

# Bed Load Erosion Patterns and Their Effect on the Structural Strength of Rigid Pipes Made of Homogeneous Materials

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**Abstract:** This manuscript concerns the decrease of the structural strength of rigid pipes made of homogeneous materials due to bed load erosion, caused by different concentrations of hard solid particles transported by the sanitary sewage, or surface runoff. Such a phenomenon has been observed in combined sanitary systems (CSS), as well as in force mains delivering domestic sewage due to the penetration of hard solid particles into the sanitary sewerage system. Field observations have indicated that the low concentrations of solid particles transported with the sewage lead to the formation of a groove at the pipe invert. Experiments carried out in this study have shown that the width of the groove mainly depends on the concentration of the solid particles. Numerical simulations have indicated that bed load erosion resulting from very low concentrations of solid small particles may be more detrimental to the structural strength of the pipe than higher concentration of such particles.

**Keywords:** Bed load erosion; concrete pipe failure; bed load erosion pattern; pipe collapse; pipe rupture; pipe crushing; pipe structural strength.

## INTRODUCTION

This study has originated from a case history of asbestos-cement (AC) force main collapse occurring in Tel-Aviv area, Israel [1]. At the invert of the collapsed pipe a 20 mm deep and 40 mm wide groove has been found. This groove has been the result of around 30 years of bed load erosion caused by low concentrations of hard solid particles (sand and gravels) transported with the domestic wastewater delivered through that force main.

In comparison to the broad range of themes connected with sediment transport in water sheds, open channels, canals and pipes, the subject of pipeline abrasion by sediments has been attracted very moderate attention. However, this topic is very much relevant to transport of various types of slurry through pipes [e.g., 2, 3] because the transported high concentration of solid particles of the slurry causes continuous decrease of the pipe wall thickness due to its abrasion. Also pipe selection for sewerage and drainage systems considers the pipe resistance to abrasion, because this parameter is assumed to affect costs of maintenance and there is a need for rehabilitation or replacement of such systems.

Manufacturers of pipes often support studies aiming to show the advantage of their products by carrying out tests in which the high abrasion resistance of their products is demonstrated [e.g., 3, 4].

With regard to waste water flow, The Engineering Tool Box:

([www.EngineeringToolBox.com](http://www.EngineeringToolBox.com)) provides the guidelines referring to settling and sedimentation of solids in sewage piping and pumping systems, as well as effects of erosion and abrasion *via* the linkage:

[http://www.engineeringtoolbox.com/sewage-piping-systems-d\\_568.html](http://www.engineeringtoolbox.com/sewage-piping-systems-d_568.html)

According to the recommendations of this source the flow velocity of the wastewater should exceed certain trouble free operation and avoid settling and sedimentation of solids as follows:

- For horizontal wastewater pipe systems with solids the speed should exceed 0.9 m/sec.
- For wastewater system with organic solids the speed should exceed 0.6 m/sec.

The flow velocity of wastewater must not exceed certain limits to reduce the potential for wear and tear due to the effects of erosion and abrasion as follows:

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- The speed in high-grit sewage handling systems should not exceed 3.6 m/sec.
- The flow velocity in sewage systems with low grit concentrations should not exceed 5.4 m/sec.

It should be noted that The Engineering ToolBox does not refer to patterns of erosion due to bed load or suspended load, and only provides warning against erosion of the pipe wall that originates from the high speed of the sediments moving with the water that is subject to high flow velocity. Further, The Engineering ToolBox gives the impression that the increase of the grit concentration increases the risk of abrasion; and if wastewater flow velocity is lower than 3.6 m/sec then there is no (or very limited) abrasion risk.

The Project Development Design Manual [5] of The Federal Lands Highway identifies two primary causes of early failure in drainage pipe materials:

- 1) Corrosion, and
- 2) Abrasion.

