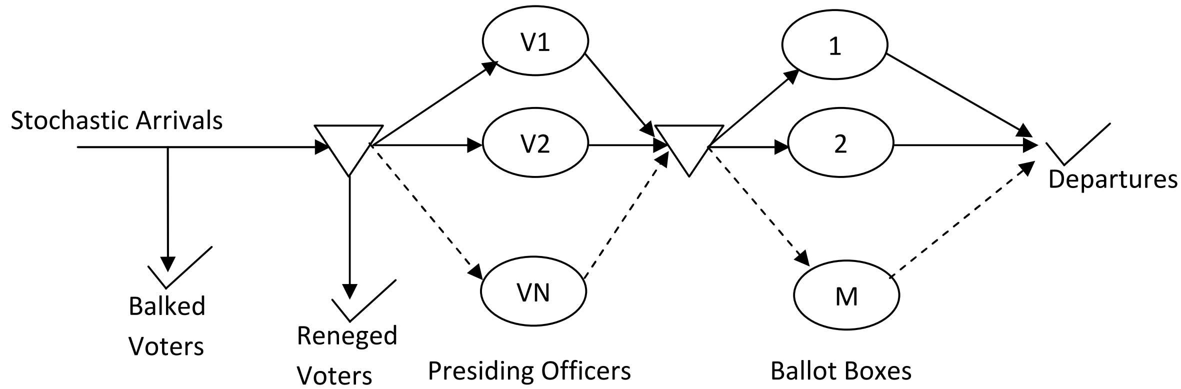

Fig. (2). The Actual Voting Exercise.

exercise (see Fig. **1**). Upon arrival at the polling center, a voter may join the queue if any, or he is immediately served by any of the (free) poll officials. Thereafter, he indicates his political preference by thumb printing on the appropriate section of the ballot paper. Then he proceeds to cast his ballot in any of the available ballot boxes and leaves the system (see Fig. **2**). Owing to the mismatch between demand (arrival influx) and available service capacities (ballot equipment and personnel), there is usually congestion at polling stations. As mentioned earlier, there are a number of factors that render queueing modeling of voting systems analytically intractable. Non-stationary (time-dependent) arrival process, non-steady-state queue dynamics, balking and reneging attributes of the prospective voters are some of the reasons why we have resorted to simulation modeling. The simulation model (as depicted in Figs. **1**, **2**) is implemented using SIMUL8 (and Visual Logic Programming Language) with the input parameters obtained from both data analysis and expert advice. The simulation model provides a tool to evaluate both the current situation and proposed improvement schemes.

4.2. The Stochastic Simulation Optimization Model

The ultimate, perhaps, in analyzing a simulation model is to find a combination of the input factors that optimizes a key output performance measure. For example, there may be an output of direct impact on voter satisfaction, such as waiting times and queue length or of direct economic importance, such as a profit or cost, which we would like to minimize or maximize over all permissible values of the input factors. In general, the input factors of interest include discrete quantitative variables such as the number of personnel at a service stage, continuous quantitative variables such as the mean processing time at a service stage, or a qualitative variable such as the choice of a queue discipline. Although it would be possible in a simulation study to seek optimal values of both controllable and uncontrollable input factors, the primary focus in this research study is on the input factors that are controllable as part of system design and operational policy [2, 7, 9, 11-13].

It is helpful to think of this problem in terms of classical mathematical optimization, e.g., linear or nonlinear programming. We have an output performance measure from the simulation, say *R*, whose value depends on the values of input factors, say $\upsilon_1, \upsilon_2, ..., \upsilon_k$; these input factors are the *decision variables* for the optimization problem. Since *R* is the output from a simulation, it will generally be a random variable subject to variance. The goal is to maximize or minimize the objective function $E[R(\upsilon_1, \upsilon_2, ..., \upsilon_k)]$ over all possible combinations of $\upsilon_1, \upsilon_2, ..., \upsilon_k$. There may be constraints on the input-factor combinations, such as range constraints of the form

$$l_i \le \upsilon_i \le u_i$$

for constants l_i (lower bound), and u_i (upper bound), as well as more general constraints, perhaps *p* linear constraints of the form

$$a_{j1}\upsilon_1 + a_{j2}\upsilon_2 + \cdots + a_{j2}\upsilon_2 \le c_j$$

for constants a_{ji} and c_j for $j = 1,2, \ldots, p$. Let $\upsilon_1, \upsilon_2, \upsilon_3$ and υ_4 be the number of personnel types that we need to hire. Suppose a_{1i} is the salary of type *i* staff member and c_1 is the amount budgeted for salaries and wages, then in choosing the values of the $\upsilon_i's$ we would have to obey the staff-budget constraint

$$a_{11}\upsilon_1 + a_{12}\upsilon_2 + a_{13}\upsilon_3 + a_{14}\upsilon_4 \le c_1$$

In general, if the output *R* is, say, average waiting time of voters that we seek to minimize, or a fixed number of human resources that we seek to optimally allocate across the various voting centers, the problem can be formally stated as

$$\min_{\upsilon_1, \upsilon_2, ..., \upsilon_k} E[R(\upsilon_1, \upsilon_2, ..., \upsilon_k)]$$

Subject to

$$l_1 \le \upsilon_1 \le u_1$$
$$l_2 \le \upsilon_2 \le u_2$$
$$\vdots$$
$$l_k \le \upsilon_k \le u_k$$

$$a_{11}\upsilon_1 + a_{12}\upsilon_2 + \cdots a_{1k}\upsilon_k \le c_1$$
$$a_{21}\upsilon_1 + a_{22}\upsilon_2 + \cdots a_{2k}\upsilon_k \le c_2$$
$$\vdots$$
$$a_{p1}\upsilon_1 + a_{p2}\upsilon_2 + \cdots a_{pk}\upsilon_k \le c_p$$

