

# Research on Tooth Modification of Spur Bevel Gear

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**Abstract:** Isometric modification is proposed as a new method of axial modification of spur bevel gear. Based on the gear geometry theory and the normal meshing motion equation of gear pairs, changes of meshing points and angles are analyzed and the effect of axial modification on meshing movement of gear pairs is discussed. After that, a method of drawing spherical involute with the aid of SolidWorks software is achieved, by which the accuracy of solid modeling of spur bevel gear is improved. After solid modeling, bevel gear analysis is carried out by ANSYS/LS-DYNA software so that the contact stress and acceleration changes of driven wheel during the meshing process which was used to guide the modification of spur bevel gear can be calculated. The simulation results show that the stress distribution of gear surface is controlled by tooth modification. The load concentration, agglutination and pitting of the gear can be avoided effectively.

**Keywords:** Solid modeling, tooth modification, spur bevel gear, finite element analysis (FEA).

## INTRODUCTION

Spur bevel gears can be utilized to transmit rotary motions between the orthogonal intersecting axes and they are usually applied to decelerate, differential operations, etc. The special use of spur bevel gear calls for high driving accuracy and transmission stability. However, it is difficult to avoid the impact and load concentration of a pair of meshing gears due to many complex factors such as the variety range of bearing amplitudes, manufacture and installation tolerances, contact deformations, bending deflections and torsional deformations of the shaft and bearing clearances, etc. To abate these problems, one can increase the size of shaft, the hardness of bearing or support as well as restrict the width of every gear. Apart from these, another effective method of axial modification was developed recently.

Dynamic simulation analysis will be operated explicitly with the nonlinear software ANSYS/LS-DYNA in this paper. Compared with static analysis, the results of dynamic analysis will reveal much more specific meshing situation [1-4]. In spite of this, accurate solid modeling remains the basis of dynamic simulation. The spherical involute can not be obtained directly through 3D software and therefore the plane involute or the three-point arc is often used instead. In this sense, the accuracy of solid modeling can not be guaranteed. This paper will endeavor to propose a method of drawing spherical involute and to get an accurate model of spur bevel gear.

## ISOMETRIC MODIFICATION OF SPUR BEVEL GEAR

Isometric modification, as the name implies, is a method of axial modification of the standard tooth with the aim to

create a new surface in normal direction [5, 6]. The new surface will contact with the standard tooth instead of the original one. Therefore, the contact area of the gear can be controlled by both the position and the size of isometric modification. Fig. (1) is the schematic diagram of isometric modification. As is shown in Fig. (1),  $\Sigma$  is the non-modified tooth surface,  $\Sigma'$  is the modified tooth surface,  $p$  is a point in surface  $\Sigma$  and point  $p'$  is in surface  $\Sigma'$ ,  $h$  stands for the size of modification, that is, the distance between  $P$  and  $p'$ .

The advantages of isometric modification include improving the contact area, avoiding the contact at the edge and reducing the impact caused by load change and the sensitivity of assembling error.

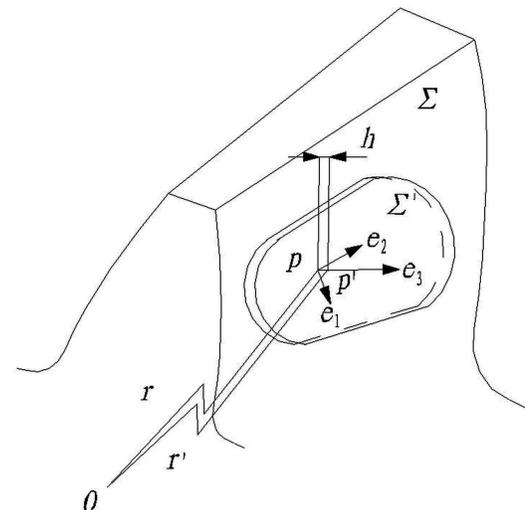


Fig. (1). Schematic diagram of isometric modification.

When a gear pair is in normal contact, the ideal contact area will take up 40-70% along the tooth height and 35-65% along the tooth length. The size and position of the contour of isometric modification can be determined on the basis of the ideal contact area. Fig. (2) is the schematic diagram of the contour shape and position of isometric modification.

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The size of  $h$  plays a decisive role in the effect of isometric modification. If the size is too big, the contact area will

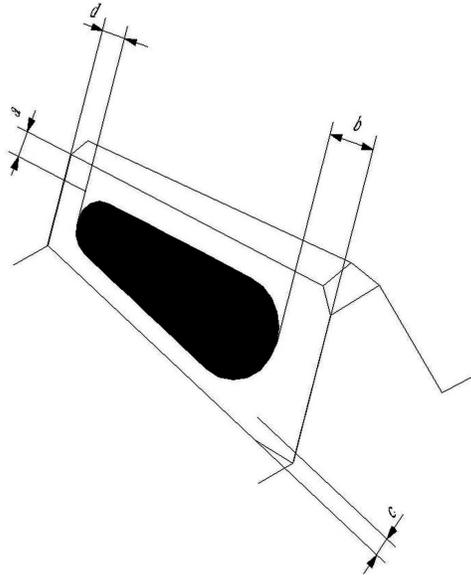


Fig. (2). Schematic diagram of the contour shape and position of isometric modification.

decrease and the load concentration will increase, on the contrary, the target of modification can not be reached. From the above, many factors should be considered to determine the size, including manufacture and installation tolerances, contact deformations, bending and torsional deformations of the shaft, hot deformations in high-speed operation, etc. Mechanics of Materials, elastic mechanics theory and numerical method can be utilized to calculate the size but all of the three methods are very complex.  $H$  is always determined by one's experience in practical work and the recommended size is 0.03~0.05 mm or 0.004~0.005 $m$  ( $m$  is the modulus) [7].

