Time Domain Modelling of First Return Stroke of Lightning

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Abstract: The four most important factors that govern the return stroke evolution can be identified as: (i) electric field due to charge distributed along the channel, (ii) transient enhancement of conductance by several orders at the bridging regime (iii) the non-linear increase in channel conductance at the propagating current front and (iv) the associated dynamic electromagnetic field which support the evolution of current along the channel. For a more realistic modelling of the lightning return stroke, the present work attempts to consider these aspects in suitable manner. The charge simulation method is employed for evaluating the quasi-static field due to (i). For the dynamic field, the problem involves conduction along a thin structure with open boundary on one side. Further, in order to efficiently represent a vertically extended grounded strike object, as well as, channel of quite arbitrary geometry, boundary based approach is believed to be the ideal choice. Considering these, a time-dependent electric field integral equation (TD-EFIE) along with a sub-sectional collocation form of the method of moments (MoM) is chosen for the numerical field evaluation. The dynamic variation of conductance in the channel other than the bridging zone is modelled by a first order arc equation. For the bridging zone, arc equation which explicitly portray in some sense, accumulation of energy is considered. Accordingly, formulations given by Barannik, Popovic and Toepler were scrutinized for their suitability. After some preliminary simulation studies, a self contained model for the first return stoke of a lightning flash is presented. The stability of the model is verified by running the program for longer durations with different cloud base potentials and cloud base heights. Simulation results are in agreement with the field data on current and velocity decay rate for the first one kilometer height. Also, the relation between the charge density at channel tip and the return stroke current peak favorably compares with the literature.

1. INTRODUCTION

Lightning is known to be a luminous, high current, natural electric discharge produced in the atmosphere, of length extending up to kilometers. Amongst different types of lightning, the cloud-to-ground discharge has gained maximum importance because of its detrimental effects on livestock, ground based structures and sensitive electrical and electronic equipments. Due to the large magnitude and rate of rise of current associated with the return stroke phase of a lightning discharge, it is basically responsible for most of the lightning induced damages and hence, has gained maximum prominence. In general, the return stroke modelling is essential for: (i) the study of return stroke current evolution, (ii) understanding the interaction of lightning with tall strike objects (iii) accurate description of the fields in the vicinity and (iv) the analysis and design of a suitable lightning protection system (LPS). The necessity of a theoretical model has arisen from the facts that: (i) experimenting in the field is very time consuming, as well as, expensive and probably, it takes a few decades to get reliable data (due to the random nature of the phenomenon) (ii) it is impractical to conduct realistic laboratory experimentation due to the large differences in the phenomenon.

With regard to the intended modelling, the following point would be worth mentioning. For the electromagnetic aspects of lightning discharges, it would be adequate to reliably emulate the return stroke current evolution along with the charge neutralization and to pertinently represent the electromagnetic interaction of lightning stroke with a vertically extended grounded strike object. At the same time, it is also worth reiterating here that it is the electric field produced by the charge deposited initially on the channel which excites and supports the return stroke current evolution. In other words, it is the electromagnetic field which dominates over other physical processes and from the application perspective it is the same electromagnetic aspect of lightning that assumes importance. Therefore, it may be adequate to macroscopically emulate other associated physical processes leading to transient change in conductance, diffusion of charge from the core into the corona sheath etc.

In our considered opinion, any realistic model for the return stroke current evolution should necessarily incorporate the above mentioned aspects. This forms the basic goal of this research work. The paper is organized in the following manner; a brief review of the existing lightning return stroke models along with their suitability and limitations will be presented in the next section (Section 2), which is followed by time domain electromagnetic modelling (Section 3), simulation results (Section 4) and conclusion (Section 5).

2. REVIEW OF EXISTING RETURN STROKE MODELS

Classically the lightning return stroke models are broadly classified into four classes [1-3]. They are Gas Dynamic models (physical models), Distributed Circuit models (Transmission Line models), Electromagnetic models and Engineering models. Gas Dynamic models [1] are primarily concerned with the radial evolution of a short segment of the lightning channel and its associated shock wave with the

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input being an assumed channel current versus time. Principal model outputs include calculation of physical parameters of the lightning channel such as temperature, pressure and mass density as a function of radial coordinate and time. These models are not intended for depicting the return stroke current evolution.

Engineering models [1-3] are the models for radiated electromagnetic fields rather than modeling of the evolution of lightning current along the channel. In these models, for a specified spatio-temporal distribution of current along the channel, the remote electromagnetic fields are computed by using Maxwell's equations. In these models, the physics of the lightning return stroke is deliberately downplayed and the emphasis is placed on achieving good agreement between the model-predicted electromagnetic fields and those observed at distances from tens of meters to hundreds of kilometers. As the engineering models require specification of current throughout; their extension to include tall strike objects requires specification of current even on them. For this, the transmission line like behavior was envisaged and time invariant reflection coefficients were employed at either ends of the tall strike object. These coefficients were deduced from measured currents on those particular grounded objects or by assuming transmission line like behavior along with correspondingly estimated channel impedance. First of all, it was not very clear whether fixed reflection coefficients would be valid for TEM mode and with non-linear channel dynamics. Further, whether the deduced coefficients are applicable to all types of strokes and more importantly, for all kinds of vertical extended grounded strike objects which needs a detailed examination. As the engineering models are not designated for modelling return stroke current evolution, they are not considered for the intended study.

The distributed circuit models and the existing electromagnetic models can emulate in some sense the evolution of channel current and hence, will be discussed a little more in detail.