Solving such a problem in a real simulation context will usually be truly daunting. First, as in any optimization problem, if the number of decision variables (input factors in the simulation) k is large, we are searching for an optimum point in a k-dimensional space; of course, a lot of mathematical-programming research spanning decades has been devoted to solving such problems. Second, in simulation we cannot evaluate the objective function by simply plugging a set of possible decision-variable values into a simple closed-form formula – indeed, the entire simulation itself must be run to produce an observation of the output R in the above notation. Finally, in a stochastic simulation we cannot evaluate the objective function exactly owing to randomness in the output; one way to ameliorate this problem is to replicate the simulation, say, n times at a set of input-factor values of interest and use the average value of R across these replications, $\overline{R}$, as an estimate of the objective function at that point, with larger n leading to a better estimate and, of course, to greater computational effort [12, 13].

Simulation is great for evaluating proposed decisions but it cannot help to uncover the "right decisions" to evaluate. To find the exact combination of conditions that will give the 'best' possible system performance we need to examine multiple scenarios. Each scenario requires some modification of the simulation model. To investigate every possible scenario can be very expensive and time consuming. Simulation Optimization algorithms (and programs like OptQuest for SIMUL8), automate this search procedure in an intelligent manner saving time and providing superior results. Stochastic Simulation Optimization is an extremely difficult problem [10]. The goal here is to allow the stochastic simulation optimization algorithm to automatically alter the system parameters and variables by changing the value of the variables (for example, the replications of each service station, the arrival and service process parameters).

With a clearly defined objective function and carefully specified scenario variables and associated model constraints, a simulation optimization tool works with the simulation model to automate the search for an optimal solution. A typical simulation optimization algorithm will take the steps below. Fig. (**3**) depicts the interactions below the simulation model and the stochastic optimization algorithm.

1. Use the history of previous solution attempts (if any) to choose the next set of values for all scenario variables taking into account the bounds on the variables and any existing constraints.
2. Pass scenario information to the simulation model for evaluation.
3. Receive results for the most recent scenario from the simulation model.
4. If a stopping criterion has been satisfied, STOP. Otherwise, return to step 1.

Simulation thrives as a technique for modeling complex systems, making random variations of parameters and dynamic interactions a natural part of the study discipline. With even a few scenario variables, the list of all possible scenarios to evaluate rapidly becomes unwieldy. Considering the fact that each scenario takes a substantial time to evaluate, and that each scenario must consist of multiple runs, it is easy to see that simply enumerating every scenario is not a very feasible option. Simulation optimization algorithms deal with these difficulties by searching the space of possible solutions in an extremely intelligent manner. The stochastic simulation optimization model of our case study problem (the Nigerian Voting exercise) is implemented using the OptQuest for SIMUL8 optimization tool. OptQuest for SIMUL8 integrates metaheuristic and classic optimization methods at very efficient speeds to produce excellent solutions [7, 11-13].

In this section of the research, our goal is to develop a mathematical tool which will be used to evaluate the efficiency and equity parameters of any given poll resource (materials and personnel) allocation policy. Considering the stochastic nature of the input-factors and output variables of the simulation model, coupled with the stochastic interactions between the simulation model and the optimization package and the computational effort required, we have resorted to implementing the simulation-optimization model using the OptQuest for SIMUL8 plugin. As mentioned earlier, OptQuest for SIMUL8 automates the search for an optimum in an intelligent manner saving time and providing superior results.

5. INPUT PARAMETER ESTIMATIONS

5.1. The Arrival Process

Voters' arrival at the polling location is in an uncoordinated manner, leading to a random pattern. This randomness is of critical importance because it often leads to highly variable waiting times. Yet, the arrival process is constrained in that it is not possible to have many more arrivals than there are possible voters living in the area covered by the polling location, leading to a finite calling population queueing system. Results from data analysis on the arrival process have shown unequal proportions of arrivals across the time intervals during the Election Day. Tables **1** and **2** below show the distribution of the proportions of voters' arrivals during accreditation process and voting exercise on the Election Day.

Tables **1** and **2** portray that the arrival process of voters during the Election Day is (time-dependent) non-stationary. The arrivals within the time intervals (every one hour) follow a Poisson process since the interarrivals within each time interval is exponentially distributed (with coefficient of variation ≈ 1). The fluctuating number of (prospective) voters arriving during different time intervals shows that there are arrival surges at certain times of the day. The arrival process is modeled to reflect all the above mentioned characteristics.

