

Application of Cavity Expansion Theory in Predicting Centrifuge Cone Penetration Resistance

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Abstract: In this paper, a simple model for the prediction of centrifuge cone penetration tip resistance in sand is presented. The proposed method, which relates the tip resistance to the pressure to expand a spherical cavity, refines and simplifies Greeuw's original equations. The method assumed the failure mode of a spherical cavity expansion pressure given as a function of shear stiffness, angle of friction and compressibility of the soil. The proposed method provides a better prediction of tip resistance by taking into account soil compressibility and the decrease of angle of friction with increasing mean normal stress. The proposed approach is verified *via* the comparison of the predicted results with the results from a series of centrifuge cone penetration tests.

INTRODUCTION

Inadequacy of conventional laboratory testing and inevitable disturbance of soil samples have led to the development and growth of in-situ testing devices. One of these devices, the cone penetrometer, has undergone tremendous development in recent years and has been transformed from a somewhat primitive tool into a highly sophisticated instrument. Penetrometers can be used to study the stratification of soil because of their capability of providing a continuous measurement over the whole depth of penetration. Simplicity of the test procedure, increased reliability of the test data and increased pushing capacity of the equipment has made it a very popular device, especially in offshore site investigation work.

A rigorous theoretical analysis of cone penetration is extremely difficult because of the large strains and material non-linearity associated to the problem [1]. There are many approximate theoretical correlations proposed since the early 1960. Some of them have been widely used in practice, for example, [2-4]. As summarized by Yu and Mitchell [1], the analysis of cone penetration in undrained clay and fully drained sands can be divided into five categories, i.e. bearing capacity theory, cavity expansion theory, steady state deformation, incremental finite-element analysis, and calibration chamber testing.

Based on the observation that the pressure required to produce a deep hole in an elastic-plastic medium is proportional to the required pressure of expanding a cavity of the same volume under the same conditions, the analogy between the cavity expansion and cone penetration was presented in [5]. The use of cavity expansion theory to predict

cone penetration resistance involves the determination of the cavity expansion limit pressure, and then relating the limit pressure to the cone penetration resistance. The method has been used by many researchers [6-17].

The spherical cavity expansion theory has seldom been adopted by centrifuge modelers in analyzing their test results, except Lee [9]. One of the reasons is the highly non-linearity nature of the test that is associated with the centrifuge environment. It is difficult to determine the correct stress state and soil properties for analysis of data from centrifugal tests. The cone penetration resistance profile obtained from a calibration chamber test normally shows a constant tip resistance value with penetration depth. Thus, it is more certain to obtain the stress state and soil properties for the analysis of calibration chamber data than the centrifuge data.

In this paper, based on the cavity expansion theory, a closed form analytical solution for the analysis of cone penetration resistance in cohesionless material is presented. First, a brief of the centrifuge cone penetration is given. Then, the derivation of the close form solution is presented. The predicted results were then compared with the results obtained from four cone penetration tests conducted in the centrifuge.

CENTRIFUGE CONE PENETRATION TESTS

Centrifuges have been widely applied to model geotechnical problems because the behavior of a foundation can be observed in a soil specimen of known parameters without the expense and delay of doing full-scale tests. Four cone penetration tests had been conducted in geotechnical centrifuge using the Fontainebleau sand from France. The diameter in the particle-size distribution curve corresponding to 50% finer of the sand (D_{50}) was 0.181 mm and the coefficient of uniformity of the sand (d_{60}/d_{10}) was 1.69. The average specific gravity of the sand (G_s) was 2.644. The average values of maximum (e_{max}) and minimum (e_{min}) void ratios were 0.55 and 0.92 respectively.

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Four sets of data obtained from centrifuge cone penetration tests have been chosen for analysis in this study. The test specimens were prepared by hand pluviating Fontainebleau sand from a single-hole hopper into a cylindrical tub of 850 mm in diameter. All the test specimens have an average height of 350 mm. Only one cone penetration test was conducted at center of each specimen under a nominal acceleration of 70g. All the four cone penetration tests were performed using a 10 mm diameter cone penetrometer. Detailed description of centrifuge cone penetration tests can be found in Bolton *et al.* [18]. The test results for all the four cone penetration tests are presented in Fig. (1). For test T1: dry density = 1663 kg/m³, void ratio $e = 0.59$, and relative density $I_D = 89\%$; for test T2: dry density = 1631 kg/m³, void ratio $e = 0.62$, and relative density $I_D = 81\%$; for test T3: dry density = 1552 kg/m³, void ratio $e = 0.70$, and relative density $I_D = 58\%$; and for test T4: dry density = 1538 kg/m³, void ratio $e = 0.72$, and relative density $I_D = 54\%$. The calculation of the void ratio was based on the overall weight of the specimen and its volume measured after the specimen was mounted on to the centrifuge arm. All the tests, except test in specimen with relative density $I_D = 54\%$, exhibit a uniform tip resistance profile, which means these specimens have a consistent relative density throughout the specimen depth. As for test T4 ($I_D = 54\%$), the non-uniform profile shows that the specimen has an inconsistent relative density throughout its depth.