Corrosion gradually wears away at the pipe walls by chemical action, and can occur from both the soil and water sides of the pipe. Abrasion wears away at the interior pipe wall by friction from suspended or bed-load sediment. The design manual of polyethylene pipes [6] adds that abrasives – such as stones or debris – can result in mechanical wearing away of the pipe. The extent of the problem depends on the type of abrasive, frequency that the material is in the pipe, velocity of the flow, and the type of pipe material. The effect of abrasives may be seen in the pipe invert where exposure is most severe. Over time, abrasives can result in a loss of pipe strength or reduction in hydraulic quality as they gradually remove wall material. Abrasion is a precursor to accelerated corrosion. The Federal Lands Highway Project Development Design Manual [5] has designed measures of abrasion for typical flow conditions in drainage pipes (rather than a particular design flood) as follows:

- *Level 1.* Nonabrasive conditions exist in areas of no bed load and very low velocities. This is the condition assumed for the soil side of drainage pipes.
- *Level 2.* Low abrasive conditions exist in areas of minor bed loads of sand and velocities of 1.5 m/sec or less.
- *Level 3.* Moderate abrasive conditions exist in areas of moderate bed loads of sand and gravel and velocities between 1.5 m/sec and 4.5 m/sec.
- *Level 4.* Severe abrasive conditions exist in areas of heavy bed loads of sand, gravel and rock, and velocities exceeding 4.5 m/sec.

Again it should be noted that like The Engineering ToolBox, The Federal Lands Highway Project Development Design Manual [5] does not refer to patterns of erosion due to bed load or suspended load erosion. It considers that abrasion risk of the pipe wall increases with the increase of the water flow velocity and the sediment concentration. Further, The Federal Lands Highway Project Development Design Manual [5] gives the impression that abrasion risk always takes place in drainage systems and provides levels of abrasion, without specifying features typical of the different levels of abrasion.

The Department of transportation of California has supported a comprehensive study concerning an evaluation of abrasion resistance of pipe and pipe lining materials [7]. However, the study only refers to corrosion and abrasion of culverts. Further, since effects of pH of the surface runoff are often crucial in most observed cases, the effect of abrasion alone is not clear, and the involvement of all 4 different levels of abrasive conditions (in different years and seasons) in most observed cases makes the particular evaluation of the phenomenon of pipe wall abrasion (without corrosion) impossible.

The Department of Transportation of Colorado has issued corrosion/abrasion guidelines for selecting culvert pipe materials [8]). The report includes a comprehensive literature review of previous studies, handbooks and guidelines concerning durability, corrosion and abrasion of drainage pipes and culverts, which had been carried out and issued by departments of transportation of various states. Efforts in all these studies, including the study carried out in Colorado are focused on predicting and determining the service life of the pipe that is subject to corrosion and abrasion. Such an interest is based on considerations of the pipe wall corrosion and time variation of the pipe wall thickness. Further, none of the studies considers phenomena particularly typical of abrasion caused by the sediments. The report [8] shows the applicability of some computer codes to predict the service life of various types of pipes.

Testing the effect of abrasion on the wall thickness of a pipe test section and thereby evaluating the abrasion resistance of the pipe material can be carried out by several methods. One method is of measuring the loss of the wall thickness due to the effect of sediments moving along the pipe or covered trough test section subject to swinging for a certain time at a certain rate in a Darmstadt swinging apparatus [e.g., 4, 9, 10]. According to another method we measure the loss of the wall thickness of a pipe test section subject to rotation at a certain speed for a certain time [e.g., 11]. The objective of such measurements is usually to compare the abrasion resistance of different pipe materials without taking into account parameters of the sediments, like particle size, density and concentration. The abrasion resistance in studies concerning this issue is usually defined as the rate of loss of the pipe wall thickness due to the abrasive action.