**INVOLUTE CHARACTERISTICS OF ISOMETRIC MODIFICATION GEAR**

Fig. (3) is the schematic diagram of the characteristics of the involute. According to the principle of involute forming,

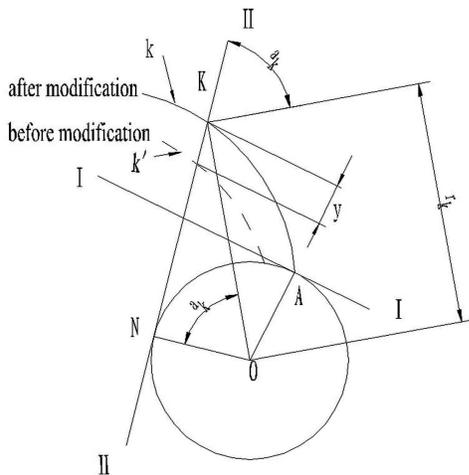


Fig. (3). Schematic diagram of involute characteristics.

the generating line  $KN$  is the normal of the involute at the point  $N$  and the size of modification  $y$  is uniform in the normal direction. Therefore, the new profile is still an involute profile that meets the requirements of modification.

**MESHING OF ISOMETRIC MODIFICATION SPUR BEVEL GEAR**

On the basis of the gear geometry theory and the normal meshing motion equation of gear pairs, changes of meshing points and angles can be analyzed with the help of isometric modification.

**Position of Meshing Point**

As is shown in Fig. (1),  $e_1, e_2$  are unit vectors in tooth depth and width of point  $P$ ,  $e_3$  is surface  $\Sigma$ 's unit vector in the normal direction, point  $O$  is in the rotation axis,  $r, r'$  are vectors of  $O$  to  $P$  and  $P'$ .

The changes of the normal direction are caused by different modification sizes  $h$ . After isometric modification, surface  $\Sigma$  is parallel to surface  $\Sigma'$  with  $h$  which is a constant magnitude, that is to say, the normal direction of the gear surface does not change.

Denote  $\delta_1, \delta_2, \delta_3$  as the variable quantities of the meshing point in the direction of  $e_1, e_2, e_3$ . The variable position of the meshing point caused by the change of the tooth thickness is shown as follows:

$$r' = (r + e_3 h) + dr + e_3 \delta_3 + e_3 dh \tag{1}$$

With the theory of differential geometry, calculate the second derivative of radius vector  $r(u,v)$ .

$$d^2r = r_{uu} du^2 + 2r_{uv} dudv + r_{vv} dv^2 \tag{2}$$

$n$  is the unit normal vector. Define  $d^2r \cdot n$  as the second basic form of surface [8], thus  $n = r_u \times r_v / \sqrt{(r_u \times r_v)^2}$ . The second form can also be expressed as follows:

$$\varphi_{II} = c_{11} \delta_1^2 + c_{12} \delta_1 \delta_2 + c_{22} \delta_2^2 \tag{3}$$

In Eq. (3),  $c_{11}, c_{22}$  respectively represent the induced normal curvatures in the direction of  $e_1, e_2$ ;  $c_{12}$  is the negative induction geodesic torsion in the direction of  $e_1$ .

Therefore,  $\delta_3$  can be expressed as follows:

$$\delta_3 = \frac{1}{2} \varphi_{II} = \frac{1}{2} (c_{11} \delta_1^2 + c_{12} \delta_1 \delta_2 + c_{22} \delta_2^2) \tag{4}$$

Known from Eq. (4),  $\delta_3$  is the second-order small element and the changed position of the meshing point has little effect on the rotational angle of the gear pair.

**Rotational Angle**

The angle tolerance of a gear can be measured through a comprehensive survey instrument. Denote  $\Phi_1$  as the angle of the master gear,  $\Phi_2$  is the theoretical angle of the measured gear before modification,  $\Phi_2'$  is the actual angle of the measured gear after modification,  $\Delta\Phi_2$  is the angle variation caused by modification. Obviously,

$$\Phi_2' = \Phi_2 + \Delta\Phi_2 \quad (5)$$

Denote  $r_1', r_2'$  as the vectors of the master gear and the measured gear respectively. The gear pair will reach the actual meshing point  $P_1, P_2'$  when  $r_1', r_2'$  turn around the certain angle  $\Phi_1, \Phi_2$ . Known from Eq. (1)

$$p_1' = p_1 + e_3 h^{(1)} + (e_1 \delta_1^{(1)} + e_2 \delta_2^{(1)}) + e_3 \delta_3^{(1)} \quad (6)$$

$$p_2' = p_2 + e_3 h^{(2)} + (e_1 \delta_1^{(2)} + e_2 \delta_2^{(2)}) + e_3 \delta_3^{(2)} + e_3 \delta_3^{(2)} + e_3 dh^{(2)} + k_2 \times p_2 \Delta\Phi_2 \quad (7)$$

Where  $P_1, P_2$  are respectively the points on the tooth surfaces of the master gear and the non-modified measured gear;  $k_2$  is the equivalent gear's rotation vector of the measured bevel gear and it is in the same direction with  $e_3$ .

The modified gear pair should meet the requirement as follows:

$$p_1' - p_2' = p_1 - p_2 \quad (8)$$

After finishing, to

$$(k_2 \times p_2) \Delta\Phi_2 e_3 = h^{(2)} + (\delta_3^{(1)} - \delta_3^{(2)}) \quad (9)$$

Known from Eq. (4),  $\delta_3$  is a second-order small element and the term  $\delta_3^{(1)} - \delta_3^{(2)}$  in Eq. (9) can be neglected.

$(k_2 \times p_2) e_3$  stands for the basic circle radius and  $\Delta\Phi_2$  can be expressed as follows:

$$\Delta\Phi_2 = \frac{h}{R} \quad (10)$$

The establishment of the relationship between angle change and modification size provides not only the basis for calculation and the selection of the modification size, but also reference for the detection of the modification effect in the future work.

### ACCURATE SOLID MODELING OF SPUR BEVEL GEAR

The basic idea of modeling is solving spherical involute equation, generating curve in the 3D software SolidWorks, filling surface and creating tooth space entities, arraying and cutting entities combined with gear matrix [9, 10]. The modeling details of differential planetary gear will be listed in the following example.