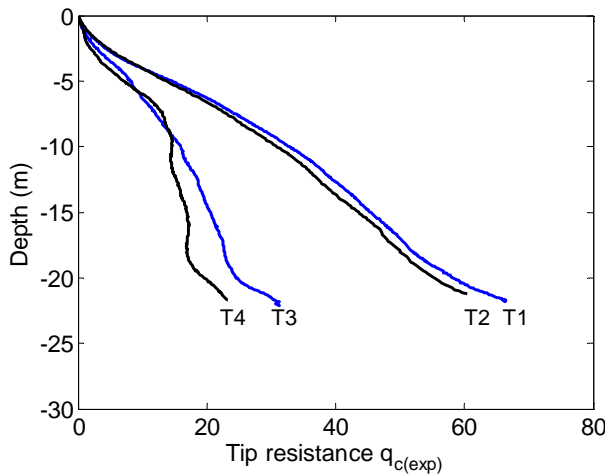


Fig. (1). Result of the centrifuge cone penetration tip resistance.

In this study, the tip resistance results back estimated from the cavity expansion theory are compared with those obtained from the centrifuge tests. Since it is impossible to measure the *in situ* mean effective stress in the test specimens, experimental tip resistance $q_{c(exp)}$ and vertical stress σ_v are used here to estimate the angle of friction ϕ [19] for used in the following analysis:

$$\phi = 3 \left(I_D \left(10 - \ln \sqrt{q_{c(exp)} \sigma_v} \right) - 1 \right) + \phi_{crit} \quad (1)$$

$$\psi = \phi - \phi_{crit} \quad (2)$$

where ψ is the angle of dilation and ϕ_{crit} is the critical angle of shearing. ϕ_{crit} of the Fontainebleau sand used is taken to be

32°. Equation (1) relates the relative density, tip resistance and overburden stress to the angle of friction of a soil. As seen from Fig. (2), the calculated angle of friction ϕ exhibits non-linearity with the mean effective stress. Another parameter, the secant shear modulus at 50% ultimate shear stress G_{50} , had been determined from a series of laboratory triaxial test as:

$$G_{50} = \frac{237.5 \sqrt{\sigma_o}}{e^4 (1 + \nu)} \quad (3)$$

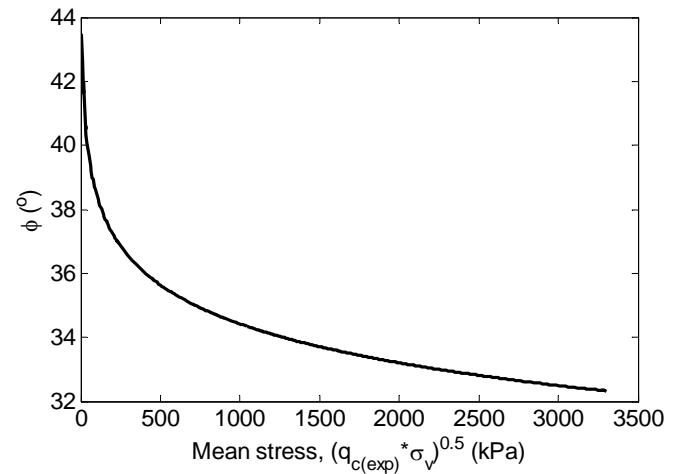


Fig. (2). Non-linear variation of angle of friction with mean effective stress for specimen with $I_D = 58\%$.