(A) Distributed Circuit Model

In this type of modelling, the lightning channel is considered to be a transmission line with distributed series R-L and shunt C parameters. The equivalent transmission line is assumed to be charged by the preceding leader phase to a specified potential and then closed at the ground end with a switch to initiate the return stroke. In general, each of the transmission line parameters representing the return-stroke channel, if can be defined pertinently, would be a function of both space and time [4,5]. This approach, which is quite appealing to electrical engineers and representable in EMTP, can describe in some sense the evolution of channel current. Incidentally, very limited efforts could be seen towards the modelling of channel-tall strike object interaction. However, distributed circuit model [4-7] suffers from serious inherent limitations. It assumes Transverse Electro-Magnetic (TEM) mode of propagation of return stroke current wave which is difficult to accept as (i) there is a large component of electric field in the direction of propagation all along the wavefront and most importantly at the bridging zone, (ii) for TEM mode there should be atleast two conductors with total charge at any wavefront section equal to zero [8] and further, in general, the separation distance between them should be very small compared to the associated wavelengths. This condition even under the assumption of perfectly conducting ground can not be satisfied. Therefore, TEM mode cannot exist and in particular, even a quasi-TEM approximation fails for the time regime spanning up to the current peak (which forms the critical time period). At the best, as significant conduction is solely limited to the channel core, a Transverse Magnetic (TM) mode can exist (for the wavefront regime). Further, due to the reasons quoted above, extension of distributed circuit approach would be erroneous for modelling strike to tall strike objects.

(B) Electromagnetic Models

The present day electromagnetic models simulate the propagation of return stroke current for the evaluation of fields and current in tall strike objects. They invariably employ thin wire antenna approximation to the channel with the lumped source excitation at the bottom of the channel. Solution of the governing field equations is envisaged without resorting to any simplification of the field structure (for example, no assumption is made on the mode of propagation). In other words, the inherent limitations associated with distributed circuit model are overcome. The governing fields are numerically solved by using Electric Field Integral Equation (EFIE) along with Method of Moments (MoM) [9]. Incidentally, both frequency domain and time domain approaches [10] have been employed, where the former approach seems to be more frequently employed. For realizing the reduced velocity of propagation, either distributed R-L loading [11] is employed or the channel is embedded in a dielectric medium (other than air) that occupies the entire half space above ground [12,13] or in a dielectric cylinder of finite radius [13]. A thorough review of the above models has been reported in [13,14]. In the frequency domain approach, more realistic ground has also been simulated [11]. These works have demonstrated fairly good agreement between the model-predicted and typical measured electric fields at distances ranging from tens of meters to tens of kilometers. Also, computed currents for tall strike objects were satisfactorily agreed. The frequency domain modelling has also been employed in the analysis of lightning surge response of transmission lines [15], as well as, lightning protection systems [16,17]. These models are basically good for evaluating the lightning electromagnetic fields. They appear to be not originally intended for the simulation of lightning current evolution and hence, are not considered for modelling due to the following reasons.

In actual scenario, the electric field produced by the charge deposited on the channel mainly forms the excitation; however, in the above works a lumped source model is employed. In addition, the lumped current source modelling imposes almost 100 % reflection at its junction and further, almost completely isolates the channel impedance with that of the tall strike object. In order to overcome this discontinuity, some literatures employ voltage source modelling with smaller series impedance. There exists an uncertainty with regard to the impedance mismatch at the channel-tall strike object junction and that at the attachment point. Also with the voltage source modelling, artificial manipulation is necessary to ensure a desired channel base current waveform. This effort invariably produces current in tall strike objects which is same as that given by the current source model.

Finally, the significant amount of dipolar electric field produced by the lumped source is unrealistic and placement of the source is a big question for elevated attachments points. The lumped source modelling at the best, charges the channel rather than discharging it.

(C) Essential Aspects to be Considered in Return Stroke Modelling

For a realistic return stroke modelling, apart from considering accurately the associated electrodynamics, the other important aspect that needs to be considered is the dynamic channel conductance. The streamer ahead of the descending main leader meets the upward moving streamer / leader at the attachment point. The mechanism occurring at the bridging zone is very complex and involves change in conductance of many orders of magnitude, which occurs in a very short interval of time. This transient change in conductance is considered to be the main cause for the return stroke current initiation. The non-linear variation of the channel conductance at the wavefront supports the return stroke current evolution. The associated transient processes involve variation in the conductivity and radius of core. Perhaps, for a macroscopic representation of the core dynamics, it would be possible to describe the combined effect of above two parameters in terms of channel conductance. Whether it is the bridging zone or any other portion of the channel, it is the prevailing electric field which is responsible for the processes leading to a change in conductance. The prevailing electric field serves as a prime mover for the processes leading to enhancement of the conductance.

With regard to the solution of the dynamic electromagnetic field, time domain approach possesses several advantages over the frequency domain approach even for a linear problem. Further, the non-linearity in the channel conductance can be conveniently handled in the time domain approach giving it an edge over the frequency domain approach.

Incidentally, in [18] these aspects have been fully considered and Finite Difference Time Domain (FDTD) method was employed for the numerical field evaluation. Braginskii's spark law based formulation for the streamer regime and first order arc equation for the arc regime had been implemented. A limited comparison is provided for the spatio-temporal current distribution and results were very satisfactory. Although axi-symmetric problem geometry was employed, very special efforts are made in modelling the thin core. The open boundary was truncated by perfectly matching layer. Due to the use of domain based approach, large number of variables had to be handled at every time step. The computational efforts required for modelling even an inclined geometry and tower like tall strike objects would be increased by an order; thereby, further, increasing the complexity.