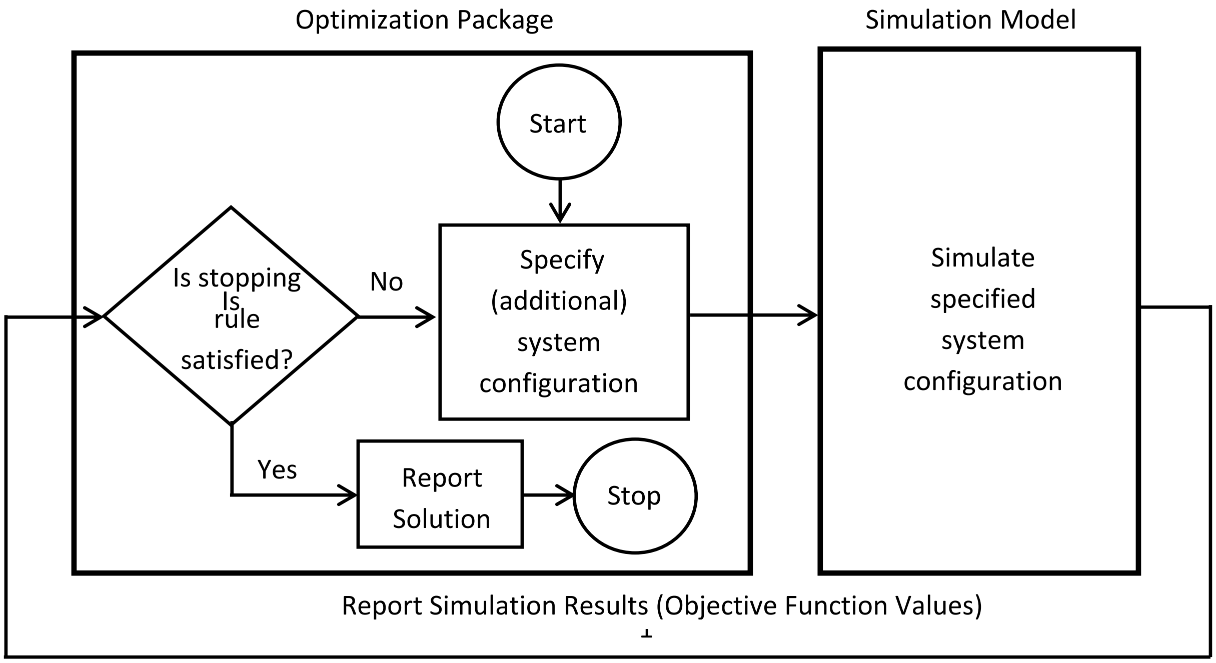

Fig. (3). Stochastic interactions between the optimization algorithm and the discrete-event simulation model.

5.2. The Impatient Voters

Information from recorded data (through double entries) have shown that an appreciable number of (prospective) voters tend to balk upon arrival or renege after waiting in queue for a prolonged length of time during the accreditation and voting processes. Tables **3** and **4** summarize the balking and reneging characteristics of intending voters. Some retrials usually occur after a variable amount of time; 30 minutes on average while the rest completely abandon the system without receiving the desired service.

Table 1. The Distribution of the Number of Arrivals of Prospective Voters During the Accreditation Process

Time Interval	Percentage of Voters' Turnout	Number of Voters	Interarrival Times ($1/\lambda$)
Before 8am	18	107	Pre-Loaded Queue
8am – 9am	37	207	0.29
9am – 10am	7	39	1.56
10am – 11am	25	136	0.44
11am- 12noon	13	72	0.84
After 12noon	Arrivals Rejected	Arrivals Rejected	Arrivals Rejected

5.3. The Voting System: Current Configuration

The general framework of the Nigerian voting system (from the standpoint of a queue analyst) is described in section 4 (see Figs. **1**, **2**). The capacities (in terms of human and inanimate resources) allocated to the polling units across any given state is at the discretion of the state electoral board. From records, it is evident that polling units with the same expected voters' turnout do not usually have the same resources allocated to them. In our case study polling unit, there were 7 electoral officers spread across the 3 stages of the accreditation process and the same officers manned the polling unit during the voting process. The service times at each process stage and the waiting spaces in-between process stages during the accreditation and voting exercises are displayed in the Table **5**.

Table 2. The Distribution of the Number of Arrivals of Voters During the Voting Exercise

Time Interval	Percentage of Voters' Turnout	Number of Voters	Interarrival Times ($1/\lambda$)
Before 1pm	15	83	Pre-Loaded Queue
12noon – 1pm	21	116	0.52
1pm – 2pm	12	66	0.91
2pm – 3pm	28	154	0.39
3pm- 4pm	8	44	1.36
4pm – 5pm	10	55	1.09
5pm – 6pm	6	33	1.82
After 6pm	Arrivals Rejected	Arrivals Rejected	Arrivals Rejected

Table 3. Information on the Balking and Retrial Attributes of (Prospective) Voters

Accreditation Process				Voting Exercise			
Balked	10%	Retrials	80%	Balked	5%	Retrials	85%
		Exit	20%			Exit	25%

Table 4. Information on the Reneging and Retrial Attributes of (Prospective) Voters

Accreditation Process				Voting Exercise			
Reneged	After about 80 minutes	Retrials	73%	Reneged	After about 100 minutes	Retrials	68%
		Exit	27%			Exit	32%

6. RESULTS AND DISCUSSIONS

6.1. Originality of the Obtained Results

The model input parameters were obtained from the statistical analysis conducted on the raw data collected at the various poll centers across the state on the Election Day. The accreditation and voting process models were developed (using SIMUL8) to represent a generic form of the electioneering system configuration. Figs. (**1**, **2**) represent the process configuration and the models were developed in exact form using the input parameters obtained from the data analysis. The results obtained from this study are original since both the statistical and simulation results are firsthand outcomes and have never been obtained elsewhere.

6.2. Simulation Results for Current Settings

In this section of the paper, we present results of the stochastic simulation and optimization models of the voting system. Figs. (**4**, **5**) below depict the graphical representation of the major queues (of an arbitrary simulation run) during the accreditation and voting exercises. The queues are outcomes of the arrival and service processes on the Election Day.