#### Solution to the Involute Equation

The spherical involute equation in Cartesian coordinate system is presented as follows:

$$\begin{aligned} x &= l(\sin \varphi \sin \phi + \cos \varphi \cos \phi \cos \theta) \\ y &= l(\sin \varphi \cos \phi \sin \theta - \cos \varphi \sin \phi) \\ z &= l \cos \varphi \cos \theta \end{aligned} \quad (11)$$

Where  $\sqrt{x^2 + y^2 + z^2} = l$ ;  $\phi = \varphi \sin \theta$ ;  $l$  is the initial radius of the gear;  $\theta$  is the base angle;  $\varphi$  is the angle between the starting line which is on the contact surface and the

instantaneous rotor. When the starting point of the involute is on the base circle bevel,  $\varphi = 0$ .

Take a differential bevel gear for instance. Its parameters are shown in Table 1.

**Table1. Parameters of Differential Bevel Gears**

Parameters	Planet Gear	Half Axle Gear
Shaft angle	90°	
Tooth number	10	15
Module	4.438	
Pressure angle	22.5	
Reference diameter	44.38	66.57
Outer cone distance	40±0.025	
Reference circle cone angle	33.69°	56.31°
Tip circle cone angle	46°	66°
Root circle cone angle	24°	44°
Addendum	4.98	3.28
Tooth depth	9.17	9.17

Solve the coordinates of  $x, y, z$  under the condition that  $\varphi$  is in  $[0, \pi/3]$  according to Eq. (11). The corresponding Matlab program is as follows:

```

clc; clear;
z1=10; % Tooth number of planetary gear
z2=15; % Tooth number of axle gear
m=4.438; % Gear module
r=m*(sqrt(z1^2+z2^2))/2; % Outer cone distance
l=r+5; % Initial radius of gear
alpha=22.5*pi/180; % Pressure angle
theta=33.69*pi/180; % Pitch angle
gamma=asin(sin(theta)*cos(alpha)); % Base angle
counter=0;
for t=0:0.01:1 % Step is 0.01
phi=t*pi/3; % Angle between the starting line which is on
the contact surface and the instantaneous rotor
counter=counter+1;
x=l*cos(phi*sin(gamma))*cos(gamma)*cos(phi)+l*sin(phi)*
sin(gamma)*cos(phi);
y=l*cos(phi*sin(gamma))*sin(gamma)*sin(phi)-
l*sin(phi*sin(gamma))*cos(phi);
z=l*cos(phi*sin(gamma))*cos(gamma);
D(counter,1)=phi;
D(counter,2:4)=[x y z];
End
    
```

The calculation results are shown in Fig. (4).

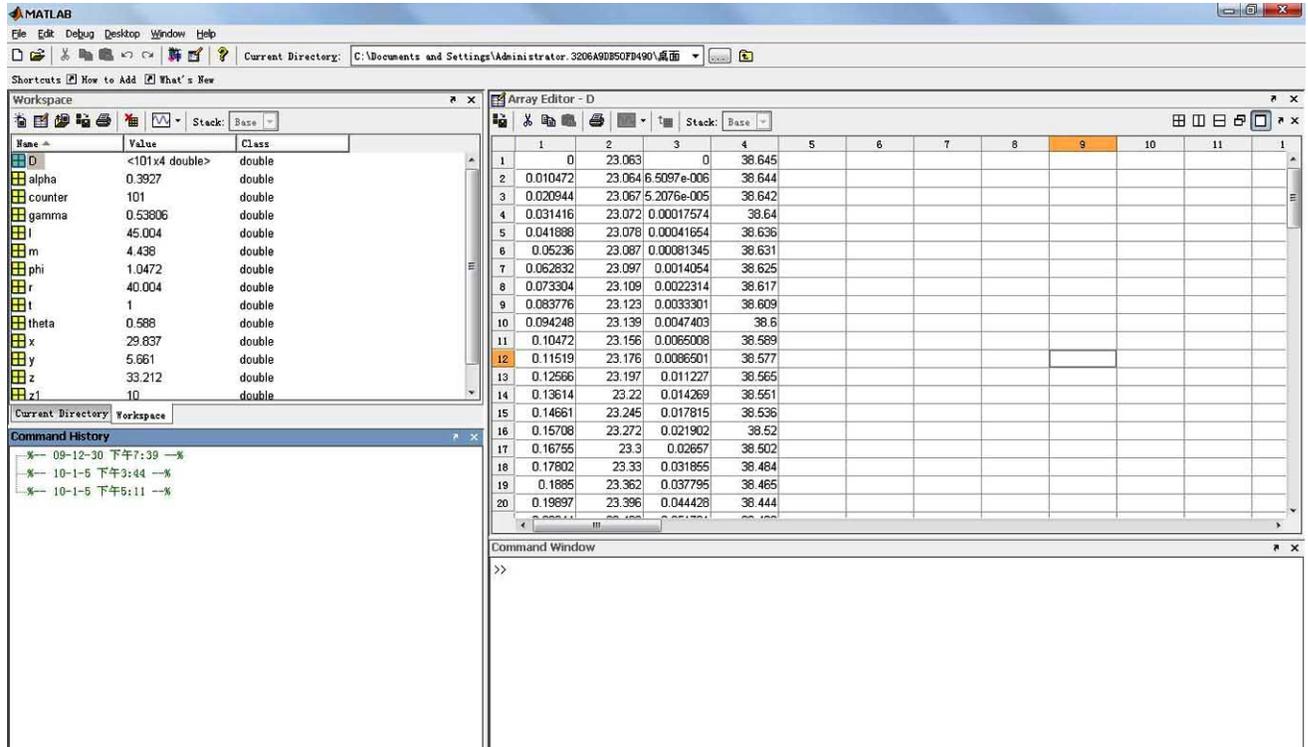


Fig. (4). Calculation results.