3. PRESENT WORK

It was envisaged that the difficulties in the above work could be overcome by employing a boundary based numerical field solution approach. This approach can automatically handle the open boundary, as well as, arbitrary geometry of the channel and tall strike object. The present work basically aims to develop a model for the lightning return stroke incorporating all the essential features enumerated above. Accordingly, the charge on the channel would form the excitation, transient change in the conductance leads to current evolution and the dynamic electromagnetic field, which is numerically evaluated by boundary based approach, supports the return stroke phenomenon. In this paper, the work carried out on the modelling of channel conductance would be emphasized.

Even though a boundary based numerical field computation approach can enable return stroke model to handle complex geometries, as a first step in this direction, only axi-symmetric geometry will be considered in this paper. There are other simplifying assumptions made which are deemed to have minimal impact on the capabilities of the proposed model. They are: (i) the lightning channel is considered to be without any branches. Excluding the case of simultaneous termination on more than one point, the influence of branch can be considered to be insignificant for the critical time period, (ii) cloud dynamics is not known precisely and hence, neglected. For typical channel lengths, the influence of clouds if any, could be seen only at the later half of the tail portion and hence, is not important. (iii) Explicit reference to dynamically varying channel radius, temperature and the air density was not made. However, it is believed that the arc equation employed to describe the temporal variation in conductance would adequately take care of their influence, (iv) earth is considered to be perfectly conducting. Based on the field observations presented in the literature, it can be concluded that soil electrical parameters have little influence on the stroke parameters.

The return stroke modelling is categorised into two parts: the electromagnetic aspects and dynamic channel conductance, which will be dealt below.

(A) Electromagnetic Aspects

Lightning channel can be considered as both an electrically and a geometrically thin structure. An EFIE, mostly applied for the analysis of thin wire structures is used to determine the time-dependent current distribution on the structure excited by an arbitrary time varying electric field. In this work, the numerical solution methodology of Time Domain Electric Field Integral Equation (TD-EFIE) given by Burke [9] is employed. The EFIE enforces the following equation along the wire:

$$\hat{s}\cdot\vec{E}^{A}(\vec{r},t)=\frac{\mu_{O}}{4\pi}\int_{C(\vec{r})}\begin{bmatrix}\dfrac{\hat{s}\cdot\hat{s}'}{R}\dfrac{\partial}{\partial t'}I(s',t')+\\ c\dfrac{\hat{s}\cdot\vec{R}}{R^{2}}\dfrac{\partial}{\partial s'}I(s',t')-\\ c^{2}\dfrac{\hat{s}\cdot\vec{R}}{R^{3}}q(s',t')\end{bmatrix}ds'+I(s',t')Z_{S} \quad (1)$$

where, $\vec{r}$ = observation point location with respect to the origin, t = observation time, C($\vec{r}$) = path along which the current I($\vec{r}$,t) is flowing, R = distance between observation and source point, $\vec{r}\,'$ = source point location with respect to the origin, s´ = s($\vec{r}\,'$) = distance between the source point and ground end of the wire structure, $\hat{s}'$ = unit tangent vector to C($\vec{r}$) at $\vec{r}\,'$, $\hat{s}$ = unit tangent vector to C($\vec{r}$) at $\vec{r}$, c = velocity of light in vacuum, $t' = t-(R/c)$, $\vec{E}^A$ = the field produced by the charge deposited along the leader channel, $\vec{E}$ = the reaction field generated due to the return stroke current $I(s', t')$ and the associated linear charge density $q(s', t')$ and $I(s', t')$ Zs = resistive drop across the segment under consideration. The above equation basically states that the resultant tangential electric field comprising of the incident and scattered fields must be equal to the residual field required by the resistive drop. The sub-sectional collocation form of the MoM [9] is used to reduce this integral equation to a form that can be evaluated on a computer as an initial value problem. Sub-sectional collocation method comprises of (i) discretising the structure into a number of subsections (or segments) whose union may approximately or exactly represent the whole (in the present code, the number of segments *Ns*, considered for a 2.5 km channel is 500) (ii) the unknown variable, in this case, the segment current, is expanded on each segment using Lagrangian interpolation function in two dimensions i.e. in space and time [9] (iii) boundary condition is enforced in a point wise manner at the centre of every segment spanning the structure. The charge and current is related by the continuity equation given by:

$$\frac{\partial}{\partial s'} I\left(\vec{r}', t'\right) = -\frac{\partial}{\partial t'} q\left(\vec{r}', t'\right) \tag{2}$$

Hence, a quadratic interpolation function in space for current yields a linear interpolation function in space for charge. Special care was necessary for handling charge so as to be in consistent with the interpolation function. After an elaborative simplification the final equation is as follows,

$$\frac{\mu_o}{4\pi}\sum_{i=1}^{N_S}\hat{s}_u\cdot\int_{-\Delta_i/2}^{\Delta_i/2}\left[\begin{array}{l}\frac{\hat{s}'_i}{R_{iu}}\frac{\partial}{\partial t''_j}I_{ij}\left(s''_i,t''_j\right)+\\ c\frac{\vec{R}_{iu}}{R_{iu}^2}\frac{\partial}{\partial s''_i}I_{ij}\left(s''_i,t''_j\right)-\\ c^2\frac{\vec{R}_{iu}}{R_{iu}^3}q_{ij}\left(s''_i,t''_j\right)\end{array}\right]ds''_i$$

$$+I_{ij}\left(s''_u,t''_v\right)Z_S = \hat{s}\cdot\vec{E}^A\left(s_u,t_v\right) \tag{3}$$

where, $u = 1,\ldots, N_S$, $v = 1,\ldots, N_T$, N_T = number of time steps, $R_{iu} = \left|\vec{r}_u - \vec{r}_i - \vec{s}_i\right|$, $\vec{s}_i \equiv s''_i\hat{s}_i$, $t''_j = -s''_i/c$ for measurement above source point, $t''_j = s''_i/c$ for measurement below source point and for self segments $t''_j = -s''_i/c$ for s''_i varying from 0 to $\Delta/2$, $t''_j = s''_i/c$ for s''_i varying from $-\Delta/2$ to 0.