THEORY OF CAVITY EXPANSION

Carter *et al.* [7] assumed that the cavity is expanded in an infinite medium that is initially in a hydrostatic stress state, i.e. $\sigma_1 = \sigma_2 = \sigma_3 = p_o$, where σ_1 , σ_2 , and σ_3 are the principal stress components. They pointed out that the analysis is applicable to deep cavities since any variation in stress as body forces can be neglected. In addition, the analysis allows a one-dimensional description of the problem because the displacements in the radial direction are more important than others. This assumption simplifies the analysis.

Herein, following [7], consider a spherical cavity of initial radius a_o that can be expanded by a uniformly distributed internal pressure p_o . At a particular time t , the cavity enlarged to radius a (Fig. 3), while the internal pressure increased to p . In the absence of body forces, the total stress at this point must be in equilibrium and satisfies

$$\frac{d\sigma_r}{dr} + \frac{2(\sigma_r - \sigma_\theta)}{r} = 0 \quad (4)$$

where σ_r and σ_θ are the major and minor principal stresses during cavity expansion, and r is the radial distance to the center of the cavity. The boundary conditions are $\sigma_r = p$ at $r = a$ and $\sigma_r = p_o$ at $r = \infty$. For $r > R$ the soil mass remains in a state of elastic equilibrium.

In this study, soil was modeled as an isotropic, elastic-perfectly plastic material where the soil initially behaves elastically and obeys Hooke's law until the onset of yielding which is defined by

$$\frac{\sigma_r}{\sigma_\theta} = \frac{1 + \sin \phi}{1 - \sin \phi} \quad (5)$$

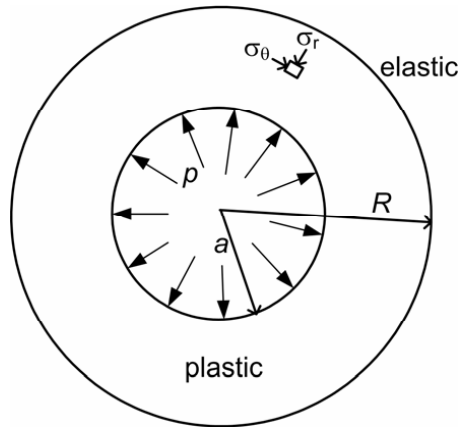


Fig. (3). Cavity expansion problem (after [7]).

Determination of Limit Pressure

To estimate the tip resistance, the limiting pressure caused by the expansion of a spherical cavity must be calculated. For an infinite deformation Carter *et al.* [7] found that the cavity pressure approaches a limiting value. They assumed that a steady state deformation mode is approached at very large deformations. They employed the non associated Mohr-Coulomb yielding criterion into their formulation in order to take into account the dilatancy behaviour of the sand in the analysis. Detailed derivation of their formulation can be found in Carter *et al.* [7]. For convenience of referencing, their relationship between the limiting pressure P_L and the mean stress σ_o is repeated here:

$$\frac{2G}{\sigma_o} = \frac{N-1}{N+2} \left[T \left(\frac{P_L}{\sigma_R} \right)^\gamma - Z \left(\frac{P_L}{\sigma_R} \right) \right] \quad (6)$$

where G is the shear modulus of the soil and

$$M = \frac{1 + \sin \psi}{1 - \sin \psi}, N = \frac{1 + \sin \phi}{1 - \sin \phi}, Z = \frac{6X}{\alpha + \beta} \quad (7)$$

$$X = \frac{2(1 - \nu) - 2\nu(M + N) + MN}{(1 + \nu)MN} \quad (8)$$

$$T = 3 \left(1 + \frac{2X}{\alpha + \beta} \right), \gamma = \frac{1 + \alpha}{1 - \beta} \quad (9)$$

$$\sigma_R = \frac{3N}{N + 2} \sigma_o, \alpha = \frac{2}{M}, \beta = \frac{2 - N}{N} \quad (10)$$

The above expressions can be used to determine the value of P_L/σ_o for specific values of G/σ_o , ν , ϕ and ψ . Using an iterative procedure, Greeuw *et al.* [8] adopted Eqs. (6)–(10) to obtain the limit pressure for the analysis of their calibration chamber cone penetration data. This study overcomes the iterative procedure by further simplifying and rearranging Eq. (6) into the following analytical solution

$$\frac{P_L}{\sigma_o} = \left[\left(\frac{N + 2}{3N} \right)^{1-\gamma} \left(1 - \frac{3}{T} \right) \frac{G}{\sigma_o} \frac{3 - \sin \phi}{T \sin \phi} \right]^{(1/\gamma)} \quad (11)$$

