Research on Grey System Model and Its Application on Displacement Prediction in Tunnel Surrounding Rock

Xiaobo Xiong*,1,2,3

[1]School of Civil and Architecture Engineering, Nantong University, Nantong, 226019, China
[2]State Key Laboratory of Geohazard Prev. and Geoen. Protection, Chengdu 610059, China
[3]Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China

Abstract: In the process of the highway tunnel construction, the stability and the reliability of tunnel rock is an important guarantee to ensure safety construction. Tunnel surrounding rock deformation monitoring is an important method for obtaining information on surrounding rock and controlling tunnel stability in the period of construction. Forecasting deformation of surrounding rock is the key to estimate shoring types, parameter and longtime stability after being commissioned for use. Considering the non-linear characteristic of deformation of the tunnel, the grey system prediction models were proposed. Based on the displacement of Guankouya rock tunnel, grey models of the GM(1, 1) and the Model DGM(2, 1) were established for the tunnel grey forecasting model of the rock tunnel displacement. The calculated results show that two models for tunnel displacement generally were predictable. GM(1, 1) and DGM(2, 1) models are similar to the tunnel displacement development model. Application examples demonstrate that it has extraordinary adaptability to the tunnel displacement forecast and all types of surrounding rock displacement can be predicted better by the grey model and the model has high simulation and prediction accuracy.

Keywords: Displacement prediction, GM(1, 1), DGM(2, 1), Grey system model, Tunnel.

1. INTRODUCTION

Tunnel deformation monitoring plays an important role in ensuring tunnel safety. Numerical modeling has been used widely in mining and construction industries in recent years. The most important issue in engineering projects designed with numerical modeling is the degree of accuracy in the modeling of rock mass behavior. If the rock mass behavior is modeled accurately, fewer problems will be faced during field application of projects. Selection of the true model is a very important issue in numerical modeling for the tunnel projects.

Zhang Jun-zhong, *et al*. through data processing, have demonstrated that the unequal interval of observation sequences was transferred to interval of observation sequences for deformation monitoring of a tunnel project in Guangzhou City [1]. GM (1, 1) model was used to analyze and forecast the deformation. BiWeiguo, *et al*. collected deformation data to analyze, and predictions were established using the grey theory GM model. It was found that grey theory has certain predictive effects [2]. R.D. Dwivedi, *et al*. developed dimensionally correct empirical correlations with correlation factor of 0.94 to predict tunnel deformation for squeezing grounds [3]. Data of 63 sections from various case histories of 14 tunnels were included in the study. C.O. Aksoy, *et al*. in their research on the excessive deformations in T-35 tunnel, which is one of the 33 tunnels, compared results of the *in situ* measurements and the numerical model [4]. It is determined that the results of the numerical modeling and the *in situ* measurements are very consistent with each other.

Deformation predication of tunnel is very important in the control of construction monitoring. Considering the non-linear character of deformation of the tunnel, the prediction model of grey system has been proposed. The equal-time-span GM(1, 1) model, the grey renewal GM(1, 1) model and the grey Verhulst model of the grey system theory are applied in the tunnel for the prediction of surrounding rocks displacement [6-8]. Grey GM(1, 1) model is extensively used in grey forecasting study. Equal interval conception was introduced when the GM(1, 1) model was set up. As a result, the premise for using GM(1, 1) model is that the modeling sequence must correspond with the equal interval demand. In geo-mechanics field, non-equidistant monitoring sequence depended on time often exists. The GM(1, 1) grey majorized model was based on DGM(2, 1) background value Z(1) (k+1). The DGM(2, 1) model is provided not only with the merits of concise formation and simple calculation but also with the wider application. It can be applied to the forecasting of equidistant, non-equidistant and high growth data sequence, which is more accurate in calculating and forecasting than the traditional GM(1, 1) model.

2. TWO TYPES OF GREY MODEL

2.1. GM(1,1) Model

Suppose primary accumulating generating (1-AGO) sequence [6-8].

*Address correspondence to this author at School of Civil and Architecture Engineering, Nantong University, Nantong, 226019, China;
Tel: 86-13328090772; E-mail: thongtao2006@163.com

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$X^{(1)}=(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)$

of source data sequence

$X^{(0)}=(x^{(0)}(1), x^{(0)}(2),\ldots, x^{(0)}(n)$, $(x^{(0)}(k)\geq 0$, k=1, 2, …, n),

$X^{(0)}$, $Z^{(1)}=(z^{(1)}(2), z^{(1)}(3),\ldots, z^{(1)}(n)$

are mean generation consecutive neighbors of $X^{(1)}$,

$Z^{(1)}(k+1)=0.5(x^{(1)}(k)+0.5\, x^{(1)}(k+1)$; k=1, 2,…, n-1.