Considering Figs. (**6**, **7**), first, it is obvious that the system started with pre-loaded queues during the accreditation and voting processes. Second, the queue dynamics portray a mismatch between the average arrival rate and average service rate. In the voting system under consideration, the average arrival rate is greater than the average service rate; this mismatch could have led to “queue explosion” in traditional queueing theory, but for the fact that we are dealing with a finite calling population system, we were able to curtail the menace. The Figs. (**6**, **7**) also depict arrival surges at certain times of the Election Day. Moreover, the system cannot attain a steady state since it only runs for a short period of time. Figs. (**6**, **7**) display the waiting time distribution. For instance 30% of the prospective voters waited between 48 and 56 minutes while about 14% waited between 72 and 80 minutes before gaining access to service during the accreditation process. During the voting exercise, about 17% of voters waited between 88 and 99 minutes while 69% waited between 99 and 110 minutes before casting their votes. Recall that the accreditation and voting exercises lasted for 4 and 6 hours respectively. The figures also indicate that an appreciable number of (prospective) voters were still waiting in queue while the exercise came to an end.

Table 5. Process Configuration of the Current Voting System

	Process Time Parameters (in minutes)		
The Accreditation Process	**Process Stages**	**Number of Servers**	**Process Times**
	Stage 1	Service Capacity = 3	Uniform (1, 2)
	Stage 2	Service Capacity = 2	Average (1)
	Stage 3	Service Capacity = 2	Average (1)
	Input Buffer Sizes (Maximum Contents)		
	Process Stages	**Input Buffers**	**Input Buffers Capacities**
	Stage 1	Stage 1 Buffer Capacity	Infinite
	Stage 2	Stage 2 Buffer Capacity	5
	Stage 3	Stage 3 Buffer Capacity	5
The Voting Exercise	**Process Time Parameters (in minutes)**		
	Process Stages	**Number of Servers**	**Process Times**
	Stage 1	Service Capacity = 3	Uniform (1, 2)
	Stage 2	Service Capacity = 5	Average (5)
	Input Buffer Sizes (Maximum Contents)		
	Process Stages	**Input Buffers**	**Input Buffers Capacities**
	Stage 1	Stage 1 Buffer Capacity	Infinite
	Stage 2	Stage 2 Buffer Capacity	1

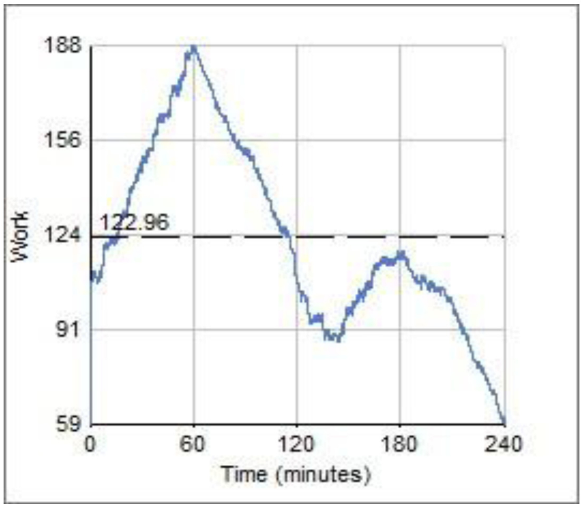

Fig. (4). The queue dynamics during the accreditation process.

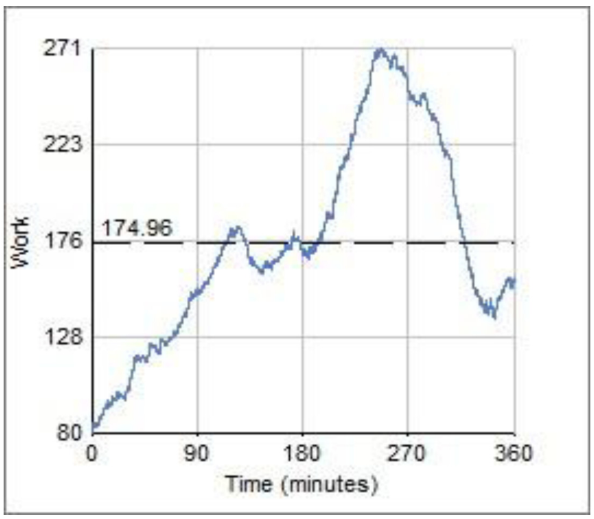

Fig. (5). The queue dynamics during the voting exercise.

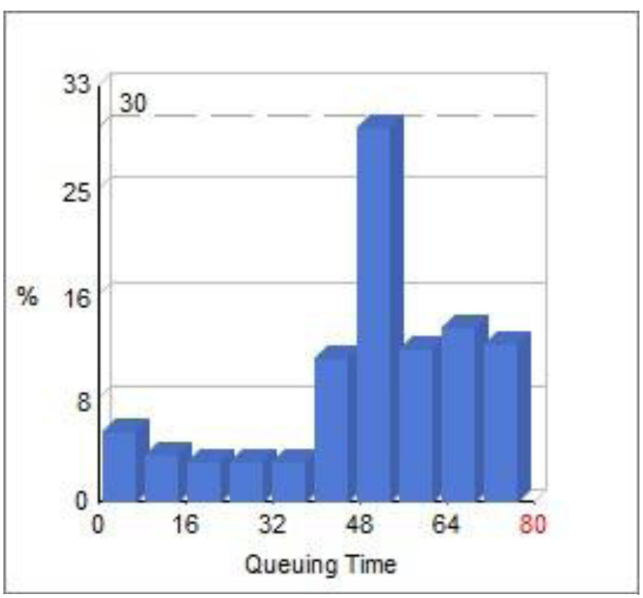

Fig. (6). Distribution of waiting times during the accreditation process.