The field coefficients in eq. (3) are analytically evaluated using MATLAB symbolic computation. The present code is validated with the results obtained by NEC for a 1m length dipole antenna of radius 0.00674m divided into 22 segments with excitation $V_s(t) = \exp[-a^2(t-t_{max})^2]$, where $a = 1.5 \times 10^9$ sec^{-1}, $t_{max} = 1.43 \times 10^{-9}$ sec and radius of 0.000674 m. This particular geometry was selected so as to be close to that used in [19] for validation. It is evident from Fig. (**1**) that matching is excellent.

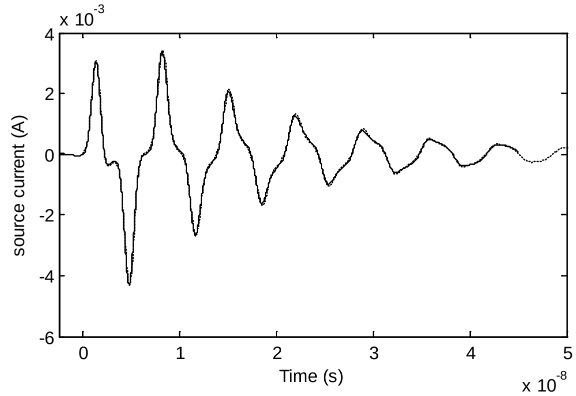


Fig. (1). Comparison of the results for validation (--- simulation result, solid line-NEC result). Time (0 – 50 ns) & Source Current (-6 to 4 mA)).

To determine ($\vec{E}^A$) in eq. (3), evaluation of initial charge deposited on the channel is required which is discussed in the next section.

(i) Evaluation of Charge Distribution on the Channel

Before the final bridging, the time variation in the global electric field is rather slow, which permits a quasi-static modelling of the associated field. The prevailing electric gradients suggested are 60 V/cm [20] for the leader portion and 5-12 kV/cm for the streamer portion [21]. Based on the works on corona, the corona sheath surrounding the core is assumed to have an internal gradient of 24 kV/cm. The initial charge on the channel can be deduced by solving for electrostatic field. The radius of the corona sheath is unknown and can be evaluated iteratively by enforcing a gradient of 24 kV/cm up to its radial boundary. For the required field computation, Charge Simulation Method (CSM) is employed with unknown charge density assumed to be linearly varying in each segment.

The other critical entity required for the modelling of return stroke is the non-linearly varying channel conductance and it will be dealt in the next section.

(ii) Dynamics of Corona Sheath

As the wavefront propagates along the channel, charges of opposite polarity will be deposited along the core. This in turn commences the neutralization of the charges in the corona sheath. Neutralization process can also be visualized as deposition of opposite polarity charges in the corona sheath. Due to the low conductivity of the corona sheath, charge on the channel core would diffuse relatively slowly into the corona sheath. The expression for the charge diffusion can be basically derived from the continuity equation and is given as follows.

$$\frac{d\lambda}{dt} + \frac{\sigma}{\varepsilon_0}\lambda = 0 \tag{4}$$

where, λ is the linear charge density. Since the radius of the corona sheath is comparable with channel segment length, for the calculation of field due to corona sheath charge, line source approximation is no more valid. Hence, field is calculated considering volume charge distribution.

(B) Modelling the Channel Conductance

It is worth recalling here that the large increase in the conductance of the channel core in a relatively short duration of time is basically responsible for return stroke evolution. The increase in conductance at the streamer section initiates the process while that along the other portion of the channel supports the current propagation. The modelling of transient build up of conductance in the channel is divided into two parts i.e. conductance in the arc regime and conductance in the bridging section.

(i) Arc Regime

Even before the inception of the return stroke, it is believed that the whole of the descending and the upward discharge except, for the streamer section, is in the arc regime [22]. The average currents are estimated to be in the range of hundreds of amperes [20]. For the leader portion prior to bridging, the dynamic variation of the conductance is described by a first order arc equation [23-25]. For rising currents, the equation is given by,

$$\frac{dg}{dt} = \frac{g_{\infty}(i) - g(t)}{\theta_r} \tag{5}$$

where, g is the conductance per unit length, θ_r is the arc time constant for rising current, g_∞ is the steady state conductance for constant current i and is related to the steady state gradient E_∞ by [25] :

$$g_\infty (i) = i / E_\infty \tag{6}$$

For falling currents, the equation is given by,

$$\frac{dg}{dt} = - \frac{g(t)}{\theta_f} \tag{7}$$

where, θ_f is the arc time constant for falling currents. $E\infty$ is related to i by the following relation [26]:

$$E_\infty = Ai^{-\alpha} \tag{8}$$

where A = 0.5e5, α = 0.4 [26]. On the other hand, experimental results on high current arcs indicate sensibly constant gradient irrespective of current. With the crossover of the return stroke current front, the corresponding portion of the channel is converted into a high current electric arc with significant enhancement of the conductance. Considering the simplicity of representation even for the high current regime, the above set of equations is deemed to be applicable. The importance of channel conductance at any point is mostly the dominant factor till the wavefront crosses it.