Suppose $\hat{a}=(a,b)^T$ are parameter matrix, and

$$B=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix},\ Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{1}$$

Then GM(1,1) grey differential equation $x^{(0)}(k)+aZ^{(1)}(k)=b$ smallest second-multiplication estimated should be met with

$$\hat{a}=(B^TB)^{-1}B^TY \tag{2}$$

The whiten equation is

$$\frac{dx^{(1)}(t)}{dt}+ax^{(1)}(t)=b \tag{3}$$

The time-responding sequence GM(1,1) grey differential equation can be written as:

$$\hat{x}^{(1)}(k+1)=(x^{(0)}(1)-\frac{b}{a})e^{-ak}+\frac{b}{a}$$

$$k=1,2\ldots n \tag{4}$$

Then the simulation value of source sequence:

$$\begin{aligned}\hat{x}^{(0)}(k+1)&=x^{(1)}(k+1)-x^{(0)}(k)\\ &=(x^{(0)}(1)-b/a)(e^{-a}-1)e^{-a(k-1)}\end{aligned} \tag{5}$$

From the above modeling, we can see that accuracy of calculation and prediction is dependent on constants a and b, which are also concerned with the structural style of background value $Z^{(1)}(k+1)$.

2.2. DGM(2, 1) Model

Suppose primary data [6, 7]

$$X^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)) \tag{6}$$

whose accumulating generating (1-AGO) sequence list is as follows:

$$X^{(1)}=(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)) \tag{7}$$

whose subtracting generates(1-AGO) sequence list is as follows:

$$\begin{aligned}\alpha^{(1)}X^{(0)}=(&\alpha^{(1)}x^{(0)}(2),\alpha^{(1)}x^{(0)}(3),\\ &\cdots,\alpha^{(1)}x^{(0)}(n))\end{aligned} \tag{8}$$

where, $\alpha^{(1)}x^{(0)}(i)=x^{(0)}(i)-x^{(0)}(i-1), i=2\sim n$

Then the grey differential equation of DGM(2, 1)

$$\alpha^{(1)}X^{(0)}+aX^{(0)}=b \tag{9}$$

which, can be transferred to:

$$\frac{d^2x^{(1)}}{dt^2}+a\frac{dx^{(1)}}{dt}=b \tag{10}$$

Estimation of Least Squares method for the grey differential equation is:

$$[a,b]^T=(B^TB)^{-1}B^TY \tag{11}$$

where, B and Y are stated by matrix

$$B=\begin{bmatrix} -x^{(0)}(2) & 1 \\ -x^{(0)}(3) & 1 \\ \vdots & \vdots \\ -x^{(0)}(n) & 1 \end{bmatrix}\ Y=\begin{bmatrix} x^{(0)}(2)-x^{(0)}(1) \\ x^{(0)}(3)-x^{(0)}(2) \\ \cdots \\ x^{(0)}(n)-x^{(0)}(n-1) \end{bmatrix}$$

Then, to put out the prediction model as

$$\begin{aligned}\hat{x}^{(1)}(k+1)&=(\frac{b}{a^2}-\frac{x^{(0)}(1)}{a})e^{-ak}\\ &+\frac{b}{a}(k+1)+(x^{(0)}(1)-\frac{b}{a})\frac{1+a}{a}\end{aligned} \tag{12}$$

$$\begin{aligned}&\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k),\\ &k=1\sim n-1\end{aligned} \tag{13}$$

3. THE TYPE CURVES

Firstly, a typical rule of tunnel rock deformation curves was analyzed and the process of nonlinear regression function fitting is as follows:

Daily time series of observed horizontal convergence deformation data of a tunnel section: 1.89 mm, 3.42 mm, 4.55 mm, 5.30 mm, 5.82 mm, 6.12 mm, 6.30 mm, 6.42 mm, 6.51 mm, ...

According to the distribution of test data, the exponential relationship formula can be used to analyze.

The Exponential function is:

$u = a\times e^{-b/t}$

or $u = a\times(1 - e^{-bt})$ (14)

where, a, b are regression constants; t is the time after the first data, d; u is the displacement value, mm.

According to the convergence of the monitoring time series and the trends of deformation-time curve, $u=a\times e^{-b/t}$ can be chosen to fit the nonlinear regression function:

For $u=a\times e^{-b/t}$, by taking the natural logarithm of both sides, we get:

$$\ln u=\ln a+b(-t^{-1}); \tag{15}$$

To make, $u' = \ln u$, $t' = -t^{-1}$;

Therefore, $u' = \ln a + bt'$.