The concept of abrasion is not limited to flow of water with sediments through pipes. As an example, abrasion resistance of construction and coating materials is a common topic. Therefore, measurements of specimens of the pipe material resistance to abrasion are sometimes carried out while assuming the relevance of such tests to resistance of the pipe to abrasion caused by the flow of water with sediments. Such tests are the common hard ball test and the Taber test [e.g., 12]. Taber test involves mounting a flat specimen of the pipe material to a turntable platform that rotates on a vertical axis at a fixed speed. Two Taber abrasive wheels, which are applied at a specific pressure, are lowered onto the specimen surface. Characteristic rub-wear action is produced by contact of the test specimen against the sliding rotation of the two abrading wheels. As the turntable rotates, the wheels are driven by the sample in opposite directions about a horizontal axis displaced tangentially from



The second case history of groove development at the force main invert considered by reference [1] has occurred in Vancouver, British Columbia, Canada [15, 16]. In this case the force main has been made of prestressed concrete comprising part of the combined sanitary system (CSS). In such a sewerage system a single sewer network is used for conveying surface runoff and wastewater [15]. Therefore, every rain storm leads to flow of surface runoff with hard solid particles mixed with the raw wastewater through the force mains. However, before arriving at the force main the wastewater mixed with surface runoff has passed through a settling system. Therefore the concentration of hard solid particles in wastewater flowing through the force main is very low. The flow velocity range of the wastewater through the force main has been 1.4 – 3.3 m/s. In this case, after 20 years of operation the force main has been subject to several failure events, namely breakage of pieces of the pipe wall and eruption of sewage on the street. These events resulted from the development of grooves at the invert of the force main as shown in Fig. (2). In this case the pipes have been made of prestressed concrete, and the developed groove has allowed direct contact between the steel bars and the flowing wastewater. This contact has led to intensive corrosion of the steel bars and wires leading to fractures and breakage of the concrete pipe wall. Also in this case the maximum depth of the observed groove has been approximately half a circle.

## LABORATORY EXPERIMENTS

Abrasion resistance tests of pipe materials have been reported by various studies [e.g., 4, 6, 9, 10, 11] in which the Darmstadt rocker apparatus with various test procedures have been applied.

In the present study we have constructed the apparatus shown in Fig. (3) to carry out the specific set of experiments required to demonstrate the effect of small concentrations of solid hard particles characterized by various variables on the parameters of the bed load erosion developed at the pipe invert.

It should be noted that experiments carried out in this study with the apparatus shown in Fig. (3) do not simulate the sediment abrasion in flow through pipes, for example

fine sediments, like sand which are usually transported as bed load or even suspended load in pipe flow, are not subject to rocking in our experimental setup. However, by carrying out tests with the apparatus shown in Fig. (3) it is possible to follow some basic characteristics of the process leading to the groove formation at the pipe test section invert, as described in following paragraphs.

The pipe test section of the apparatus shown in Fig. (3) has been made of Plexiglas; with length 1,250 mm, internal diameter 54 mm, wall thickness 3 mm. Both ends of the pipe test section have been equipped with caps allowing complete filling of the pipe test section with water and solid particles before carrying out the test and cleanup of the test section from air and impurities. The pipe test section has been subject to a rocking angle of  $\pm 22.5^\circ$  resulting from the upward and downward movements of the vertical shaft. During the test section rocking, the water in the test section is at rest, and only the hard solid particles are moving at the invert of the test section from one end to the other end and vice versa.

Our objective has been to identify the effect of the density, size and number of hard particles on the size and on the rate of development of the groove that develops at the invert of the pipe test section. Before starting the experiments the internal side of several Plexiglas pipe test sections has been painted with several mixtures of acrylic paint layers whose total thickness 0.6 mm. For our experiments we use the following types of solid spherical particles, to which surface fine sand is glued to increase their roughness:

- Spheres made of steel whose diameters were: 4, 6, 15 mm.
- Spheres made of glass whose diameters were: 13, 18 mm.
- Spheres made of lead whose diameter was: 9 mm.

The spherical particles with various numbers have been inserted into the pipe test section, and then the pipe test section is completely filled with water and connected to the apparatus for carrying out the rocking experiment. Results of the experiments are summarized in Table 1. Fig. (4) shows some examples of final grooves formed during the experiments.