For the simulations, unless otherwise stated, the following values were chosen for the parameters: initial gradient of the leader portion (E_L) = 6 kV/m [20], initial gradient of the streamer portion (E_{str}) = 400 kV/m [25], arc time constant for rising currents (θ_r) = 50 µs [25], arc time constant for falling currents (θ_f) = 500 µs [26] and conductivity of corona sheath (σ) = 40 µS/m [18]. Generally, a lightning cloud is located at a distance of 2-3 km [27] and the estimated cloud potential is of the order of 20 MV-100 MV [20]. In this work, the adopted cloud base height and cloud base potential is taken to be 2.5 km and 50 MV respectively with a channel core radius of 2.5 mm [28]. The entire channel was divided into 500 segments resulting into a segment / step length of 5 m (except for the 4.5 km channel length where the channel was divided into 900 segments to keep the segment length constant). The time step represented the time taken to travel one step length and it was found to be approximately 17 ns. The minimum streamer conductance (g_{ms}) and the minimum leader conductance (g_{mL}) was taken to be 0.25 mS/m and 0.0167 S/m respectively by assuming a uniform current of 100 A [20] flowing through the leader and streamer section before the attachment and dividing it by their corresponding gradient. The maximum value of the channel conductance (g_{ML}) was set to 2 S/m which is in the lower range of that measured for the power frequency arcs in [29].

In the numerical simulation, lower limit on conductance is essential to ensure build up of current in all the cases considered and also, to minimize the initial delay. As only stroke to ground is considered, significant upward leader activities are not envisaged; however, upward leader is expected to exist during bridging. At the same time, for the prevailing situation, the length and conductivity profile of the leader portion of upward discharge is not accurately known. In view of this an average representation is sought.

(ii) Bridging Section

The bridging zone is spanned by streamers from both ahead of the main descending leader and the upward leader. Similar to that in long air gaps, the streamer section is weakly ionized and possesses low conductivity. The prevailing strong electric field initiates processes which, cumulatively enhances the conductance. It is this transient enhancement of the conductance which initiates the return stroke current. Therefore, its modelling is of prime importance. The time spanned by the transition from streamer to leader is rather small, and hence, we assume that the energy loss compared to the input during this period is insignificant and therefore, can be neglected. In view of this, suitable formulation for the dynamic variation of spark / arc resistance in which accumulation of energy is explicitly seen for arbitrary time varying current is sought. In [30], a comparison of eight different types of arc-resistance equation has been made. Amongst them, the formulations given by Barannik, Popovic, Rompe & Weizel, Vlastos and Toepler have explicitly indicated the accumulation process. However, Vlastos's formulation requires radius of the conducting core explicitly which is, not readily known while Rompe & Weizel's formulation seem to neglect the variation in radius. On the other hand, the remaining three formulations evaluate the resistance without demanding any other dynamic quantities. Before experimenting with these formulations, for the sake of completeness a very brief description [30] of them is provided.

Following the formulation of Barannik, the arc resistance per unit length is given by,

$$R(t) = \frac{C_b \, \rho_o^{1/3}}{\int_0^t i^{2/3} \, dt} \tag{9}$$

where, C_b is a constant, ρ_o is the initial gas density (kg/m^3). Although the above equation takes into account the arc radius effect, it assumes constant arc-channel conductivity.

According to Popovic, arc resistance follows the following equation,

$$R(t) = \frac{C_p}{\left[\int_0^t i^2 \ dt\right]^{np}} \qquad (10)$$

where, np = 0.33, Cp is a constant. Similarly, Toepler's formulation provides the following equation for the arc-resistance, where C_t is a constant.

$$R(t) = \frac{C_t}{\int_0^t i \ dt} \qquad (11)$$

While Barannik's formulation is analytically derived, the remaining two formulations are empirical in nature. By referring to a particular experimental data, all the three formulations have shown to exhibit good matching for the initial time period involving kiloamperes of current.

Numerous simulations have been carried out to assess the suitability of the above three arc equations for the problem in hand [31]. It is worth noting here that the goal is not to critically evaluate the ability of these arc equations in predicting the dynamic conductance of the channel core in natural lightning or to evaluate their ability to describe the variation in conductance for a specified current, but to scrutinize them for their ability to work in tandem with the present model in describing the evolution of return stroke current.

The present work does not explicitly refer to the gas dynamics and ionization phenomenon occurring in the channel; but attempts to macroscopically represent the channel arc dynamics and hence, it invariably contains model parameters. Even though, the range of these parameters is reasonably known, in the simplified representation, it becomes necessary to suitable tune them so as to not only ensure the building up of current but also to get a better resemblance with the field data. In order to check with the different arc formulations, following parameters are varied: streamer gradient and the steady state arc gradient are taken as the main variables while, the minimum value of streamer conductance is considered to be a secondary variable.

Inspite of using an arc resistance formulation which cumulatively builds the conductance without any appreciable loss, it is possible to have no return stroke current evolution when the rate of build up is rather slow. This occurs when the charge, and hence, the electric gradient in the bridging zone depletes considerably before an appreciable build up of conductance. Simulations carried out for verifying the suitability of these models gave the following results [31].

Barannik's formulation yielded a relatively slow building up of conductance. Therefore, the proposed return stroke model imbibing Barannik's formulation was subjected to a critical test when minimum gradient of 4 kV/cm was specified for the bridging zone. As a consequence of low gradient, as well as, lower rate of increase in conductance, simulation results indicated a considerable delay in the evolution of current. However, after this initial delay there was a sudden build up of current, inspite of the fact that this formulation yields slower build up of conductance. Probably, the simultaneous build up of conductivity in the leader section adjacent to the bridging zone is responsible for the above. The basic drawback with this formulation is the fast building up of current and considerable reduction in the current amplitude in the initial portion of the channel. Also, things did not improve even with higher gradient of 12 kV/cm specified for the streamer section. In view of the same, this formulation was not chosen for the present work.