Determination of Cone Tip Resistance

Vesic [6] applied the spherical expansion model to analyze the tip resistance of a flat-ended pile. From the observations on models and full-size piles, he concluded that the advancement of the pile in dense soil is made possible by lateral expansion of soil as well as by any possible dilation/compression. The stress induced at the base of the pile can be assumed to be equal to the limiting pressure needed to expand a spherical cavity in an infinite soil mass. The formula for the resistance at the tip of a pile q_o has been given by Vesic [6] as:

$$q_o = N_{pile} \sigma_o \quad (12)$$

where

$$N_{pile} = \frac{3}{3 - \sin \phi} \exp \left[\left(\frac{\pi}{2} - \phi \right) \tan \phi \right] \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) I_{rr}^{\frac{4 \sin \phi}{3(1 + \sin \phi)}} \quad (13)$$

where I_{rr} is a rigidity index, and σ_o is the mean normal ground stress, which is correlated to the effective vertical stress σ_v via

$$\sigma_o = \left[\frac{1 + 2(1 - \sin \phi)}{3} \right] \sigma_v \quad (14)$$

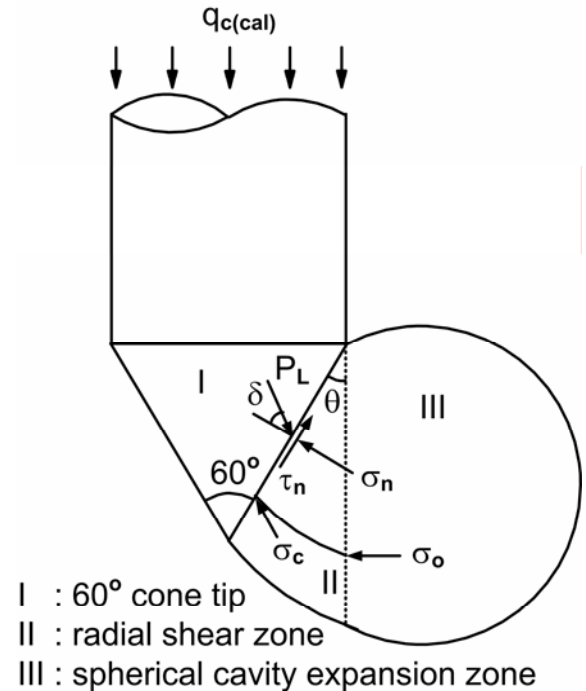


Fig. (4). Failure pattern of a cone in sand, where σ_n and τ_n are the normal and shear stresses acting on the surface of the 60° cone, θ is the angle of the radial shear zone, σ_c is the mean stress acting on the 60° cone in the plastic zone, and δ is the surface roughness of the cone (after [9]).

Greeuw *et al.* [8] used Eqs. (12) and (13) without the rigidity index term I_{rr} , to predict their calibration chamber cone penetration resistance. However, the calculation was, perhaps, incorrect because N_{pile} was derived based on the geometry of a flat-ended pile. In order to incorporate the geometry and surface roughness of the cone (Fig. (4)),

Vesic's [6] formula for the pile-tip resistance – Eqs. (12) and (13) – must firstly be modified. The modification involved the assumption that the soil adjacent to the cone is highly sheared and full strength is mobilized [9]. For a smooth cone, the mean stress σ_c acting on a 60° cone in the plastic zone is

$$\sigma_c = \sigma_o \exp(2\theta \tan \phi) \tag{15}$$

where σ_o is the mean stress at the transition surface and $\theta = 60^\circ$. Combining the Haar and von Karman's hypothesis:

$$\sigma_o = \frac{2\sigma_\theta + \sigma_r}{3} \tag{16}$$

and the Mohr-Coulomb failure criterion, Lee [9] obtained σ_o as

$$\sigma_o = \frac{3 - \sin \phi}{3(1 + \sin \phi)} \sigma_r \tag{17}$$

As a result of putting σ_r approaches the limit pressure P_L at the plastic state Lee [9] obtained the theoretical tip resistance $q_{c(cal)}$ as

$$q_{c(cal)} = N_{cpt} P_L \tag{18}$$

where

$$N_{cpt} = \frac{\cos(\delta - 60^\circ)}{\cos 60^\circ} \frac{\sin(\Delta + \delta)}{\sin \Delta} \frac{3 - \sin \phi}{3(1 + \sin \phi)} e^{2\theta \tan \phi} \tag{19}$$

and $\Delta = \sin^{-1} \left(\frac{\sin \delta}{\sin \phi} \right)$. The surface roughness of the cone δ was taken to be a function of the angle of friction, which was $\phi/3$, in this study; hence, the cosine term in the right hand side of the equation takes into consideration the geometry and the surface roughness of the cone.