**Create Entity of Spur Bevel Gear**

First, create a new part drawn in SolidWorks, then choose “curve by xyz point” under the command of “curve” and the spherical involute will be formed by importing  $x, y, z$  coordinates. Next, generate a closed interval by connecting the starting and the end point of the involute, then fill the closed interval and the tooth surface will be obtained. Then, a base level should be determined according to the width of tooth space. The two-dimensional figures of the gear entity can be drawn on this base level. The base level can also be used as a symmetric plane of the tooth surface which is shown in Fig. (5). Fig. (6) is the schematic diagram of

addendum and tooth root surface, which is generated by rotating the two-dimensional figure.

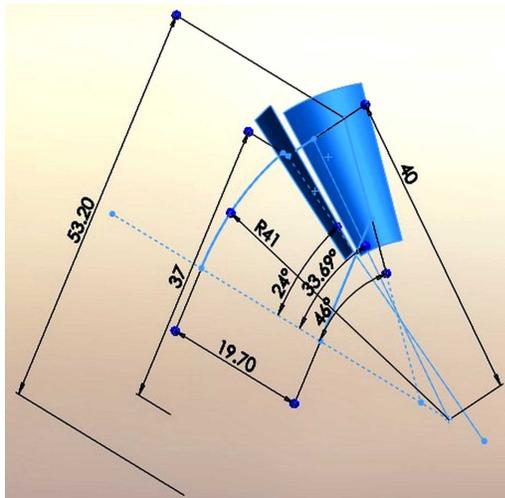


Fig. (5). Symmetric tooth surface.

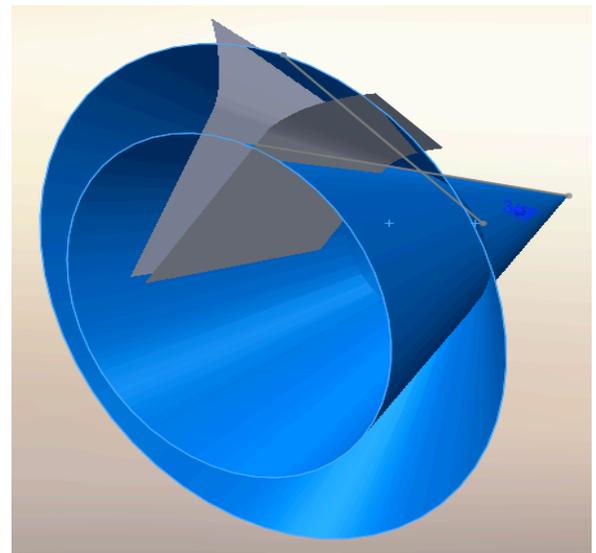


Fig. (6). Addendum and tooth root surface.

To get the entity of tooth space, the surface of addendum and tooth root should be trimmed and filled. After circularly arraying, the entity is obtained as is shown in Fig. (7).

Finally, the entity of spur bevel gear is obtained by cutting the combination of the gear matrix. Fig. (8) shows the solid model of a planetary gear.

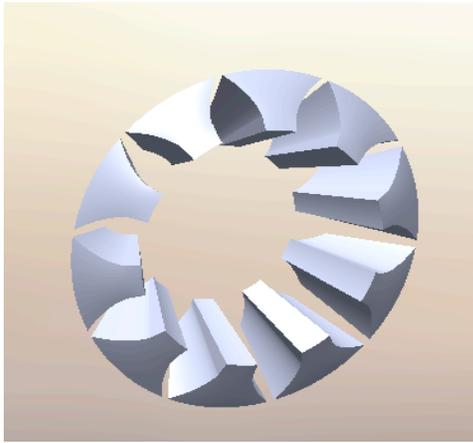


Fig. (7). Entity of tooth space.

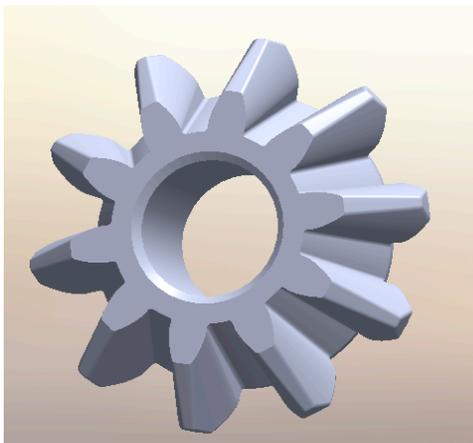


Fig. (8). Solid model of planetary gear.

**Build the Model of Isometric Modification Gear**

Draw a closed curve in tooth section in the interfical environment of sketch editing, and then withdraw it from the edit mode. Fig. (9) shows the two-dimensional schematic diagram of the closed curve.

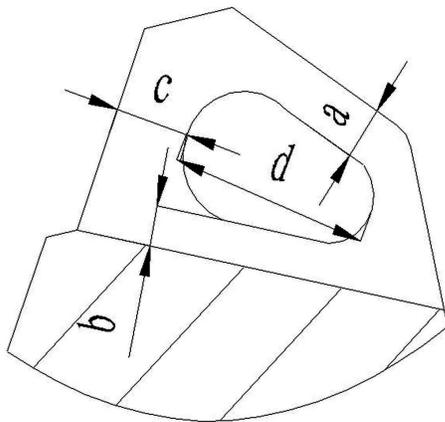


Fig. (9). Two-dimensional schematic diagram of closed curve.

Execute the order “tension-to a surface in a specified distance”. Set the distance according to the modification size *h* and the new surface is the isometric modification surface, as is shown in Fig. (10).

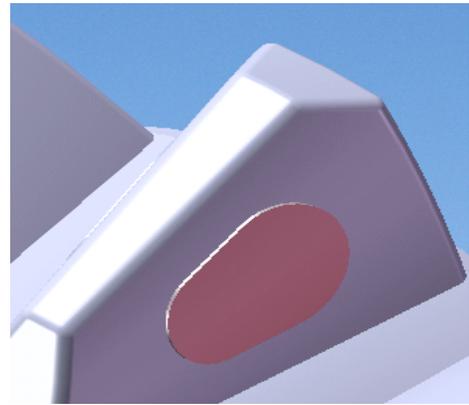


Fig. (10). Isometric modification surface.