**Fig. (2).** The groove developed at the invert of the force main in Vancouver [16].



Table 1. Experimental Results

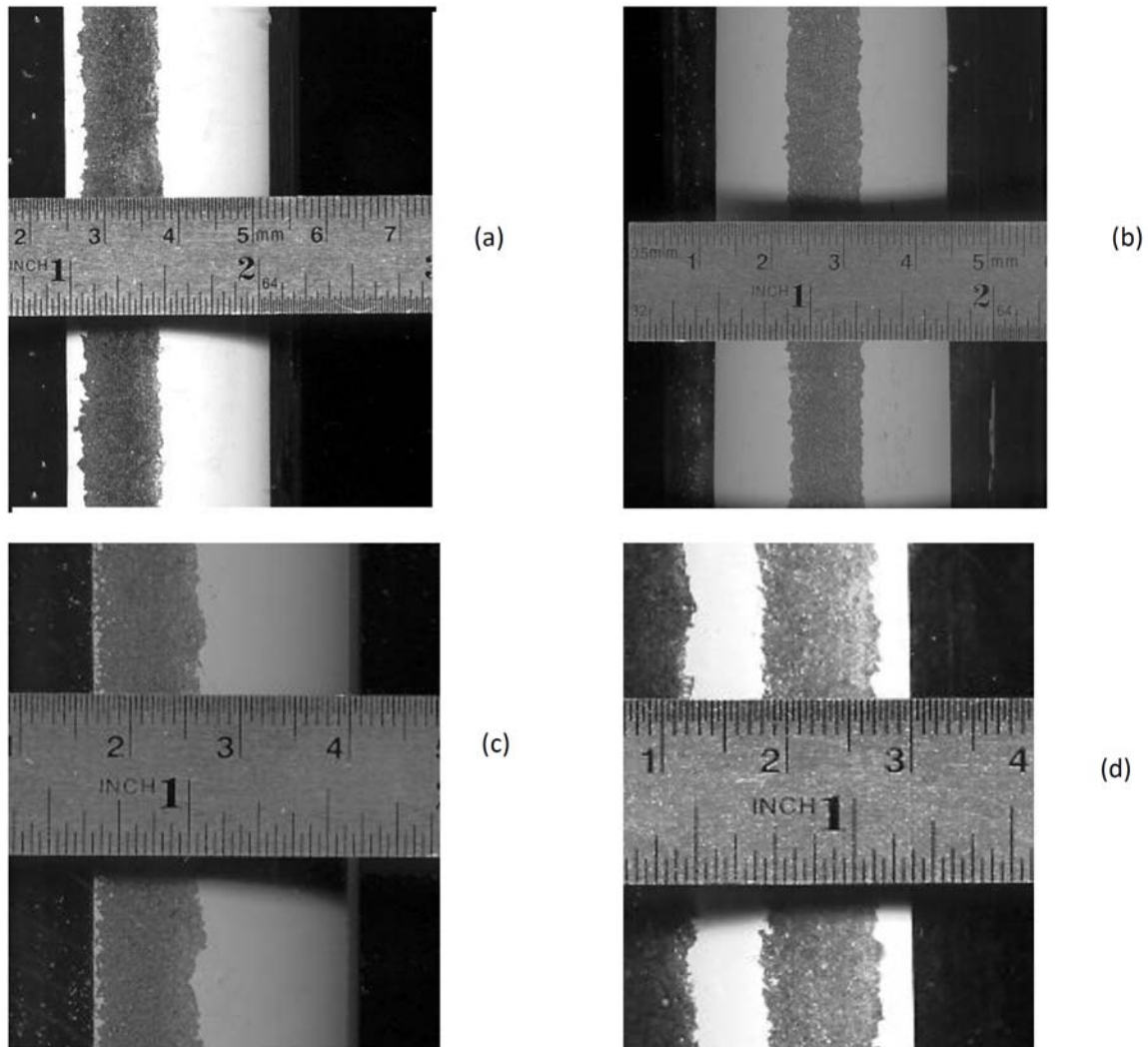
Particle Diameter (mm)	Particle Material	Number of Particles Used in the Experiment	Number of Rocking Needed to Obtain Groove Initiation	Number of Rocking Needed to Obtain Groove Completion	Groove Final width (mm)	Average Mass of a Particle after the Experiment (gr)
4	steel	1	5106	5793	6	0.45
		2	3495	4054	7	
		3	2070	2612	8	
		4	1087	1590	9	
		5	983	645	12	
6	steel	1	2274	2661	7	0.77
		2	1883	2538	8	
		3	1479	2100	10	
		4	944	1692	12	
		5	497	859	13	
9	lead	1	173	623	7	2.31
		2	143	480	11	
		3	128	420	17	
		4	113	360	23	
		5	83	308	24	
13	glass	1	169	495	9	2.31
		2	122	304	15	
		3	89	218	20	
15	steel	1	123	398	11	9.01
		2	91	261	18	
		3	58	194	21	
18	glass	1	136	425	14	6.03
		2	98	297	23	
		3	63	203	26	