Simulation with Popovic's arc-resistance formulation showed that amongst the three formulations, this gave the fastest rise in conductance. Simulation with the default value of minimum value of streamer conductance gave evolution of current however, current in the bridging zone possesses extremely fast rise time and hence, this formulation was also not selected.

Simulation with Toepler's formulation was taken up next. It was observed that compared to the above two formulations, this formulation gave much smoother current evolution at the bridging zone, as well as, relatively better rise time and lower decay rate for the current. In view of the same, this formulation is selected for describing the dynamics of channel conductance at the bridging regime. Further details of this part of the study can be found in [31].

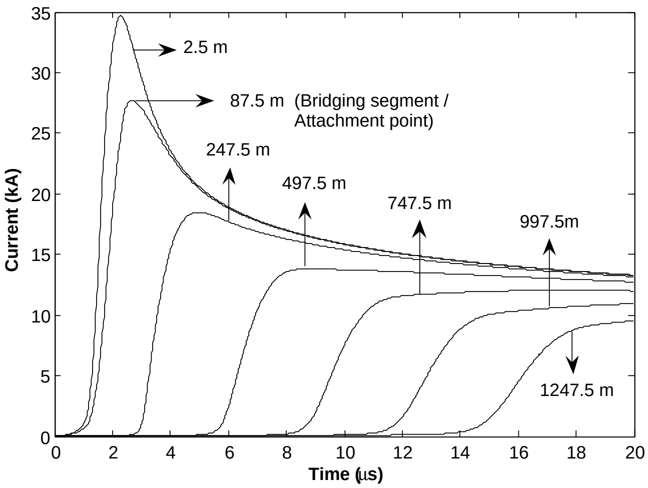
It is worth mentioning here that for a successful simulation, the steady state arc gradient in equation (6) needs to be set to a fixed value, rather than that described by equation (8). Subsequently, additional simulations with Toepler's formulation were carried out to check whether it is overly sensitive to values of the parameters like arc time constant, steady state arc gradient, etc. The study results did not show any such over sensitivity which demonstrated the suitability of the formulation for the present work.

4. SAMPLE RESULTS

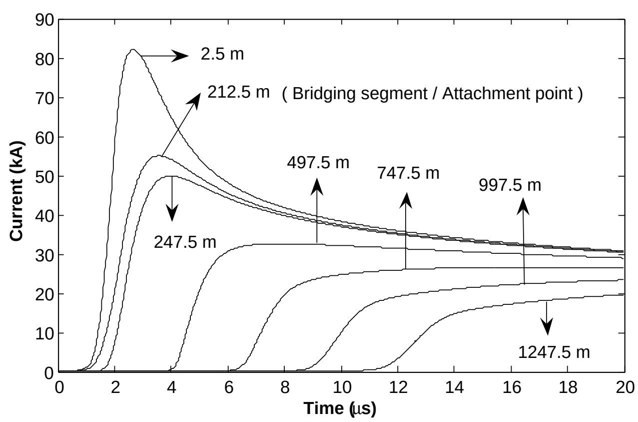
It may be worth reiterating here that for many electrical engineering applications, the region spanning up to and around the current peak forms the most crucial part of the return stroke current waveform. In view of this, main emphasis will be given only to this portion of the waveform. The simulation result for a 50 MV cloud base potential and a channel length of 2.5 km is provided in Fig. (**2a**) and that in Fig. (**2b**) for 100 MV for the same channel length using the parameter values given in previous section. Simulation was also carried out to ascertain the influence of channel core radius on the simulation results. A 20 times increase in radius has resulted in an increase of current by about only 34%. Based on the same, it is concluded that the influence of channel core radius is rather weak.

It is observed that the return stroke current has a characteristic concave shape in the rising portion with a rapid rise to the peak and a relatively slow decay [32,33]. Also, there is an increase in the rise time and a decrease in the current magnitude with height [34,35]. However, the overall current waveshape seems to deviate with altitude above 1 km, which needs further study.

The current wave shape at the channel base, spatial variation of current amplitude, rise time and velocity are used for gross validation. It is evident from the simulation results that the peak amplitude of the current and velocity decreases with height, while the rise time (10-90 % of peak value) increases with height. For a quantitative description, the peak amplitude and rise time of current extracted from results corresponding to a cloud base potential of 50 MV and channel length of 2.5 km, are presented in Fig. (**3**) and that of velocity is provided in Table **1**. The trend is in agreement with the field observation.



(a) 50 MV

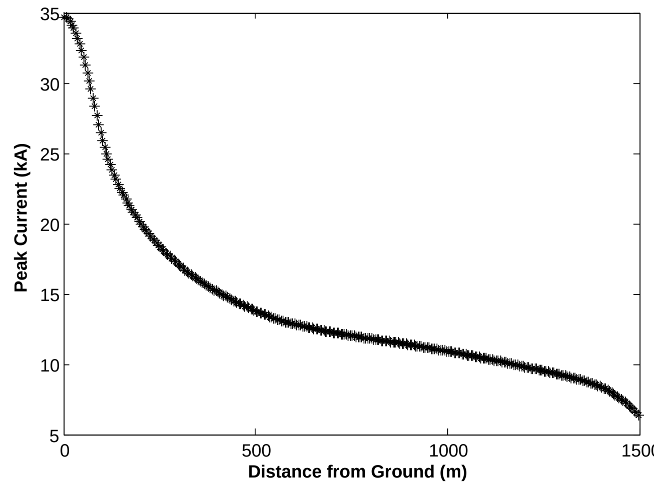


(b) 100 MV

Fig. (2). Simulation results for a 2.5 km channel length.