**DYNAMIC SIMULATION OF SPUR BEVEL GEAR**

**Basic Theory of Gear Simulation**

As for the gear pair, the meshed gears will not only have to meet the elasticity equation, but also have to satisfy the displacement of embedded conditions in the normal direction of the meshing point and obey the law of coulomb in the tangent direction. According to the contact statement, the contact surfaces can be divided into three kinds of boundary conditions as long as the input torque of driving gear does not change, namely the continuous state, the sliding state and the separation state. The gear pair can be divided into the drive and the driving gear, which are two separate objects. The finite element equations of the two gears can be established in the global coordinate. The equations are as follows:

$$\begin{aligned} K_1 U_1 &= P_1 R_1 \\ K_2 U_2 &= P_2 R_2 \end{aligned} \tag{12}$$

Where  $K_1, K_2$  are the rigidity matrixes of the drive and the driving gear,  $U_1, U_2$  are the displacement vectors of the drive and the driving gear,  $P_1, P_2$  are the load vectors force on the drive and the driving gear,  $R_1, R_2$  are the contact load vectors.

In the local coordinate ( $x, y$ ), we respectively apply  $r$  and  $u$  to represent the contact node and the displacement components of the meshing point  $i$ , which is in the direction of  $j$ . Therefore,

In continuous state,

$$\begin{aligned} r_{ij}^{(2)} &= -r_{ij}^{(1)} \\ u_{ix}^{(2)} &= u_{ix}^{(1)} + \delta_{ix} \quad (j = x, y) \\ u_{iy}^{(2)} &= u_{iy}^{(1)} + \delta_{iy} \end{aligned} \tag{13}$$

In sliding state,

$$\begin{aligned} r_{ij}^{(2)} &= -r_{ij}^{(1)} \\ u_{ix}^{(2)} &= u_{ix}^{(1)} + \delta_{ix} \quad (j = x, y) \\ R_{iy} &= \pm u |R_{ix}| \end{aligned} \tag{14}$$

In separation state,

$$r_{ij}^{(2)} = -r_{ij}^{(1)} = 0 \quad (j = x, y) \quad (15)$$

Where  $u$  is the friction coefficient,  $\delta_{ix}$  is the initial clearance in the normal direction of the meshing point  $i$ ,  $\delta_{iy}$  is the initial clearance in the tangent direction of the meshing point  $i$ ,  $R_{ix}$  is the contact load in the normal direction of the meshing point  $i$ ,  $R_{iy}$  is the contact load in the tangent direction of the meshing point  $i$ .

**Fundamental Assumptions of Gear Meshing Procession**

Before simulation, a collection of fundamental assumptions, which the simulation scheme has to satisfy, should be listed:

1. The material of the gear has the characteristics of continuity, linear elasticity, uniformity and isotropy.
2. The deformation of the meshing point is along the normal direction of the tooth profile. The contact load is along the normal direction of the tooth surface. The contact interface is in a smooth and continuous fashion.
3. The gear that used in simulation is in an ideal model in which manufacture and installation tolerances are excluded. The mechanical and geometric boundary conditions of the contact interface can be denoted by node parameters.
4. Thermal deformation of the transmission system and the elastic hydrodynamic lubrication mechanism are out of consideration. Lubrication is represented by the friction coefficient and the friction coefficient should obey the law of coulomb.

**Simulation Procedures**

Dynamic simulation analysis of spur bevel gear is operated explicitly with the nonlinear software ANSYS/LS-DYNA [11, 12]. Fig. (11) is a procedure chart of the application of Solid Works- ANSYS-DYNA.

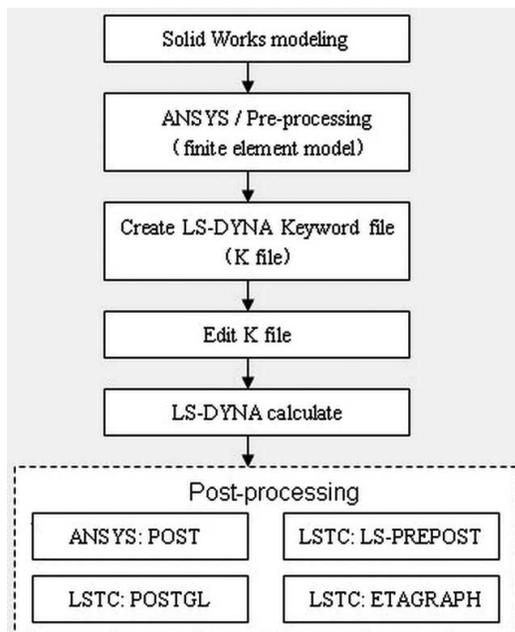


Fig. (11). Procedure chart of the application of Solid Works- ANSYS-DYNA.

The assembly model of a gear pair which is created by SolidWorks can be imported into ANSYS/LS-DYNA in the manner of “Parasolid”. In ANSYS/LS-DYNA, penalty function is used to deal with contact information and its principles are as follows: Check whether the associate node penetrates the main surface. If it does, a contact force should be introduced between the associate node and the main surface. In physics, there is a normal spring between the associate node and the main surface, which is used to avoid penetrating. The size of the contact force is proportional to the penetration depth and the hardness of the main surface. The contact force is expressed by the value of the penalty function. Penalty function method has many advantages such as little hourglass effect, little numerical noise and exact algorithm conservation of momentum. Because of these factors, dynamic analysis results can clearly show the specific meshing situation.

The simulation procedures consist of three main steps: Pre-processing, Solution to dynamic analysis and Post-processing.

Pre-processing includes defining element type, setting real constants, choosing material model, meshing and defining contact, and load and constraints.

When the assembly model was introduced into ANSYS/LS-DYNA, the properties of the material should be defined. Set the differential gear as an example. Its material is 20CrMnTiH, its elastic modulus is 207GPa, the density is  $7.8 \times 10^3 \text{kg/m}^3$  and the Poisson ratio is 0.25.