RESULTS AND DISCUSSION

Arm with Eqs. (7)-(11) and Eqs. (18)-(19), it is possible to predict each of the profile of the centrifuge cone penetration tip resistance. The four predicted profiles together with the corresponding centrifuge cone penetration profiles are shown separately in Fig. (5a)-(5d) for ease of comparisons. The predicted profiles for specimens with relative density I_D of 89% (Fig. 5a) and 81% (Fig. 5b) seem to underestimate

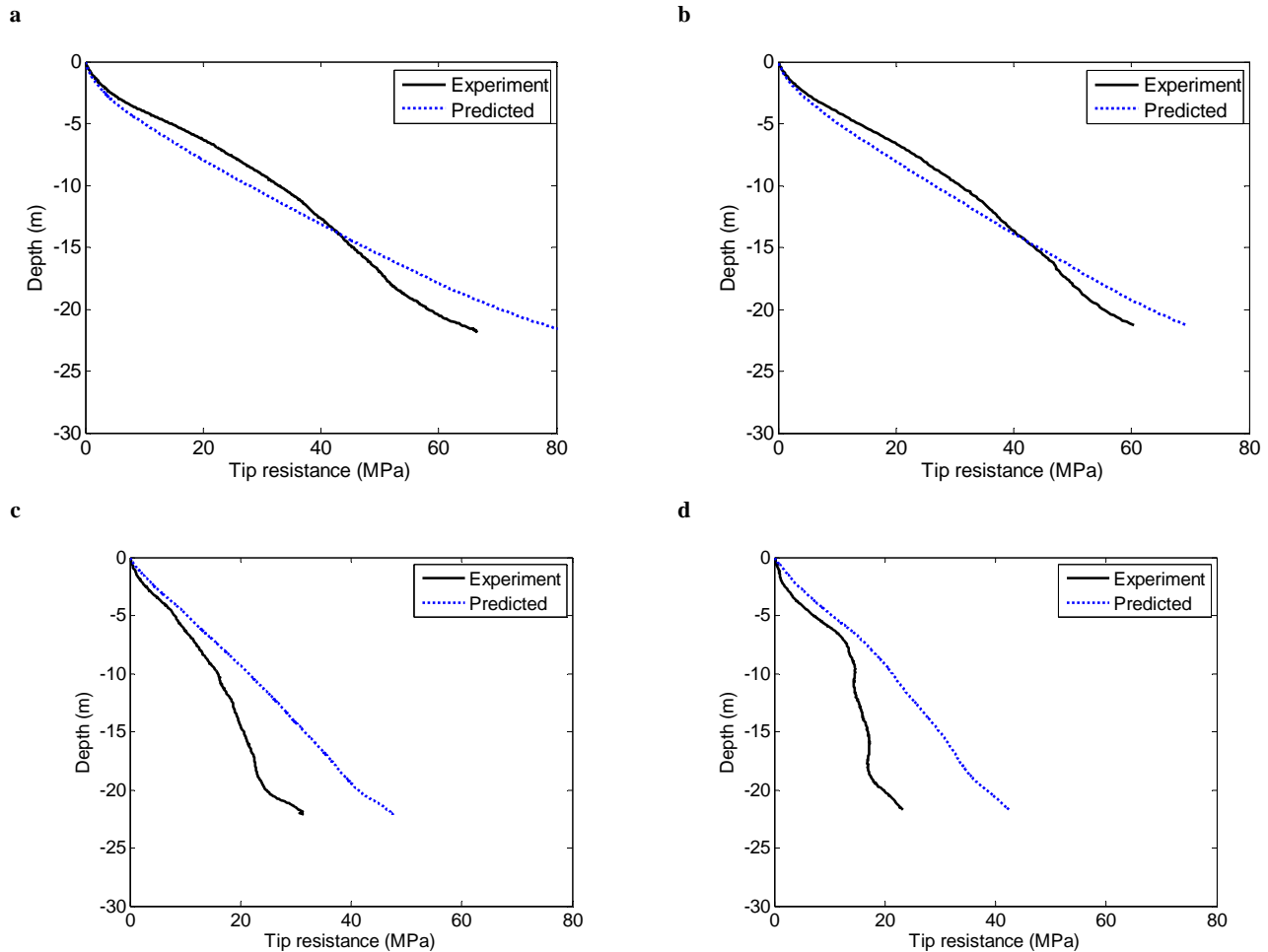


Fig. (5). (a). Comparison of predicted and measured cone resistance profiles for specimen with $I_D=89\%$. (b). Comparison of predicted and measured cone resistance profiles for specimen with $I_D=81\%$. (c). Comparison of predicted and measured cone resistance profiles for specimen with $I_D=58\%$. (d). Comparison of predicted and measured cone resistance profiles for specimen with $I_D=54\%$.

the centrifuge results for the first 14 m of penetration. Thereafter, the predicted profiles begin to overestimate the centrifuge results. For specimens with relative density I_D of 89% and 81%, the predicted result was, respectively, roughly within $\pm 25\%$ and $\pm 15\%$ of the centrifuge result.

For specimens with relative density I_D of 58% (Fig. 5c) and 54% (Fig. 5d), the predicted results overestimated the centrifuge results as soon as the penetration started. For specimens with relative density I_D of 58% and 54% the predicted result overestimated the centrifuge result by a maximum of 68% and 100%, respectively. The reason for the underestimation of the centrifuge results for the first 14 m of penetration in the very dense specimens is not so clear but one of the