Table **6** below summarizes the values of the key performance indicators of the situation of the voting system during the accreditation and voting exercises. The results were obtained from the simulation experiment of 2000 trials.

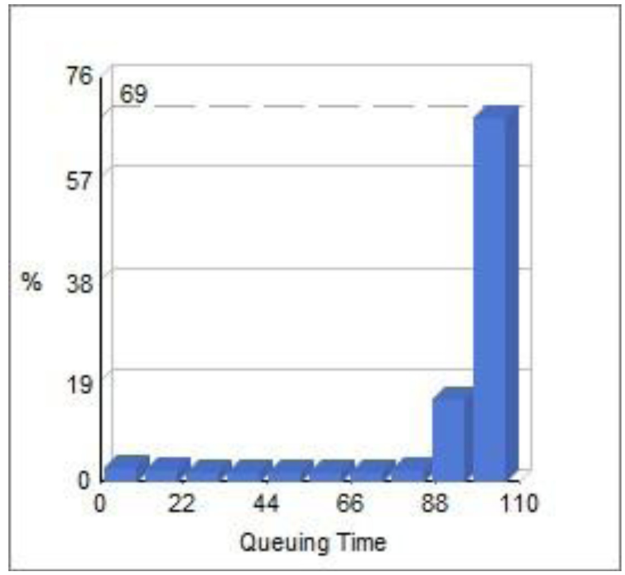

Fig. (7). Distribution of waiting times during the accreditation process.

6.3. Model Validation

The simulation model of our case study polling unit is validated by comparing the simulated results (for example, Average Time in System) with the actual data obtained during the data collection exercise. To further ascertain the credibility of the simulation model, we conducted multiple comparative simulation runs (experiments) using the empirical data (interarrival times and service times) versus the fitted probability distributions. The outcomes (i.e. system performance indicators) of these two scenarios were close; within ±0.2 error margin. It also shows that the probability distributions used to model the interarrival and service times were the right modeling options. Table **7** below portrarys the outcome of the validation exercise.

7. WHAT-IF SCENARIO ANALYSIS

In this section of the paper, we present results on some simulated results of the what-if scenario analysis conducted on the accreditation and voting processes.

7.1. What if the Length of the Election Day is Increased by 2 Hours?

In this section, we investigated the impact of increasing the length of the Election Day period (i.e., adding an hour each to the accreditation period and voting period) on the performance measures of the Nigerian voting system. We shall compare the results (shown in Table **8**) of this what-if scenario analysis with the results displayed in Table **7**. It is assumed that all process parameters and conditions (about the arrival and service processes, voters' routing and system capacities) remain unchanged except for the fact that arrivals are assumed not to occur during the last additional one hour of the extended Election Day.