Elements SOLID164 and SHELL163 are adopted in this paper. SOLID164 is used for 3D modeling of solid structures. It is defined by eight nodes which have the following degrees of freedom at each node: translations, velocities, and accelerations in the nodal x, y, and z directions. SOLID164 does not have rotational degrees of freedom, so SHELL163 is needed in order to apply angular velocity and torque to gear pair. SHELL163 is a 4-node element with both bending and membrane capabilities. It is permitted both in-plane and normal loads. The element has 12 degrees of freedom at each node. They are translations, accelerations, and velocities in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes [13].

Many factors must be taken into consideration before meshing, such as the number of grid, the meshing density, the unit order, the meshing quality, the interface and demarcation point of grid, the grid distribution and the coordination of displacement. To improve the accuracy of the simulation, the grid in both the tooth root and the mating surface should be designed more sophisticated. Fig. (12) is the schematic diagram of the meshed gear pair.

The torque and the speed are all parameters of time. As a result, the array of time, the torque and the speed should be defined and assigned. After that, angular velocity is applied to planetary gear and the negative torque is applied to axle gear. The axial displacement of the two gears must be restricted when a gear pair is meshing. The load command is “EDLOAD, Option, Lab, KEY, Cname, Par1, Par2, PHASE, LCID, SCALE, BTIME, DTIME”.

Control parameters should be assigned before getting a solution, such as the control of the calculation time, the

control of the output file and the advanced control of the solution. The termination time of calculation is set according to angular velocity of the gear (termination time is very short in the physical process, generally in milliseconds). In time-step control, the calculation time of CPU will increase if the time step is too small. Mass scaling is an effective way to control minimum time step and to cut down the calculation time of CPU. Mass scaling is only used in elements which are smaller than minimum time step, that is to say, the minimum time step should be a negative value. The types of output file are “.RST”, “.HIS”, “d3plot” and “d3thdt” and they can be used in post-processor POST1, POST26 of ANSYS and LS-Prepost of LS-DYNA.

The purpose of isometric modification is to control the stress distribution and to compensate tooth deformation. The additional dynamic load is the direct result of tooth deformation,

and therefore, messages of stress distribution and acceleration are needed. The special post-processor Ls-prepost can meet these requirements.

Fig. (13) is the dynamic stress nephogram of the meshing of a gear pair. Known from Fig. (13), the stress distribution is not uniform, and the load concentration is in the big end. For these reasons, pitting, wearing and other tooth injuries will appear and the lifespan of gear will also be reduced. These aspects are usually caused by tooth deformation, which is mainly caused by the different curvature in the big end and the small end. In addition, manufacture and installation tolerances, bending and torsional deformations of the shaft, hot deformations in high-speed operation will exacerbate them. In order to improve these aspects, one can apply axial modification as a good way.

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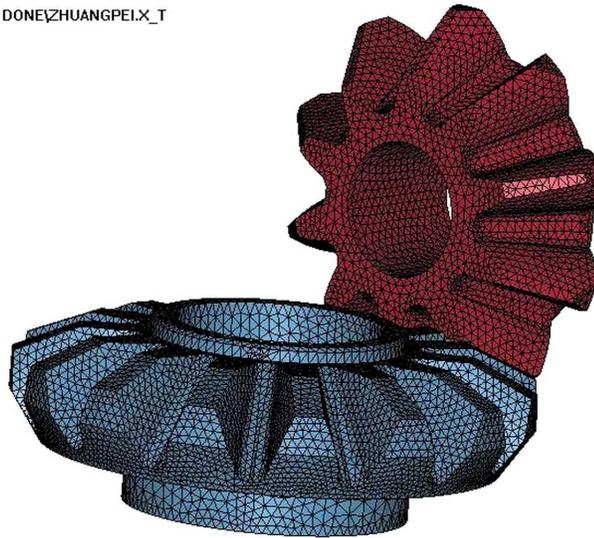


Fig. (12). Schematic diagram of meshed gear pair.

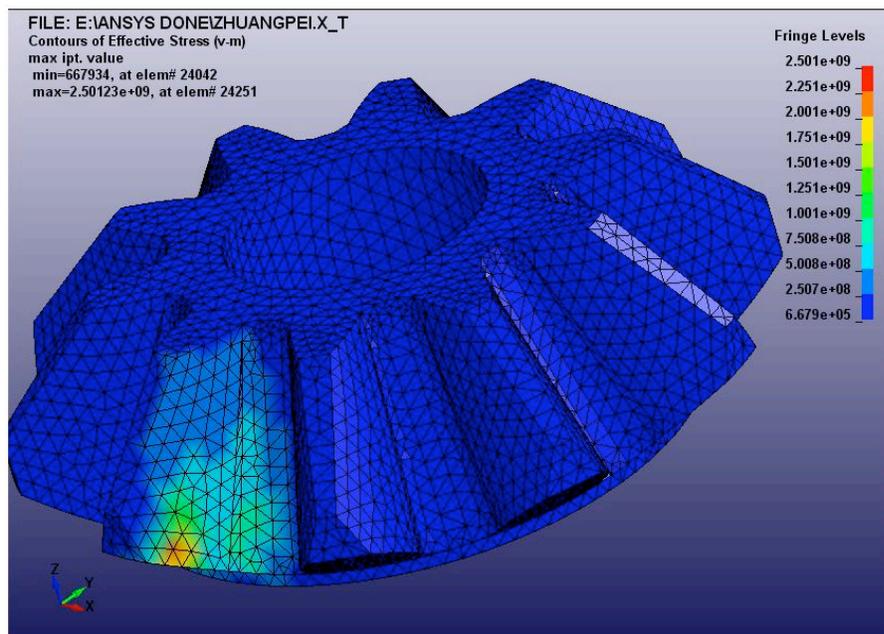


Fig. (13). Diagram of stress distribution before modification.

**EFFECTS OF ISOMETRIC MODIFICATION**

The characteristics of the strain and stress of planetary gear and axle shaft gear are similar. Only planetary gear’s messages are listed in this paper.