reasons for the overestimation in the medium dense specimens could be the overlooking of the effect of compressibility of the medium dense soil in Eq. (18). The significance of compressibility on the bearing capacity of shallow foundations on Fontainebleau sand had been pointed out by [20] and this effect will be incorporated and examined in the following section.

Effect of Compressibility

To account for the effect of compressibility in cone penetration resistance, the modified bearing capacity formula proposed by Vesic [21] is adopted here

$$q_{c(comp)} = q_{c(cal)} F_{comp} \tag{20}$$

where $q_{c(comp)}$ is the tip resistance after considering the effect of soil compressibility and F_{comp} is the soil compressibility factor, which is given by [21]

$$F_{comp} = \exp \left\{ \left(\frac{(3.07 \sin \phi)(\log 2I_r)}{1 + \sin \phi} \right) - 3.8 \tan \phi \right\} \leq 1.0 \tag{21}$$

where I_r is the rigidity index, which is given by

$$I_r = \frac{G_{50}}{\sigma_v \tan \phi} \tag{22}$$

The rigidity index varies with the stress level and the characteristic of loading. A higher value of I_r implies a relatively incompressible material while a lower value implies a relatively compressible material.

Figs. (6a)-(6d) show the effect of the compressibility on the calculated tip resistance profiles. Fig. (6a) shows that for a very dense specimen ($I_D = 89\%$), there is virtually no compressibility effect ($F_{comp} = 1$) because the soil is too dense to allow for any compression to occur. Fig. (6b) shows that for a slightly looser (in relative to $I_D = 89\%$) specimen with $I_D = 81\%$, the effect of compressibility begin to appear, albeit