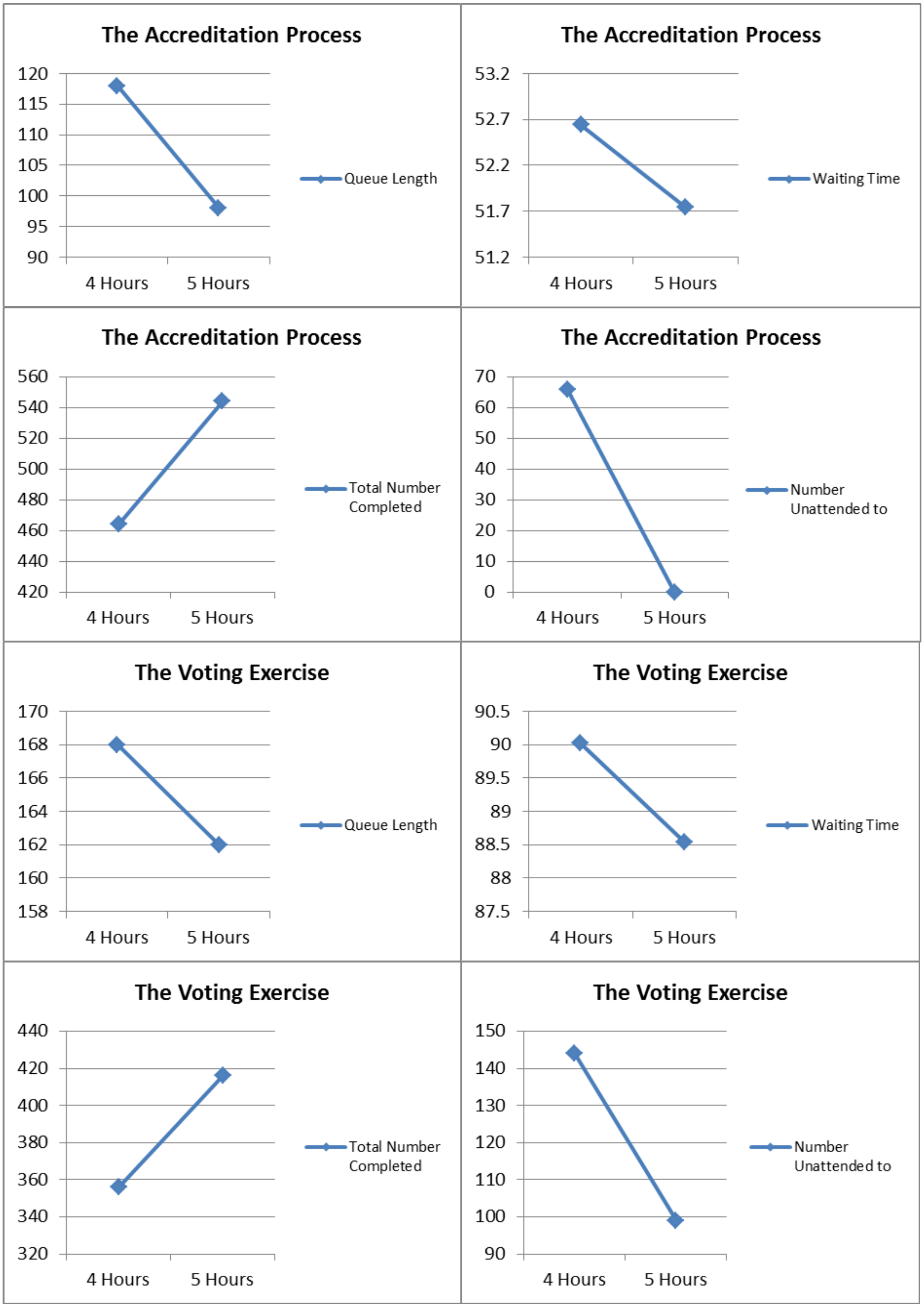

Fig. (8). The response of the voting system to the 2 hours increase on the length of Election Day.

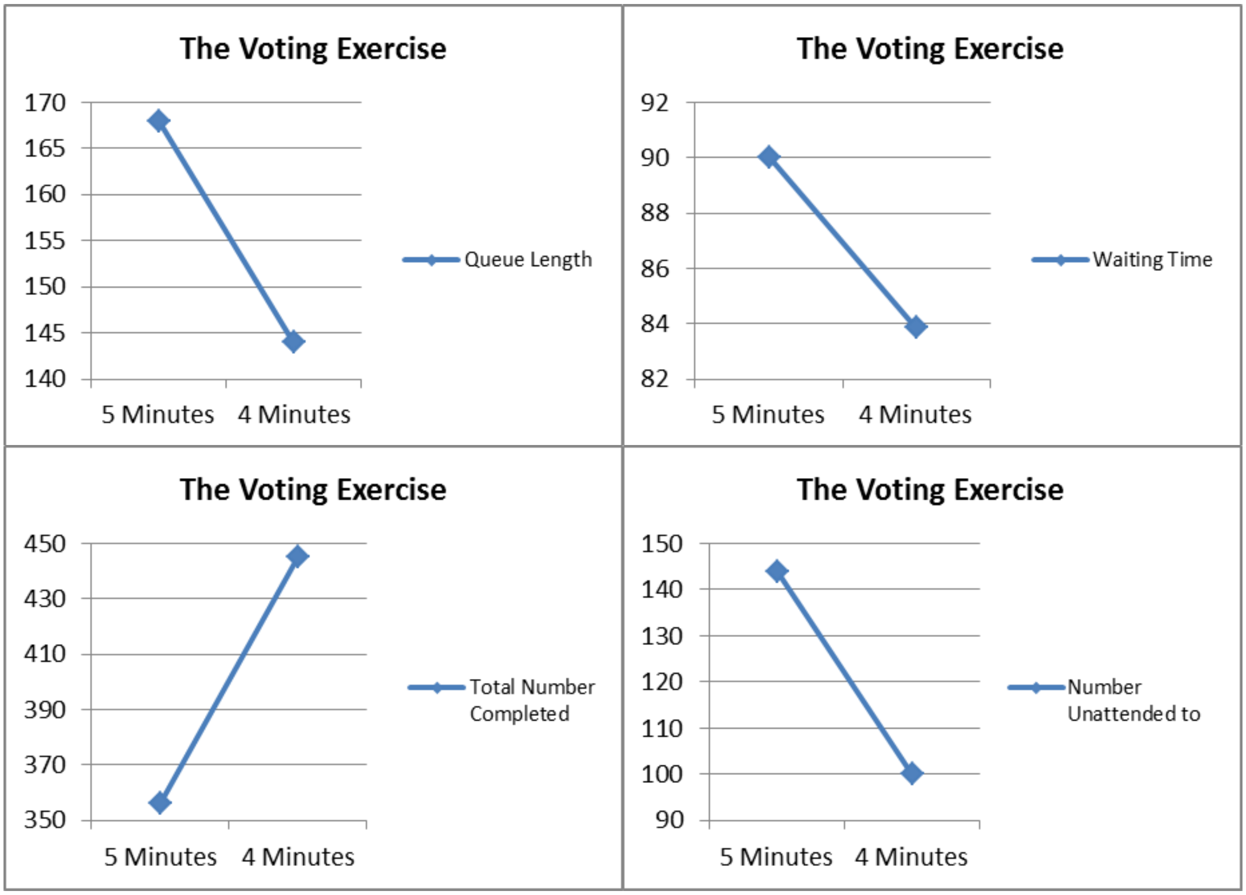

Fig. (9). The impact of the one minute reduction in the ballot box process stage service time on the performance of the voting system.