Then, the nonlinear relationship equation becomes linear equation. By applying the above average level displacement time series data of the tunnel section as a parameter of the fitted regression equation's raw data that is available, the following equation can be obtained:

$\ln a = 2.027,\ a = 7.6,\ b = 1.43.$ (16)

To obtain the regression equation of horizontal convergence observed data:

$u = 7.6 \times e^{-1.43/t}$ ($R = 0.99$). (17)

When, $t \to \infty$, to obtain the final value of convergence displacement, u is equal to 7.6 mm.

Then, we can find that the development trend of tunnel rock deformation has a negative exponential relationship, in line with the grey forecasting system theory. Therefore, it can be analyzed using grey theory.

Both the engineering practice and theory analysis indicate that the types of tunnel displacement-time curves can be classified as shown in Fig. (**1**).

3.1. Model I Displacement-Time Curve

Model I curve often reflects the displacement character of Tunnel. Its characteristic is: the curve is a protruding type (Fig. **1**, curve □). The speed curve is monotonically decreasing (Fig. **2**, curve □). The Model I displacement-time curve is the reflection that the tunnel is becoming stable gradually.

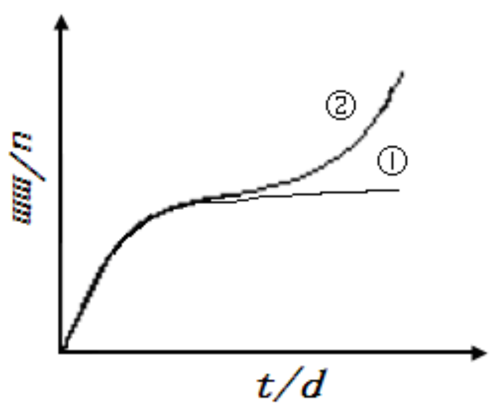

Fig. (1). Curve of *u~t*.

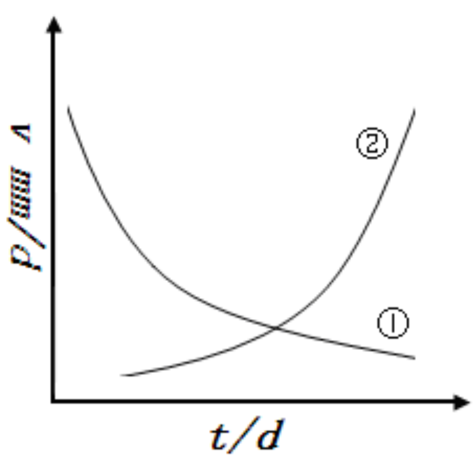

Fig. (2). Curve of *v~t*.

3.2. Model □ Displacement-Time Curve

Model □ curve takes place in the weak rock, soft rock or creep deformation aroused by high geo-stress. The curve changes from the protruding type into the concave type (Fig. **1**, curve □). The speed increases from the monotonically decreasing the speed curve that has the polar value (Fig. **2**, curve □). Model □ curve is the creep instability curve. It can be divided into three stages. The first stage is in increasing state, the second stage is in steady-state creep, and the third stage is in accelerating instable-state creep. Model □ curve is the reflection of the creep instability of tunnel.

Mostafa Sharifzadeh, *et al*. in their research on time-dependent behavior of Shibli twin tunnels, investigated using laboratory testing, monitoring data, and finite difference numerical simulation approaches [5]. Numerical modeling was verified by its comparison with tunnel displacement monitoring results. The creep behavior of the rock mass was predicted during tunnel service life based on back analysis results.

It also proves that the grey majorized model has more general suitability and very high simulation precision in predicting the deformation character of the rock.

4. CASE ANALYSIS OF THE GREY MODELS

In this paper, the restraint displacement of the ZK73+834 section in Guankouya tunnel is taken as the primary source data sequence. The duration is from the 1st day to 10th day, for establishing grey prediction and comparing the measured data series with the prediction series and analyzing the simulation and prediction accuracy [9-14].

According to the characteristics of the two grey system theory, the constraint displacement of each day of the construction model of GM(1, 1) with the primary time series data was adopted. But for DGM(2, 1) model, accumulating generating (1-AGO) sequence for the construction model with the primary time series data was adopted. At the same time, simulate and predict the following several days of displacement, and define relative error and synthetic error as follows:

$$E_r = \frac{\hat{u}(t) - u(t)}{u(t)} \times 100\% \tag{18}$$

$$E_g = \frac{\sqrt{\sum_{i=1}^{n} (\hat{u}(t) - u(t))^2}}{\sqrt{\sum_{i=1}^{n} u^2(t)}} \times 100\% \tag{19}$$

where, the prediction value is $\hat{u}(t)$, the measured value is $u(t)$.