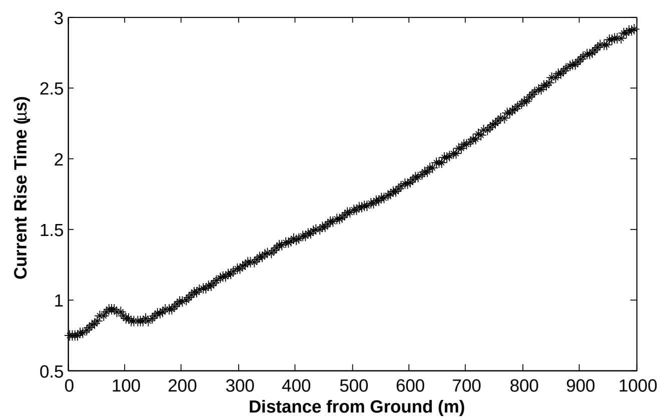
A comparison of the linear charge density at the tip of the channel and the resulting return stroke current peak would provide, in our opinion, stronger validation. It is to be noted here that for a given channel length, the charge density is related to the cloud base potential. For a fixed channel length of 2.5 km, three cloud base potentials 25 MV, 50 MV and 100 MV are considered. The corresponding tip charge density and the peak base current amplitude as deduced from the simulation are compared in Table **2** with the relationship given in [3]. It is evident that the comparison is favorable.

(**a**)



(a) Peak current magnitude versus height along the channel

(**b**)



(b) Current rise time (10 − 90% peak value) versus height along the channel

Fig. (3). (**a**) Current magnitude versus height along the channel (**b**) Current rise time versus height along the channel (All the figures present results corresponding to the case of Fig. (**1a**).

Table 1. **Velocity of Current Wave Extracted from Results Corresponding to a Cloud Base Potential of 50 MV and Channel Length of 2.5 km (Values are Represented in Terms of *c* = Speed of Light in Vacuum)**

Distance from Ground (m)	247.5	497.5	747.5	997.5	1247.5
Velocity (m/s)	0.315c	0.311c	0.303c	0.298c	0.295c

The numerical stability of the model was checked by running over longer time duration, for different cloud base

Table 2. Comparison of the Linear Charge Density at the Tip of the Channel for a Given Current (Results are for Channel Length of 2.5 km)

Sr. No.	Peak Current at the Channel Base (kA)	Tip Charge Density from Simulation (mC/m)	Tip Charge Density According to [3] $\rho = 5.767 \times 10^{-5} I_p^{0.81}$ (mC/m)
1	5.5	0.15	0.229
2	34.7	0.65	1.02
3	82	1.8	2.047

potentials and cloud base height from the ground; the results of which are displayed in Fig. (**4**). The current waveform for a 50 MV cloud base potential and a channel length of 4.5 km is provided in Fig. (**4a**) and that in Fig. (**4b**) for 100 MV cloud base potential. This exercise has clearly demonstrated the numerical stability of the model, as well as, the consistency of the model results. A difference in the peak value of the computed currents presented in Figs. (**2**) and (**4**) could be seen due to the associated change in charge density on the channel.

For further validation of the model, vertical electric fields on the ground at radial distances of 50 m, 1 km and 2 km are also evaluated. Larger distances were not considered due to the limitations in the modeling of the cloud end termination. For the case dealt in Fig. (**4a**), the computed fields for the above distance are presented in Fig. (**5a**, **b**, **c**) respectively. The characteristics of the computed field can be seen to have good resemblance with that presented in the literature, which adds further support to the model developed [4,36-38].

5. CONCLUSION

For a more realistic return stroke modelling it is necessary to consider the following essential features: the associated dynamic electromagnetic field including that due to the initial charge on the channel (prime mover for the return stroke), the dynamics at the attachment point (initiator of the return stroke current) and the nonlinearly varying channel conductance (return stroke current evolution). Such a modelling has been attempted in this work with an aim to simulate return stroke current evolution.

A time domain electric field integral equation for thin wire structure is employed with sub-sectional collocation form for the numerical field solution of the dynamic field and the initial charge distribution is deduced by static field solution along with specified corona sheath gradient. The transient enhancement of channel conductance is the key factor in return stroke current evolution. This paper has laid more emphasis on modelling of the same.

The increase in conductance of the bridging zone, which is an important event, is macroscopically represented by suitable arc/spark resistance formulations. For this, formulation by Toepler, Barannik and Popovic were considered with streamer gradient and the steady state arc gradient taken as the main variables of the parametric study. For the modelling