Fig. (14) and (15) are the diagrams of stress distribution after isometric modification and the planetary gear’s stress distribution curve in tooth width after isometric modification. Known from Fig. (14) and (15), the contact area and stress distribution are controlled effectively by axial modification in the direction of the tooth width. The maxi-

mum effective stress has a slight increase after modification while the effective stress that in the big end is reduced and the contact area is improved.

Many factors such as the manufacture and installation tolerances, contact deformations, bending and torsional deformations of the shaft, and hot deformations in high-speed operation are difficult to avoid for the reason that the base pitch is no longer equal. Transmission ratio is also changed and the additional dynamic load, vibration, noise, impact will appear [14]. Fig. (16) shows the angular acceleration

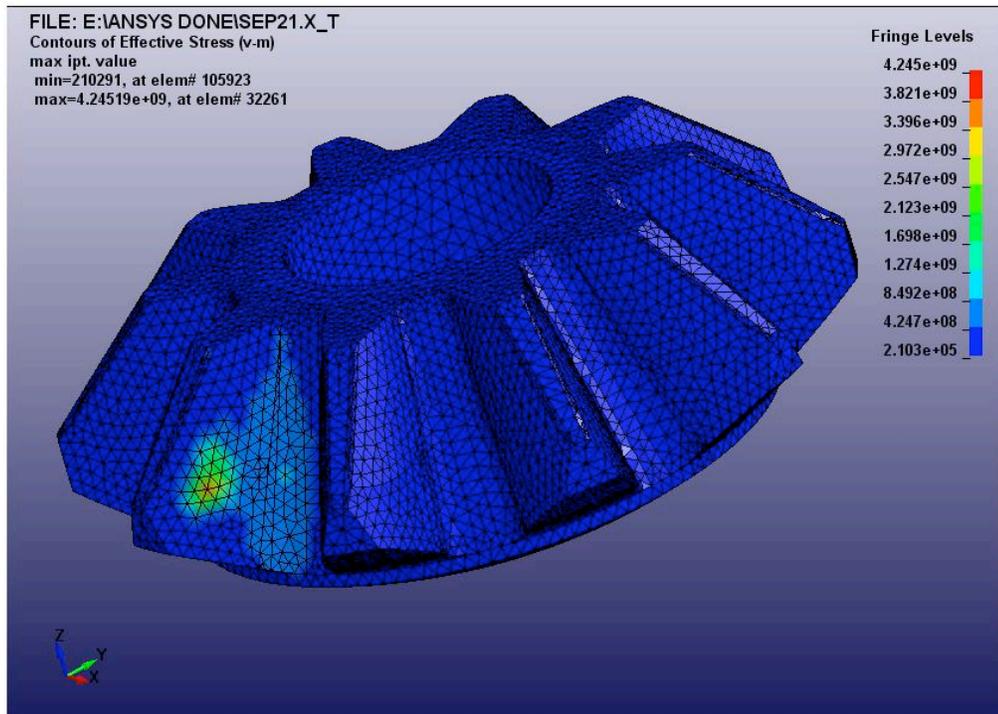


Fig. (14). Diagram of stress distribution after isometric modification.

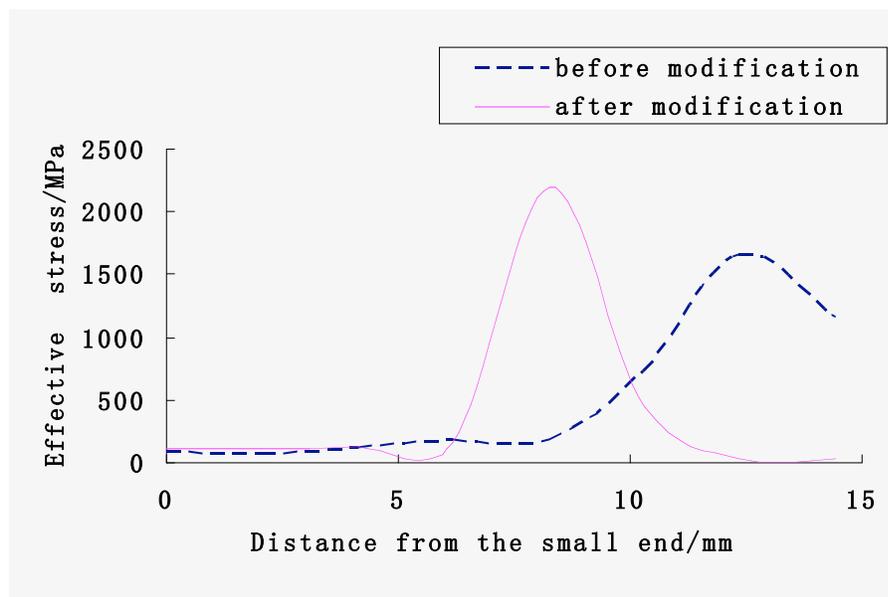
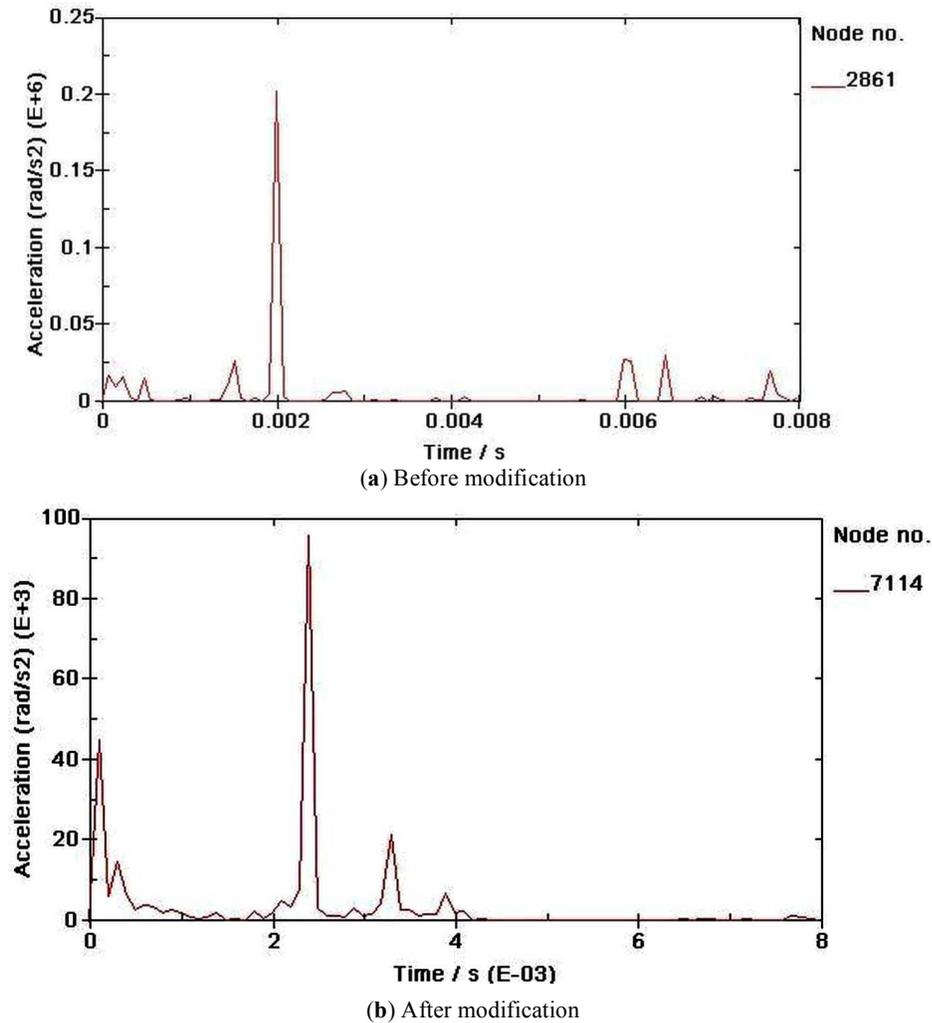