Table 6. Average Values of the Voting System Performance Measures as it were on the Election Day

	System Locations	Average Performance Measures	Average Values of Performance Indicators
The Accreditation Process	Main Queue	Queue Length	118
		Waiting Time	52.64 mins
		Number Unattended to	66
	Exit Gate (Successful)	Total Time in System	62.59 mins
		Total Number Completed	464.00
		Maximum Time in System	189.29 mins
	Exit Gate (Unsuccessful)	Total Time in System	51.28 mins
		Total Number Unsuccessful	15
The Voting Exercise	Main Queue	Queue Length	168
		Waiting Time	90.02 mins
		Number Unattended to	144
	Exit Gate (Successful)	Total Time in System	102.32 mins
		Total Number Completed	356
		Maximum Time in System	263.38 mins
	Exit Gate (Unsuccessful)	Total Time in System	114.59 mins
		Total Number Unsuccessful	81

Fig. (**8**) displays a graphical comparison between the results as it were on the Election Day and the simulated results obtained by extending the length of the Election Day (Scenario 7.1 Analysis).

Table 7. Total Time Spent in the System (Simulated Results Compared with Actual Data)

	Average Time in System (Empirical Data)	Average Time in System (Simulated Results)
Departures from the Accreditation Process (Successful Completion)	62.76 mins	62.59 mins
Departures from the Voting Exercise (Successful Completion)	101.93 mins	102.32 mins

Table 8. Average Values of the Voting System Performance Measures if the length of the Election Day is Increased by 2 Hours

	System Locations	Average Performance Measures	Average Values of Performance Indicators
The Accreditation Process	Main Queue	Queue Length	98
		Waiting Time	51.74 minutes
		Number Unattended to	0.06
	Exit Gate (Successful)	Total Time in System	62.01 minutes
		Total Number Completed	544
		Maximum Time in System	192.26 minutes
	Exit Gate (Unsuccessful)	Total Time in System	50.46 minutes
		Total Number Unsuccessful	16
The Voting Exercise	Main Queue	Queue Length	162
		Waiting Time	88.54 minutes
		Number Unattended to	99
	Exit Gate (Successful)	Total Time in System	110.33 minutes
		Total Number Completed	416
		Maximum Time in System	367.61 minutes
	Exit Gate (Unsuccessful)	Total Time in System	116.41 minutes
		Total Number Unsuccessful	84

7.2. What if the Service Time at the Ballot Boxes is Reduced by One Minute During the Voting Process?

Based on observations and discussions with experts, it is becoming increasingly important to investigate the impact of reducing the time voters spend at the ballot box voting process stage. It is generally perceived that well informed electorates spend less time to indicate their political preference at the ballot box stage of the voting exercise. It is assumed that all process parameters and conditions (about the arrival and service processes, voters routing and system capacities) remain unchanged except for the fact the average service time at the last stage of the voting process is reduced from 5 minutes to 4 minutes. The duration of the voting process remains 6 hours. The results of the what-if scenario analysis are shown in the Table **9** below. We can compare these results with values in Table **6**.

Considering the plots shown in Fig. (**9**), it is obvious that there is substantial improvement in the performance of the voting system just by reducing the voters' service time at the ballot stage of the voting exercise by one minute.

The following what-if scenarios were conceived in the process of conducting this research work:

- What-if the arrival process is stationary?
  - It is not realistic and would only be of academic interest
- What-if the accreditation and voting processes are conducted on two successive days?
- What-if early voting and e-voting is incorporated into the Nigerian voting system?
  - We did not conduct simulation experiments on the aforementioned scenarios because they are not realistic in the Nigerian electioneering settings.

8. VOTING SYSTEM RESOURCE ALLOCATION USING STOCHASTIC SIMULATION OPTIMIZATION MODELING

Voting queues violate many of the traditional queueing-theoretic assumptions: stationary arrival processes, infinite time duration, etc. So, valid analytical results for the voting-system-allocation problem require transient queueing analysis with non-stationary arrivals, which is much more complicated and limited in comparison to a steady state analysis. Hence, it is natural to turn to stochastic simulation, with its lesser reliance on over-simplifying assumptions that might render the model invalid. However, we need to apply detailed statistical analysis in order to deal with the uncertainty and randomness inherent in the simulation output, and to enable valid and precise conclusions. We

Table 9. Average Values of the Voting System Performance Measures if the ballot box service time is Reduced by 1 Minute

	System Locations	Average Performance Measures	Average Values of Performance Indicators
The Voting Exercise	Main Queue	Queue Length	144
		Waiting Time	83.85 minutes
		Number Unattended to	100
	Exit Gate (Successful)	Total Time in System	90.99 minutes
		Total Number Completed	445
		Maximum Time in System	252.67 minutes
	Exit Gate (Unsuccessful)	Total Time in System	104.11 minutes
		Total Number Unsuccessful	42

developed simulation models that allow for non-stationary voter arrivals, pre-loaded queues (some voters arrive at polling stations before they open), and non-steady-state queues (voter arrivals are cut off when the polls close). The simulation model is interfaced with the OptQuest Optimization tool in order to implement the optimization model. In this section of the paper, we present the application and results of the stochastic simulation optimization earlier explained in section 4 (see subsection 4.2).

Let $S_{ij}(x_i), i \in \mathcal{N}, j \in \mathcal{J}_i$ be the sojourn time variable associated with the j^{th} voter arrival at voting center i where $\mathcal{J}_i = \{1,2,...,J_1\}$ and $J_i > 0$ is the random variable representing the number of arrivals to voting center i and $\mathcal{N}$ is the total number of voting centers in any given state. Note that x_i is the minimum resource (human and materials) strength of voting center i. Define the random variable for the average sojourn time over a single simulation replication (i.e., a single election day) at voting center i as $A_i(x_i) = \frac{\sum_{j=1}^{J_i} s_{ij}(x_i)}{J_i}$. Let $FS_{ij}(x_i)(.)$ be the cumulative density function associated with $s_{ij}(x_i)$ and define $s_i^{sim}(x_i) = E[A_i(x_i)]$.