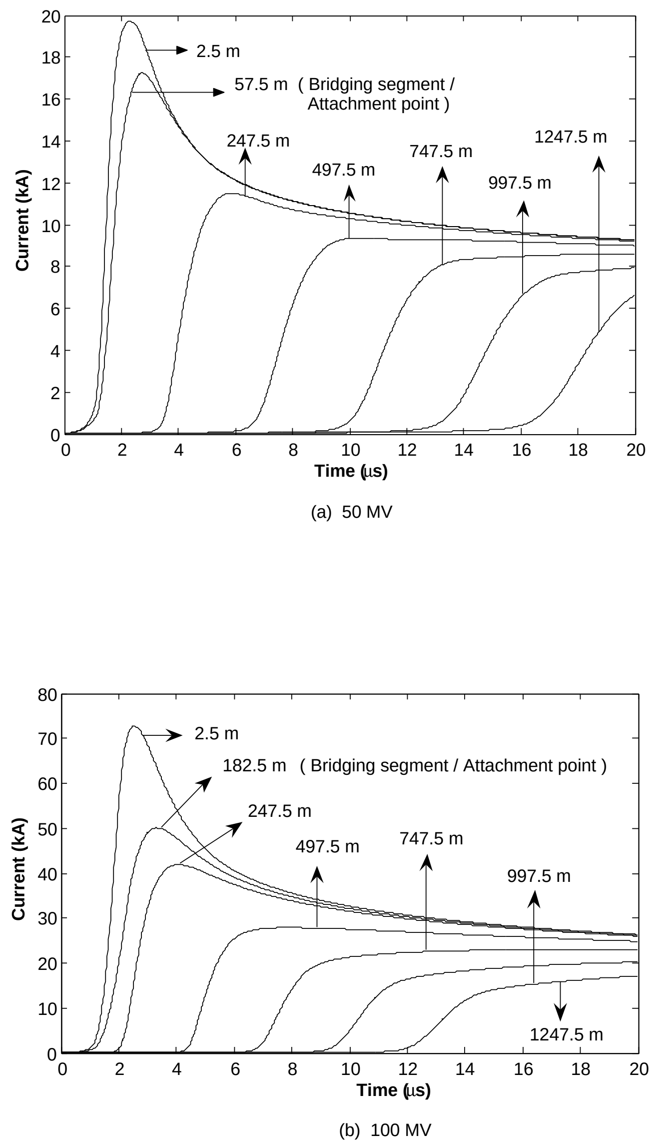
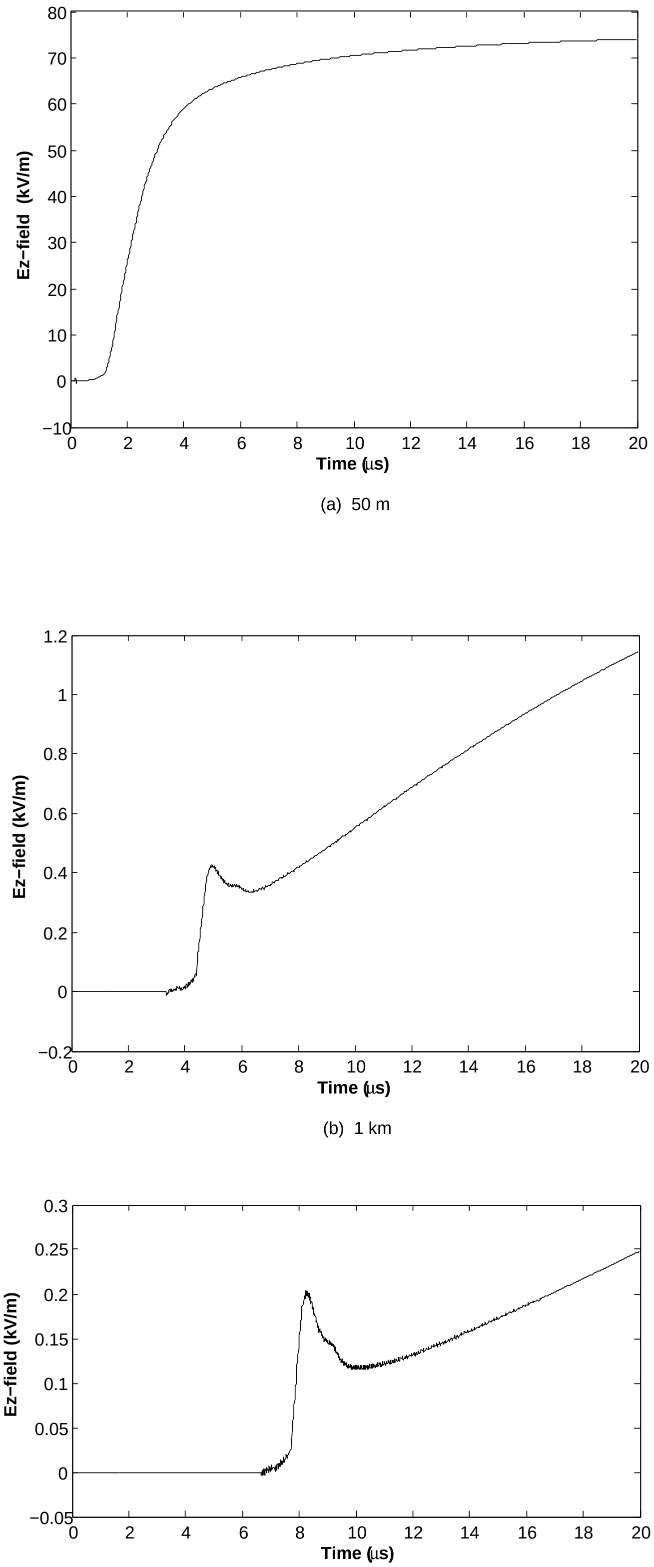


Fig. (4). Simulation results for a 4.5 km channel length.

of arc conductance, first order arc equation was chosen, however, with steady state arc gradient made independent of current. Simulation results indicate that Toepler's formulation seems to be best suited for the present modelling. The model predicted current waveshape, front time, spatial variation of the magnitude and velocity decay rate within 1 km seems to be in reasonably good agreement with the field data. Based on the same, it is concluded that a self-consistent model for the first return stroke of a lightning flash seems to have been developed in this work.



(a) 50 m

(b) 1 km

(c) 2 km

Fig. (5). Vertical electric field plots for the simulated channel current presented in Fig. (**4a**).

Work is presently under progress for accounting for the electric field due to cloud and for realising slower front time for the return stroke current. After completing the above tasks, studies on stroke to vertically extended grounded strike objects will be taken up.

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