tal results of this study and available knowledge concerning erosion and characteristics of sediment transport in pipe and open channels [e.g., 4, 9, 10, 17-22] to represent the following resulting information about the formation of the narrow groove at the pipe invert:

- 1) The rate of formation of the groove is more intensive for sediments of high density; however, the range of specific gravity of hard solid particles under field conditions is quite limited and varies around 2.4 - 2.6.
- 2) It seems that for sediments with particles of limited size, under field conditions, the width of the groove is mainly determined by the concentration of the transported sediments.
- 3) As long as sediments are transported as bed load, increasing the flow velocity probably increases the ve-

locity of the transported sediments and their momentum, which leads to increasing rate of development of the groove at the pipe invert.

- 4) Under field conditions the increased flow velocity originates from the increased discharge of the pipe flow. With increasing the discharge of the flow in the pipe, the hydraulic gradient increases and also the shear stress at the pipe wall. The increase of the shear stress means also an increase of the shear velocity. Therefore, the ratio of the shear velocity to the settling velocity of the sediment particles increases and more solid particles are transferred from the state of bed load to suspended load. This process probably leads to the increase of the width of the groove that develops at the pipe invert.



**Fig. (4).** Several examples of final groove patterns: (a) One lead particle, 9 mm diameter; (b) Two lead particles, 9 mm diameter; (c) One steel particle, 15 mm diameter; (d) Two glass particles, diameter 13 mm.

- 5) At high flow velocity probably most sediments of small particles may be transported as suspended load, and then erosion is induced almost uniformly over the entire internal circumference of the pipe wall. Therefore, at high flow velocity through the pipe erosion of the pipe wall by hard sediments can be quite intensive; the effect of groove formation and concentration of high stresses at the groove on the pipe structural strength is less significant than in cases of lower flow velocities. It means that at low flow velocity the abrasion risk on the structural strength of the pipe may be higher than at high flow velocity.

The reported field observations concerning the development of grooves at the invert of force mains [1] probably indicate that in such pipes this phenomenon is very much expected, because the settling system existing at the suction side of the wastewater pumping station leads to settlement of most quantities of grit present with the wastewater. Therefore, only low concentrations of small solid particles are transported with the wastewater that flows through the force main.

In the next section, we evaluate the effect of the groove pattern, namely geometrical parameters of the groove, on the durability of the rigid pipe, which is made of homogeneous material.