The goal of the optimization model is to minimize the maximum sojourn time in any voting center subject to the available resources. We formally present the voting system resource allocation problem as an optimization problem.

$$\min_{x_1, x_2 .. x_n} \quad \max_{i \in \mathcal{N}} s_i^{sim}(x_i)$$

Subject to

$$1 \le O_1 \le m_1$$
$$1 \le O_2 \le m_2$$
$$\vdots$$
$$1 \le O_l \le m_l$$
$$O_k > 0; \ \forall k$$

where the O_k denotes the number of (servers) officers at the different stages of service during the accreditation and voting processes and m_k denotes the maximum allowable number of servers at any service stage. Equally important is the fact that the outcome of the stochastic simulation optimization model is can also be influenced by:

- the time-dependent arrival process
- the Non-Exponential service times at process stages
- the Finiteness of the buffers in-between service stages
- the Non-steady-state Queue Dynamics due to short period of the Election Day
- by the fact that the Arrival rate is greater than the Service rate
- the fact that the Queueing Discipline is not exactly FIFO since overtaking is possible (though, FIFO is assumed in the model)
- the Balking, Reneging and Retrials in the time dependent arrival process
- the fact that the expected voters' turnout is at most the total number of registered voters
- the fact that the queueing configuration and layout of the voting centers is not the same in all voting centers

Though the aforementioned model parameters are assumed to remain statistically unchanged during simulation experiments, the decision variables of the above optimization model are the number of servers (humans and ballot boxes) at each service stage.

Considering Table **6**, it is obvious that the electorate experienced prolonged waiting times at the poll stations. The accreditation and voting processes were only open for 4 and 6 hours respectively, whereas voters spent (on average), 22% and 25% respectively of the total time the system was open, waiting in queue. Considering the maximum time spent in the system, it shows that a fraction of the total voters' turnout spent over 3 hours in accreditation process, and over 4 hours in the voting exercise. These results show that there is a pressing need for improvement on the configuration and

Table 10. The Results of the Sensitivity Analysis conducted on the Stochastic Simulation Optimization Model

	Descriptors	Best Values	Minimum Values	Average Values	Maximum Values
The Accreditation Process	Maximum Time in System	93.51 minutes	93.51 minutes	168.64 minutes	204.05 minutes
	Number of Stage 1 Officers	3	2	2.69	4
	Number of Stage 2 Officers	2	2	2.69	4
	Number of Stage 3 Officers	5	3	4.03	6
The Voting Exercise	Maximum Time in System	84.83 minutes	84.83 minutes	178.24 minutes	271.85 minutes
	Number of Stage 1 Officers	5	5	4.19	6
	Number of Ballot Boxes	10	10	7.35	10

capacity of the poll stations during the Nigerian voting process.

Suppose the election board is willing to hire more poll officers (say, 3 and 2 additional election officers respectively in the accreditation and voting processes) and also willing to purchase additional ballot boxes, in order to reduce the excessive waiting times at poll centers. The big question is "how do we optimally allocate these resources across the different process stages and across the different polling centers in any given Nigerian state?" Using the stochastic simulation optimization model implemented in OptQuest, the goal is to optimally allocate these poll resources (officers and materials) across all process stages in order to minimize the maximum time spent at polling units. Table **10** shows the results of the optimization (and sensitivity) analysis conducted on the stochastic simulation optimization model.

The results displayed in Table **6** were obtained by using common sense and experience of election board officials to distribute the poll resources (7 electoral officers and 5 ballot boxes) across the different process stages of the voting and accreditation exercises conducted in poll center with expected voters' turnout of 550 voters. In conducting the stochastic simulation optimization, we minimized the maximum time voters spend in the system. Minimizing the maximum time in the system enforces equity and efficiency. Comparing the "Maximum Time in System" values in Tables **6** and **10**, it is clear that an additional 3 employees (see "best value" column in Table **10**) to the workforce of the polling center (see subsection 5.3) will tremendously improve the performance of the voting system.

9. SUMMARY

Voting systems are complex and are thus analytically intractable. Very few papers apply mathematical models to solve the congestion and resource-allocation problems plaguing the voting processes across the globe, especially in Nigeria.

Turnout rate and voters' arrival pattern are two factors in the voting systems that are essentially impossible to control. Our experimental results indicate a tremendous improvement on the system performance if the length of the Election Day is increased by 2 hours regardless of turnout rate and voters' arrival patterns. The simulated results also indicate a significant improvement on the system performance if the vote casting service time is reduced by one minute.

Providing an efficient and equitable voting experience across all voting centers is generally perceived as an important goal in elections. Our goal is to provide equity to all voters so that no one particular group of voters is disadvantaged or disenfranchised. The maximum sojourn times across all voting centers as proposed in this paper is a performance metric for "equity." To deal with the intricacies and complexities in the voting system such as non-stationary voter arrivals, non-steady-state queues, balking and reneging attributes of (prospective) voters we propose a stochastic simulation-based optimization model to generate poll resource allocations to provide increased voter equity. It is shown that this model-based allocation technique outperforms the common sense method for allocating voting materials that is currently in use.

10. RECOMMENDATIONS

We recommend first, using methods highlighted in this paper to mitigate voters' waiting time at the polls and reduce the number of disenfranchised voters.

Second, we recommend using our stochastic simulation optimization model in planning the Nigerian electioneering process in order to achieve a cost-effective election process that will produce expeditious elections.

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CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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