Fig. (15). Planetary gear’s stress distribution curve in tooth width after isometric modification.



**Fig. (16).** Angular acceleration curve of one point on the top land.

curve of one point on the top land before and after modification. The curve can obviously reflect the influence that the modification has generated. Known from Fig. (16), the additional load is reduced significantly by isometric modification and the maximum angular acceleration is changed from  $0.205 \times 10^6$  rad/s<sup>2</sup> before modification to  $0.098 \times 10^6$  rad/s<sup>2</sup> after modification; and the size has depreciated by 52.2%. Vibration and noise will also be reduced.

## SUMMARY

Through the combination of both experience and the traditional theory of gear modification, the concept of isometric modification was proposed. By selecting the appropriate modification size and modification location, tooth deformation would be compensated and the stress distribution would be controlled in the central part of the tooth; the load concentration, agglutination, pitting of the gear could also be avoided effectively.

According to the gear geometry theory and the normal meshing motion equation of gear pairs, changes of meshing points and angles were analyzed, and then, the effect of axial modification on gear pair's meshing movement was discussed. The establishment of the relationship between angle

changes and modification size provided not only the basis for calculation and the selection of the modification size, but also a reference for the detection of modification effect in the future work.

Based on the 3D software SolidWorks, a method of drawing spherical involute was achieved, and the solid modeling accuracy of spur bevel gear was improved. After solid modeling, dynamic emulation analysis was operated by FEA software. The analytical results had shown that stress distribution was controlled by isometric modification and the additional load was reduced effectively.

This technology has been applied in small batch production of automobile differential bevel gear, and the results showed that the modified gears' kinematic accuracy and motion smoothness accuracy were up to 7 degree (GB11365-89), the lifespan of gear was increased by more than 10%. The users' feedback showed that the meshing noise of gears has significantly decreased.

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## REFERENCES

- [1] A. Kahaman, "Dynamic analysis of multi-mesh helical gear train", ASME, *Journal of Mechanical Design*, vol. 116, pp. 35-37, 1994.
- [2] V. Ramamurti, Nayak H. Vijayendra, and C. Sujatha, "Static and dynamic analysis of spur and bevel gears using FEM", *Mechanism and Machine Theory*, vol. 33, pp. 1177-1193, 1998.
- [3] R. F. Li, and C. Y. Yang, "Finite element simulation of the dynamical behavior of a speed-increase gearbox", *Journal of Materials Processing Technology*, vol. 150, pp. 170-174, 2004.
- [4] S.-W. Zhou, L.-X. Guo, S.-Q. Zhang, and C.-Y. Tang, "Study on cervical spine injuries in vehicle side impact", *The Open Mechanical Engineering Journal*, vol. 4, pp. 29-35, 2010.
- [5] N. Sigg, "Tooth profile modification of high speed duty gear", In: *Proceedings of International Conference on Gearing*, 1958, pp. 313-316.
- [6] B. W. Dudley, *Handbook of Practical Gear Design*. New York: McGraw-Hill Co, 1984.
- [7] Gear Guide Editorial, *Gear Guide*, Beijing: China Machine Press, 2000.
- [8] F. L. Litvin, *Gear Geometry and Applied Theory*, New Jersey: Prentice Hall PTR, 1994.
- [9] J. Wang, and I. Howard, "The torsional of involute spur gears", *Journal of Mechanical Engineering Science*, vol. 218, pp. 131-142, 2004.
- [10] J. Wang, and I. Howard, "FEA of high contact ratio spur gears in mesh", *Journal of Tribology*, vol. 127, pp. 469-483, 2005.
- [11] S. J. Nalluveetil, and G. Muthuveerappan, "Finite element modeling and analysis of a straight bevel gear", *Computers & Structures*, vol. 48, pp. 739-744, 1993.
- [12] J. D. Andrews, "A finite element analysis of bending stresses induced in external and internal involute spur gears", *Journal of Strain in Analysis*, vol. 26, pp. 153-163, 1991.
- [13] Livemore Software Technology Corporation, *LS-DYNA Keyword User's Manual*, 2001.
- [14] H. N. Ozguven, and D. R. Houser, "Mathematical models used in gear dynamics", *Journal of Sound and Vibration*, vol. 121, pp. 383-411, 1988.

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