#### THE EFFECT OF THE GROOVE PATTERN ON THE RIGID PIPE DURABILITY

As an example of rigid pipes, this study refers to nonreinforced concrete cylindrical pipes complying with the Israeli standard IS 27 (August 1984) "Reinforced and nonreinforced concrete cylindrical pipes". This standard has adopted (with some changes and supplements) the European standard EN 1916 (October 2002) "Concrete pipes and fittings, unreinforced, steel fibre and reinforced". Various simulations of concrete pipe loadings have been carried out by applying the ANSYS code. The assumed internal pipe diameter,  $D$  has been set to 1,000 mm (and internal radius  $R = 500$  mm). The pipe wall thickness,  $d$  has been set to 140 mm. According to the Israeli standard IS-27 (EN 1916), the pipe should be made of the B 400 type of concrete, the critical external load leading to crushing of the pipe and represented by the three



















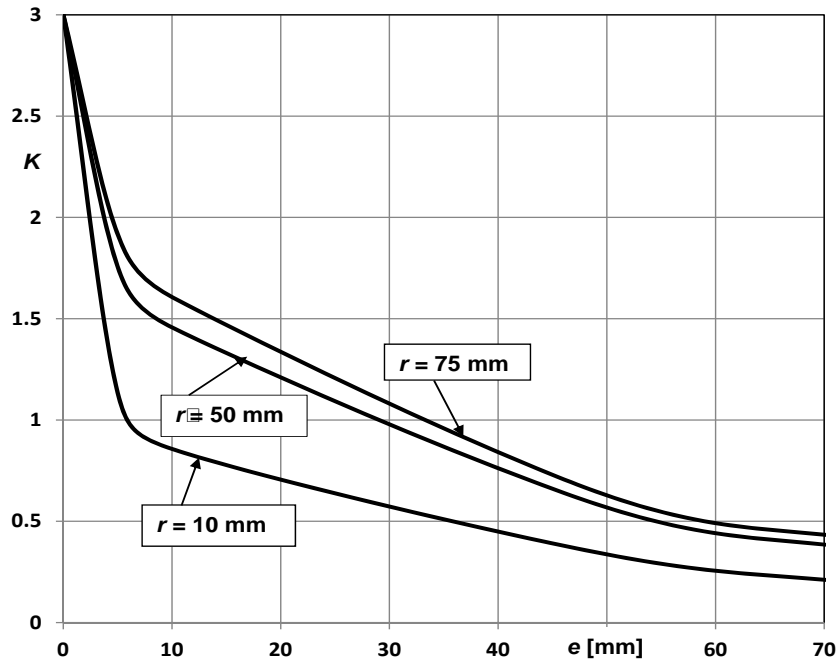


Fig. (17). The decrease of the pipe safety factor of service for variable values of the groove depths and various values of the groove bottom radius of curvature; the pipe is subject to pure internal pressure; the original safety factor is  $K = 3$  ( $D = 1,000$  mm,  $d = 140$  mm).