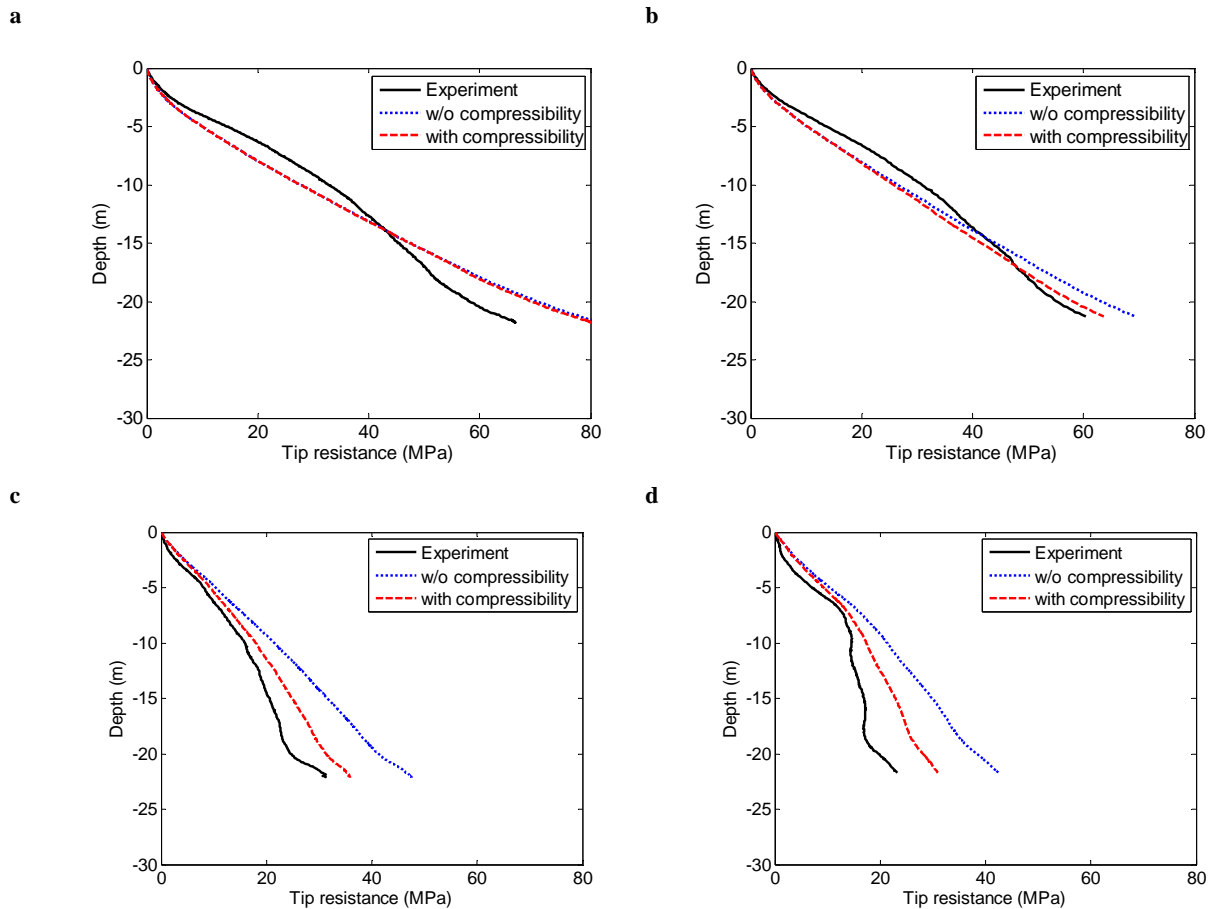


Fig. (6). (a). Effects of compressibility on the predicted cone resistance profile for specimen with $I_D=89\%$. (b). Effects of compressibility on the predicted cone resistance profile for specimen with $I_D=81\%$. (c). Effects of compressibility on the predicted cone resistance profile for specimen with $I_D=58\%$. (d). Effects of compressibility on the predicted cone resistance profile for specimen with $I_D=54\%$.

very little ($0.92 < F_{comp} < 0.96$) for penetration depth between 17 to 22m).

The contribution of the compressibility effect is more significant in the medium dense specimen than the very dense specimen. This effect can be clearly seen in Figs. (6c) and (d) for both the medium dense specimens ($I_D = 58\%$ and 54%). F_{comp} was found to be between 1.0 and 0.76 for penetration depth between 0 to 22 m for specimen with $I_D = 58\%$, and it was found to be between 1.0 and 0.73 for penetration depth between 0 to 22 m for specimen with $I_D = 54\%$. After considering the effect of compressibility the overestimation of the cone resistance is now 27% and 55% for test in specimen with $I_D = 58\%$ and 54% , respectively.

CONCLUSIONS

An analytical solution, based on the spherical cavity expansion model used by Carter *et al.* [7], has been proposed and used to predict the profile of centrifuge cone penetration resistance. The model previously proposed by Greeuw *et al.* [8] not only required an iterative procedure to obtain the limit pressure but also adopted a pile factor, instead of a cone factor, to correlate this limit pressure to the cone resistance. This study improved this shortcoming by adopting a cone factor that was derived based on the geometry of the cone. In its original form, the current analytical formulation overestimated the tip resistance of medium specimens by about 68% and 100%. After incorporating the effect of compressibility into the formulation, the overestimation of the cone resistance reduced to 27% and 55% for test specimen with relatively density I_D of 58% and 54%, respectively. The prediction of cone resistance for very dense specimen remains basically the same with or without considering the compressibility effect since the very dense specimen is relatively incompressible.

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