The measured displacement and prediction value of tunnel surrounding rock is listed in Table **1** and Fig. (**3**), based on the source time series data from day 1 to the 10th day. The two grey system model were applied to simulate and predict the displacement of 11^{th} and 12^{th} day for the tunnel.

Based on the process of the above formulation, analysis of the long-term displacement prediction in tunnel's surrounding rock, and application of the models to predict the important section of Guankouya tunnel, the grey system prediction model is shown as follows:

Table 1. Comparison between the measured and prediction displacement in tunnel by GM(1, 1), DGM(2, 1).

Time Series	Measured Displacement	GM(1, 1)		DGM(2, 1)	
		Prediction	Error	Prediction	Error
d	mm	mm	%	mm	%
d	mm	mm	%	mm	%
1	2.67	2.67	0%	2.67	0%
2	3.93	6.94	76.6%	3.68	6.3%
3	5.42	7.73	42.6%	5.61	3.6%
4	7.89	8.61	9.1%	7.42	6%
5	10.17	9.59	5.7%	9.11	10.5%
6	12.03	10.67	11.2%	10.68	11.2%
7	13.52	11.89	12.1%	12.15	10.1%
8	14.38	13.24	7.9%	13.52	6%
9	15.84	14.74	6.9%	14.81	6.5%
10	16.58	16.42	1%	16.00	3.5%
11	17.36	18.28	2.3%	17.13	1.4%
12	18.19	20.36	11.9%	18.17	1.1%

The prediction model of GM(1, l) is the following equation:

$$y = 61.1313*\exp(0.107574*t) - 58.4613 \quad (20)$$

And the long-term displacement is 27.23mm.

The prediction model of DGM(2, l) is the following equation:

$$U(k+1) = -25.2367e^{-0.09376k} + 26.5963,$$

$$y = 32.9562451029*t + 443.382619881/\exp(0.0683072446796*t) - 440.712619881 \quad (21)$$

And the long-term displacement is 26.58mm.

From the data and graph of the following Table **1** and the following Fig. (**3**), respectively, the average prediction error of GM(1, 1) is δ_1=15.58%, and the average prediction error of DGM(2, 1) is δ_2=5.5%. The prediction accuracy of the models are suitable for practical underground engineering and the accuracy is better than the traditional congress analysis model. The GM(1, 1) is suitable for short-term prediction and the DGM(2, 1) model is suitable for long-term prediction.

CONCLUSION

(1) The DGM(2, 1) model is provided not only with the merits of concise formation and simple calculation but also with the wider application. It can be applied for the simulation and forecasting of equidistant, non-equidistant and high growth data sequence, which is more accurate in calculating and forecasting than traditional GM(1, 1) model. Data sequence characters of surrounding rock displacement can be simulated and predicted better by the DGM(2, 1) model. The model had higher simulation and prediction accuracy and there are important theoretical values and practical meanings in the deformation prediction of underground surrounding rock.

(2) The grey prediction model, the GM(1, 1), which is suitable only for increasing and non-subtracting

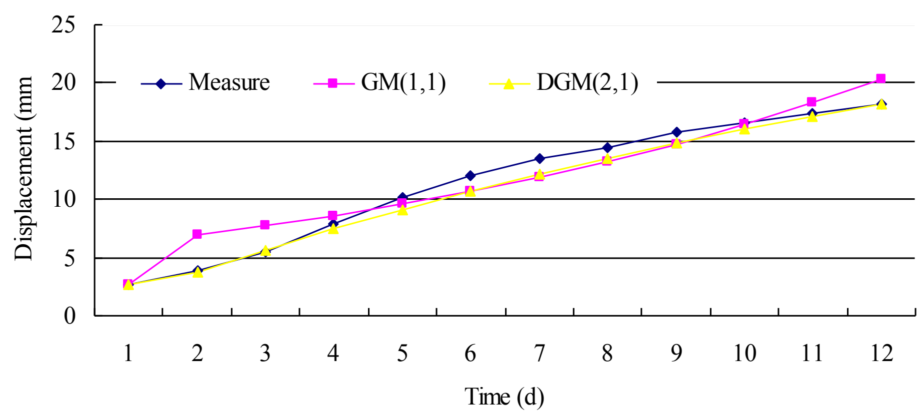

Fig. (3). Comparison of the results of simulation and prediction of the two grey models.

exponent time series, takes the average constraint velocity of the tunnel surrounding rock as the primary source data sequence. The DGM(2, 1) model is suitable for non-linear increasing time series. The Verhulst model is suitable for non-linear and non-increasing time series. So the DGM(2, 1) model and the Verhulst model both take the accumulating generating (1-AGO) sequence as the source data. The model construction process of DGM(2, 1) and Verhulst model are much simple than that of GM(1, 1) model.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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