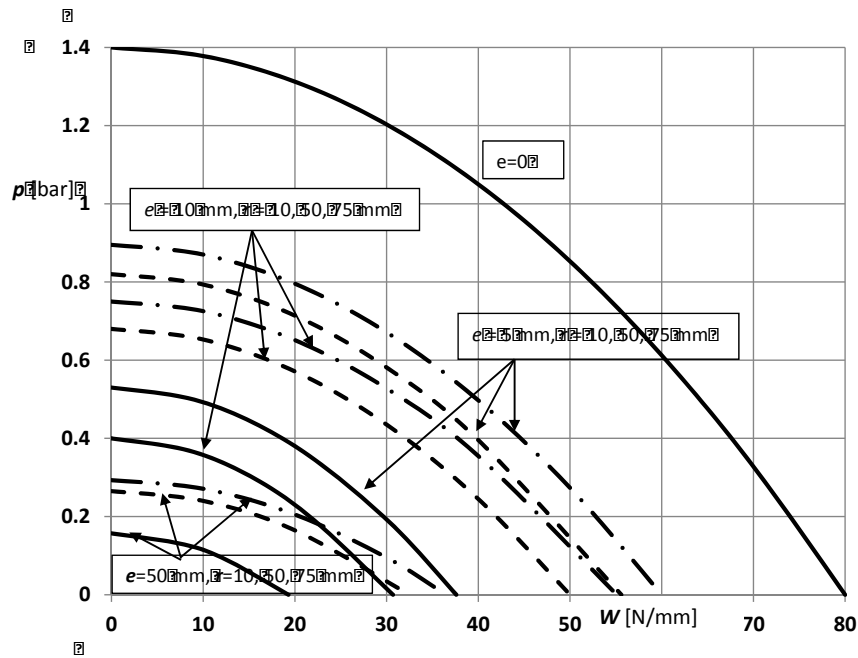


Fig. (18). Three families of failure curves for a concrete pipe characterized by  $D = 1,000$  mm,  $d = 140$  mm,  $W_{cr} = 80$  N/mm,  $p_{cr} = 1.4$  bar, for three grooves with depths  $e = 5, 10, 50$  mm; each family of failure curves incorporates reference to three values of the groove bottom curvature  $r = 10, 50, 75$  mm.

tities of grit present with the wastewater. Therefore, only low concentrations of small solid particles are transported with the wastewater that flows through the force main.

Laboratory experiments carried out within the research program of this study and available knowledge about erosion

and sediment transport in pipes and open channels suggest that under field conditions the width of the groove formed at the pipe invert, due to bed load erosion caused by small concentrations of small particle sediments, probably increases with the increase of the sediment concentration. Possibly, an increase of the water flow velocity intensifies the abrasion



effect of the transported sediments, but on the other hand the increased flow velocity may lead to transfer of particles from the state of bed load to the state of suspended load and thereby the groove width increases, and the risk of abrasion to the structural strength of the pipe decreases.

The developed groove pattern has been represented by a model useful for carrying out the structural analysis of the pipe with the groove at its invert. This model is characterized by two geometrical parameters: 1) the groove's depth, and 2) the groove's bottom radius of curvature. The groove's width is determined by these parameters according to simple geometrical relationships. We have carried out numerous numerical simulations with ANSYS code to characterize the effect of the groove parameters on the pipe durability, namely structural strength and resistance to external load and/or internal pressure. The major outcomes of our simulations are:

- 1) The groove considerably reduces the pipe resistance to external load as well as to internal pressure, due to stress concentration at the groove; the durability reduction is more effective on internal pressure than on external load.
- 2) The pipe durability reduction due to the presence of the groove is most effective for narrow grooves; therefore, very low concentrations of small particle sediments transported with the sewage or surface runoff may lead to high risks of the pipe failure due to bed load erosion.
- 3) By applying formulas based on the extension of Schlick's formula it is possible to calculate the effect of the groove developed at the pipe invert on its factor of safety for proper service.
- 4) By applying formulas based on the extension of Schlick's formula it is possible to obtain families of failure curves for pipes subject to external load combined with internal pressure, in which grooves resulting from bed load erosion have been observed. Such curves represent the decrease of the pipe serviceability due to bed load erosion and can be useful for evaluating the need for repair or replacement of a pipeline because grooves are developed at its invert due to bed load erosion.

#### CONFLICT OF INTEREST

The author(s) confirm that this article content has no conflicts of interest.

#### ACKNOWLEDGEMENTS

This study has been supported by the Umbrella Research Cooperation of RWTH Aachen University and Technion – Israel Institute of Technology devoted to “Sustainable Urban Development”.

The authors thank Eng. Gideon Zac, formerly General Manager of Dan Area Communities Consortium for providing basic field data and helpful discussions. We are grateful to Prof. Robert Levy of Ben-Gurion University of the Negev for his help and guidance with using the ANSYS code for

the numerical simulations required by this study. The authors appreciate very much the important information, as well as photos concerning issues of this study given by Eng. Paul Wilting, Senior Project Engineer, Greater Vancouver Regional District. Helpful comments and discussions with Prof. Thomas O'Rourke of Cornell University are deeply appreciated.

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Received: February 07, 2013

Revised: April 05, 2013

Accepted: April 08, 2013

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